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# *Game Theory-Based Modeling of Government–Enterprise Behavior in Exhaust Emission and Testing and Policy Optimization*

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***Abstract:*** Conflicting objectives exist between government and enterprises: the government aims to minimize detection failure rates, while enterprises seek to minimize compliance costs. Also, information asymmetry exists, where enterprises conceal actual emissions data, and regulatory agencies face resource constraints. To reveal the equilibrium state of government–enterprise behavior in emissions detection, policy effectiveness through mathematical modeling is validated. By adjusting payoff matrix parameters, transforms non-cooperative equilibria can be transformed into near-cooperative Pareto-improved states, enhancing overall social welfare. This demonstrates the optimization value of game theory for emissions detection under resource constraints. The constructed mixed-strategy payoff matrix for government-enterprise games displays that government monitoring can enhance penalty enforcement and cost-reduction efficiency improvement policies. Furthermore, incorporating long-term models is necessary to drive enterprises toward compliant emissions.

## 1. Introduction

The acceleration of industrialization and urbanization has intensified waste gas emission challenges. Enterprises, as the primary entities in economic production activities—particularly in industrialized regions—have made waste gas emissions a critical factor affecting ecological environments and public health, hindering the implementation of sustainable development principles. Despite stringent environmental regulations and emission monitoring mechanisms introduced by governments worldwide, enterprises still engage in practices such as illegal discharges, underreporting, and data falsification during actual operations, significantly undermining policy effectiveness. Traditional regulatory models primarily focus on technological improvements and legal sanctions, overlooking the complexity of strategic interactions between governments and enterprises, and thus failing to fundamentally resolve the “regulation-evasion” dilemma. On the other hand, game theory, as a mathematical tool for analyzing strategic interactions among rational decision-makers, offers a fresh perspective for deciphering the dynamic relationship between government regulation and corporate emissions behavior. Against this backdrop, the core challenge lies in revealing the underlying logic of government-enterprise behavior through game models and optimizing policy instruments to enhance emission reduction efficiency.

Theoretical foundations predominantly approach the subject through modeling, with literature concentrating on government policies and corporate behavior. Zhao et al. (2023) [1] argue that while policy convergence may cause greater profit losses in the power sector in the short term, it yields a lower profit loss rate per unit of emissions reduction compared to implementing a single policy when viewed through the lens of achieving the long-term goal of carbon neutrality. Carbon quotas, carbon pricing, penalty severity, and technology subsidies are key factors influencing the effectiveness of policy combinations. During policy optimization, implementing a benchmark carbon quota allocation mechanism, carbon price controls at 80 yuan/ton, mandatory green certificate trading, and clean technology subsidies can achieve the dual carbon goals with lower industry profit losses. Liu and Zhang (2023) [2] show that the National Key Monitoring Policy altered corporate production methods, significantly increasing clean energy inputs at the production end and markedly improving pollution control performance at the emissions end. However, the emission reduction effect of this policy primarily stems from end-of-pipe treatment mechanisms rather than clean production mechanisms. Zhao et al. (2025) [3] develop an electricity-carbon market equilibrium simulation model that accurately tracks dynamic changes in energy flows, carbon emissions, and economic flows within the market. Implementing a low-carbon optimization mechanism reduced carbon emissions by an additional 5.42% and channeled 47,900 yuan in economic flows from other sectors into the power industry, enhancing both economic and environmental benefits of the electricity market. Jiao et al. (2017) [4] analyze government and factories as primary actors, discovering that both environmental tax and environmental service fee compensation policies effectively controlled factory pollutant emissions, with environmental taxes proving more effective than service fees. Cooperative schemes among enterprises outperform scenarios where no reductions occur, offering an effective solution to the “prisoner’s dilemma.” On the other hand, a positive discount rate does not affect the Nash equilibrium, meaning the future’s importance does not exceed the present, and if all countries pursue only self-interest maximization, the ultimate outcome is that none will adopt aggregate emission reduction strategies; however, this does affect the Pareto improvement characteristics of the scheme (Liu et al., 2016) [5].

The effectiveness of policies varies under different external conditions. Peng (2013) [6] shows that pollution discharge fees can effectively address environmental pollution but hinder the development of the power industry, whereas government subsidies for pollution control promote the growth of the power sector. Sun et al. (2023) [7] conclude that both environmental protection tax policies and carbon trading policies can suppress industrial pollutant and carbon dioxide emissions to a certain extent, demonstrating the inherent synergistic effects of environmental policies in reducing pollution and carbon emissions. When stricter environmental protection tax policies are implemented, their carbon reduction effects correspondingly increase. Environmental protection tax policies and carbon trading policies exhibit synergistic leverage effects in reducing emissions from common pollutants (such as industrial solid waste and industrial exhaust gases), while stringent carbon trading policies exert a certain degree of crowding-out effect on industrial wastewater discharge reduction. The implementation of environmental protection tax and carbon trading policies can compel industrial enterprises to increase investments in environmental remediation and clean energy development. Enhanced technological investment not only improves the reduction of industrial pollutants and carbon dioxide emissions while strengthening synergistic effects but also mitigates the negative impacts of environmental regulations. Zhang et al. (2017) [8] find that reduced tax burdens led to increased industrial air pollutant emissions in eastern and western regions, significantly degrading environmental quality, while the opposite occurred nationally and in central regions. Local governments' lax environmental policies exacerbated industrial air pollution nationwide, in the eastern and central regions, creating a “race to the bottom.” Liu (2018) [9] verify that tax competition significantly curbs industrial wastewater discharge but promotes solid and air pollutant emissions, resulting in an overall trade-off. Tax competition in the economically developed eastern region more effectively suppresses wastewater discharge, while in the western region, it leads to a significant increase in air pollutant emissions. Chen et al. (2023) [10] emphasize that emission monitoring does not hinge on shifts in production levels. The true response lies in enhancing clean energy utilization efficiency, which inherently reduces reliance on fossil fuels and forms the foundation for mitigating air emissions. Simultaneously, when non-compliance costs are high and environmental compensation is low, local governments find it easier to exert influence on enterprises. Li and Shen (2023) [11] claim that when production is geographically constrained to a regulated jurisdiction, remanufacturing emerges as the dominant strategy for the producer. Once the option of relocating output is introduced, however, remanufacturing ceases to be unambiguously optimal: for emission-cap levels above the critical threshold, a partial-transfer regime yields higher pay-offs. Although remanufacturing delivers joint economic and environmental gains, partial transfer forfeits this advantage because it concentrates polluting activities inside the regulated region.

Moreover, Peng et al. (2013) [12] argue that environmental investments promote industrial emissions reductions; environmental regulations can effectively stimulate increased investment in industrial emissions control, thereby achieving emission reduction outcomes. Mohseni et al. (2025) [13] note that sustainable biofuel supply chain performance stems from both demand and supply, striving to reduce resource consumption while enhancing economic and environmental efficiency.

By examining the game-theoretic equilibrium of government-enterprise interactions in emissions monitoring, this study not only deepens the application of game theory in environmental regulation but also provides theoretical support and practical tools for addressing “regulatory failure.” By revealing the intrinsic logic of strategic interactions between government and enterprises, this research facilitates the transition of regulatory models from “passive response” to “proactive guidance.” It provides decision-making foundations for constructing an intelligent, precision-oriented modern environmental governance system, holding significant practical and policy value for achieving the “dual carbon” goals and fostering green, high-quality development.

## 2. Research Methods and Technical Approach

### 2.1 Game Analysis Framework Design

The fundamental assumptions of the model include: This scenario assumes only the government and enterprises exist, without further distinguishing the scope of government. All participants adhere to the rational economic agent assumption. The technology for emitting waste gases on the enterprise side is constant, and the net benefits of non-compliant emission reductions are consistent; differences between compliant emission reductions are not discussed. Government monitoring costs are relative values comparing strict versus lenient enforcement, with leniency defined as zero action costs. Strict regulation guarantees detection of illegal emissions (assuming 100% accurate monitoring technology).

Providing environmental protection—a public good with strong positive externalities—will be inefficient, leading to “market failure.” “Market failure” often justifies government intervention. However, once the government supervises corporate environmental protection activities, a game emerges between firms and the government involving investment protection and oversight management.

Thus, the mathematical essence of emissions monitoring is that both firms and the government minimize their costs: firms’ costs stem from emissions, while the government's costs arise from monitoring.

The strategic space for participants is defined as follows:

First, the regulatory body (government): whether inspections are conducted strictly according to regulations. Attention to detail reduces the likelihood of missed detections. Its strategy set comprises strict inspection and lenient inspection. Strict inspection aims to minimize missed detection rates and societal losses, while lenient inspection aims to minimize oversight costs.

Second, the emitter (enterprise) decides whether to comply with emission standards. If pursuing compliance, its objective is to minimize emission reduction costs; if engaging in illegal discharge or underreporting, its objective is to maximize production profits, though subject to the risk of fines.

## 2.2 Key Parameter Settings and Calculation Logic Basis

The following variables are all positive in value.

For enterprises, the compliance emission reduction cost $C_C$ for testing compliance can be calculated using the average value of listed companies, plus the operational and maintenance costs of purification equipment. The net benefit from illegal emissions $R_e = B_N - C$ (which actually saves $C_C$), where $B_N$ represents the production increase benefit. This arises from simplified waste disposal, where the opportunity cost of additional production gains is converted into compliance emission reduction measures. The difference between this and the basic treatment cost $C$ constitutes the net benefit. Penalty for detection $M$, with higher probability of subsequent spot checks and rectification. Moreover, $M = n \times R_e$, where $n$ denotes a multiplier based on maximum environmental penalties, far exceeding compliance costs.

For governments, single inspection cost $C_g$ (government procurement price) includes equipment depreciation and labor hours; single missed testing social loss $L_g$ comprises government-assessed health damages, ecological restoration costs, and liability losses; Effective regulatory performance gain $R_g$ represents the quantified increase in public satisfaction, achievable regardless of inspection rigor provided enterprises fully implement compliant emissions reductions. In other words, lenient inspections here constitute implicit gains. An additional constraint limits the regulatory budget $B$ and maximum inspection frequency $k_{\max} = B / C_g$. For example, with a budget of 2 million yuan and a single inspection cost of 200,000 yuan, 10 inspections can be conducted.

The government's payoff function can be expressed as the difference between performance gains and regulatory costs: $R_g - C_g$. The enterprise's payment function can be expressed as the difference between the benefits of illegal emissions and the risk of fines. This is because enterprises face uncertainty regarding whether the government will conduct inspections. Combined with the aforementioned multiple spot checks, the probability of encountering illegal emissions and concealment increases, necessitating the introduction of probability.

The construction of a government–enterprise game model may involve a coordination-type game where one Pareto optimum supersedes another. Therefore, achieving a Pareto-efficient outcome necessitates some form of coordination mechanism. However, the primary cause of persistent exhaust emissions despite repeated bans lies in the collusive behavior between local promotion-seeking officials and enterprises pursuing output maximization under promotion pressure (Yuan & Li, 2015) [14]. Empirical results by Li and Tan (2024) [15] indicate that when the externalities of incineration are strongly internalised by the regulator, both cost-sharing contracts and financial penalties motivate operators to intensify abatement effort. When the same externalities are instead internalised by the plants, cost-sharing dominates penalties in reducing emissions. Moreover, governmental welfare is consistently higher under cost-sharing, and the advantage widens as the municipal solid-waste stream grows. Conversely, plant-level profits are also greater when the cost-sharing regime is adopted rather than a penalty-based regime.

## 3. Model Solution and Equilibrium Analysis

### 3.1 Design of the Full Matrix (Two-Variable Decision Table)

Table 1 Static Game Payoff Matrix

| Government Enterprise | Strict Inspection ($p$) | Lenient Inspection ($1-p$) |
|---|---|---|
| Compliant Emission Reduction ($q$) | $(-C_C, R_g-C_g)$ | $(-C_C, R_g)$ |
| Illegal Discharge & Concealment ($1-q$) | $(-M, R_g-C_g)$ | $(R_e, -L_g)$ |

This example (Table 1) involves near-simultaneous decision-making, where the enterprise does not know the government's actions beforehand. The key to solving the game lies in identifying a strategy combination where neither party is willing or able to unilaterally alter their strategy. If such a combination exists and is unique, the game has a deterministic solution. Such a strategy combination where no player wishes to change their strategy alone is known as a Nash equilibrium.

This scenario lacks a deterministic Nash equilibrium solution. Introducing uncertainty conditions, $p$ and $q$ represent the probabilities of strict government inspection and corporate compliance-based emission reduction, respectively. The 2 × 2 game matrix readily yields the probabilities of lenient government inspection and corporate illegal discharge and underreporting strategies.

### 3.2 Results Presentation: Solving for Mixed Strategy Nash Equilibrium

It is observed that the expected payoff for firms engaging in illegal emissions is $-pM+(1-p)R_e$, while the payoff for compliant emissions remains constant at $-C_C$. The indifference condition is satisfied by $-pM+(1-p)R_e=-C_C$. Solving this yields the critical probability for strict inspection: $p^*=\frac{C_C+R_e}{M+R_e}$. Below this threshold, illegal emissions and concealment constitute the optimal strategy.

Similarly, the optimal reaction function for the government as the regulator yields an expected benefit of for strict monitoring $R_g-C_g$ and for lenient monitoring $qR_g-(1-q)L_g$. The indifference condition is satisfied when $R_g-C_g=qR_g-(1-q)L_g$. Solving this yields the critical probability for compliant emissions reduction: $q^*=\frac{R_g-C_g+1}{R_g+L_g}$. Above this threshold, lenient monitoring becomes the dominant strategy.

The economic implication of combined $(p^*, q^*)$ is that the government must implement strict monitoring with probability $p^*=\frac{C_C+R_e}{M+R_e}$, while enterprises must implement compliant emissions reduction with probability $q^*=\frac{R_g-C_g+1}{R_g+L_g}$.

### 3.3 Evaluation of Social Welfare at Equilibrium

Calculations of social welfare involve both parties. Among participants, the government's expected benefit is $p^*(R_g-C_g)+(1-p^*)q^*R_g-(1-p^*)(1-q^*)L_g$, while the firm's expected benefit is $-q^*C_C-(1-q^*)p^*M+(1-q^*)(1-p^*)R_e$. Total social welfare is the sum of these two expected benefits. The contradiction arises when the government's benefit is negative, indicating that incentive compatibility is not achieved under current parameters.

### 3.4 Policy Optimization Path: Dynamic Penalties and Resource Redistribution

We consider the impact of single-policy improvements on firms' responses to government actions. One approach is to increase fines *M* for improper waste disposal, i.e., raising the multiplier *n*, which lowers $p^*$ but serves as a deterrent. Another is to reduce regulatory inspection costs $C_g$, thereby increasing $q^*$. Under fixed budgets, this relative increase in inspection frequency incentivizes firms to transition to compliant emissions to avoid repeated inspections. Third, subsidizing compliant enterprises—specifically, including emissions reduction subsidies in cost calculations. This additional expenditure increases $C_g$, lowering $q^*$ and thereby weakening incentives for compliance. Enterprises, seeking to maximize their own profits, can use these subsidies to offset losses incurred from non-compliant emissions and the associated detection risk. Environmental protection taxes or pollution fees are ex ante measures and are not discussed here.

Based on the above, it is advisable to implement the first two policy measures. The dynamic penalty function adjusted according to the number of inspections can be derived as follows:

$$M_n = M \cdot e^{0.1\sum_{i=1}^{n}(E_i - D_i)}.$$

The base of the natural logarithm is set as the weighting factor. As the number of inspections increases, additional fees are incurred whenever emissions exceed the compliance threshold. The elasticity coefficient is artificially set at 0.1 to reflect the adaptive penalty model. When $p^*$ decreases sufficiently, the firm's expected profits increase relatively more; simultaneously, when $q^*$ rises sufficiently, the government's expected revenues increase relatively more, thereby promoting overall social welfare. This dynamic penalty mechanism helps address the diminishing incentive effect inherent in traditional fixed-penalty systems.

## 4. Conclusions and Outlook

### 4.1 Key Findings

As the regulatory authority, the government constrains corporate behavior by establishing emission standards and implementing monitoring and penalty mechanisms. Meanwhile, enterprises, acting as rational economic agents, often weigh the costs of emission reduction against the benefits of non-compliance, potentially leading to a "regulation-evasion" game cycle. The essence of this game lies in the existence of a mixed-strategy Nash equilibrium between government and enterprises, yet initial parameters result in social welfare losses. Game matrix analysis reveals that enhancing government monitoring, increasing penalties, and improving cost efficiency are effective approaches. Long-term models must be considered to compel enterprises to comply with emissions regulations, thereby expanding overall social welfare. Theoretically, this study integrates game theory with environmental economics, expanding the boundaries of multi-agent behavioral modeling and providing a new paradigm for analyzing complex policy interventions. Practically, the findings provide evidence for governments to design more precise emission reduction policies, such as dynamically adjusting subsidy and penalty intensities to shift enterprises from "passive compliance" to "proactive compliance." Additionally, it offers reference value for achieving coordinated government-enterprise governance under the "dual carbon" goals, facilitating the integrated development of economic and ecological benefits.

Short-term policy implementation: Dynamic penalties (progressive fines) and targeted inspections of key enterprises (top 20% enterprises accounting for 80% of emissions). Long-term implications: The government is refining blockchain-based emission monitoring in development zones (with real-time, tamper-proof data) to reduce inspection costs.

## 4.2 Research Limitations and Future Prospects

In emissions detection scenarios, governments and enterprises constitute typical parties in incomplete information games. Governments must design detection mechanisms to maximize environmental benefits within limited resources, while enterprises dynamically adjust emission strategies by balancing compliance costs and violation risks. While existing research attempts to apply game models to environmental regulation, it predominantly focuses on static games or single-stage interactions. It lacks systematic modeling of how key policy variables, such as detection probability, penalty severity, and information transparency, influence the long-term strategic evolution of governments and enterprises. Furthermore, it insufficiently explores the optimal combination of policy tools in heterogeneous enterprise scenarios.

Existing studies predominantly focus on policy effectiveness evaluations or single-agent decision analysis, lacking in-depth characterization of the dynamic interaction mechanism between government and enterprises. They particularly overlook the strategic evolution process under information asymmetry. Future research aims to construct a dynamic game framework depicting the strategic interaction between government and heterogeneous enterprises in exhaust emission detection, revealing the impact mechanisms of detection frequency, penalty intensity, subsidy mechanisms, and information disclosure policies on the behavioral evolution of both parties. By incorporating evolutionary game theory and multi-stage reputation models, this research simulates firms' compliance responses and strategy adjustments under varying regulatory environments, thereby deducing optimal regulatory combinations for governments. Key objectives include: (1) Establishing a dynamic game model for government-firm emissions inspection interactions; (2) Identifying critical policy variables influencing firm compliance behavior and their threshold effects; (3) To propose actionable policy optimization recommendations based on simulation and empirical data, achieving a balance between environmental benefits and regulatory costs.

Furthermore, a tripartite evolutionary game framework involving the central government, local governments, and enterprises is established. The central government can subsidize or penalize local governments through policy incentives, while local governments directly enforce regulatory intensity on monitored enterprises. Enterprise emission data is centrally recorded and monitored by the central government. From an evolutionary game perspective, both water pollution and air pollutant emission scale issues are addressed (Chen et al., 2023). Key steps include quantifying the impact of policy instruments (e.g., subsidies, penalties) on behavioral strategies; simulating stable states of government-enterprise behavior under different parameter scenarios to identify critical drivers; and proposing policy optimization schemes aimed at reducing emission reduction costs and enhancing governance efficiency through synergistic incentives and regulation.

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