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Research on Islamic Traditional Artistic Patterns Based on Singular Value Decomposition

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Abstract: This study focuses on geometric patterns in Islamic art, employing mathematical methods such as Singular Value Decomposition (SVD) and Principal Component Analysis (PCA) to explore the quantitative rules in terms of structural features and style classification. Initially, by preprocessing the images and representing them in a matrix format, SVD is used to extract major geometric characteristics, including energy distribution, pattern repetitiveness, and visual complexity. Subsequently, PCA and clustering analysis reveal the similarities and differences among various patterns. The experimental results indicate that the first few principal components effectively encapsulate the overall structure and local features of the patterns, while the clustering results identify three main types of decorative styles: structure-concentrated, balanced-transition, and complex-detailed.

1. Introduction

Islamic art holds a significant place in the history of world art, distinguished by its unique and intricate geometric patterns[1]. These patterns are not merely decorative elements; they embody a rigorous geometric construction and a profound symmetry that illustrate the intimate interplay between art and mathematics. From architectural niches to the adornments of handwritten Qur'ans, and from ceramic designs to carpet patterns, geometric motifs form the core visual language of Islamic art. This visual tradition transcends mere aesthetic pursuit; it profoundly reflects the concepts of unity, order, and infinity found in Islamic philosophy. Therefore, studying the structure of Islamic geometric patterns is not only a question of art history but also an important intersection of mathematics and cultural studies[2].

From a geometric perspective, Islamic art patterns frequently exhibit complex relationships of symmetry, repetitive units, and multi-layered nested structures. Star patterns, regular polygon grids, and radial constructions based on rotational symmetry are some of the most typical forms. These patterns generate nearly infinite visual effects through the combination of finite elements, reflecting an artistic expression of the philosophical concept of "infinity." Correspondingly, the pursuit of "order" and "harmony" in these designs renders these geometric structures as artistic symbols that transcend time and space. For this reason, Islamic geometric patterns are not just aesthetic objects; they serve as cultural texts infused with mathematical principles and religious symbols.

In recent years, with the development of digital image processing and computational mathematical methods, researchers have begun to attempt a quantitative analysis of the geometric features of art patterns. Compared to traditional studies that rely on manual measurements and visual observations, modern mathematical tools provide more objective and systematic analytical approaches. Among these, Singular Value Decomposition (SVD)[3], a commonly used matrix decomposition method, effectively extracts the principal structural characteristics from images, transforming complex visual information into quantifiable mathematical metrics. The introduction of clustering analysis further clarifies the similarities[4] and differences among various patterns within the structural feature space[5]. This methodological shift elevates the study of Islamic geometric patterns beyond qualitative discussions in art history and cultural studies, expanding it into the realm of computational and quantitative empirical analysis[6].Additionally, Principal Component Analysis (PCA)[7] is another commonly used statistical dimension reduction method, suitable for extracting the most representative variation directions from high-dimensional images or signals. PCA constructs a covariance matrix from centered data and performs eigenvalue decomposition[8], yielding orthogonal eigenvectors (principal components) ranked by the magnitude of the variance they explain, along with their corresponding eigenvalues. Projecting the original samples onto the first few principal components significantly reduces dimensionality while retaining most information and suppressing noise. For quantitative research on Islamic geometric patterns, PCA not only characterizes the primary structural modes and variability among patterns but also provides a robust and interpretable low-dimensional feature representation for subsequent clustering, similarity measurement, and visualization[9].

The goal of this study is to systematically analyze the mathematical structure of geometric patterns in Islamic art by integrating Singular Value Decomposition (SVD) and clustering analysis methods. By extracting key feature metrics such as structural energy distribution, pattern repetitiveness, and visual complexity, this research seeks to reveal the intrinsic law in the geometric characteristics of different types of patterns and explore their potential associations with decorative styles. This not only aids in deepening the understanding of the generative logic behind Islamic geometric patterns but also provides new data support for cross-cultural art comparisons and digital humanities research. More importantly, this study aims to underscore the unique value of Islamic art at the intersection of mathematics and aesthetics through rigorous quantitative analysis.

2. Preliminaries

In image feature analysis and pattern recognition, extracting stable and representative structural information from high-dimensional data remains a core challenge. Islamic geometric patterns, characterized by their high symmetry and intricate repetitive structures, particularly demand a mathematical tool capable of capturing both global and local features. As a fundamental method in matrix analysis, singular value decomposition (SVD)[10] offers excellent numerical stability and clear geometric interpretation. It enables dimensionality reduction and data compression while preserving key features. Consequently, SVD is selected as the primary method in this study for extracting structural energy distribution and complexity features from patterns[11].

After grayscale conversion and normalization, the pattern is represented as matrix $A \in R^{m\times n}$, whose singular value decomposition is expressed as:

$$A = U\Sigma V^T \tag{1}$$

Among these, $U \in R^{m\times m}$ and $V \in R^{n\times n}$ are orthogonal matrices, with their column vectors referred to as left singular vectors and right singular vectors, respectively. $\Sigma \in R^{m\times n}$ is a diagonal matrix whose non-negative diagonal elements $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r \geq 0$ are termed singular values, characterizing the magnitude of energy across different directions of the matrix.

In pattern analysis, to reduce redundant information, low-rank approximation is often employed to retain the top k largest singular values, yielding the following approximate matrix:

$$A_k = \sum_{i=1}^{k} \sigma_i u_i v_i^T \tag{2}$$

This representation not only significantly reduces data dimensionality but also preserves the fundamental structural features of the pattern, thereby achieving a balance between compression and reconstruction. Specifically, larger singular values correspond to overall symmetrical frameworks and dominant geometric forms, while smaller singular values primarily reflect detailed embellishments or noise.

Furthermore, the singular value distribution can be used to measure the concentration of energy within a pattern. If the total energy of the matrix is represented by the sum of the squares of all singular values:

$$E = \sum_{i=1}^{r} \sigma_i^2 \mid \tag{3}$$

The proportion of energy explained by the first k singular values is

$$\eta_k = \frac{\sum_{i=1}^{k} \sigma_i^2}{\sum_{i=1}^{r} \sigma_i^2}, \tag{4}$$

This metric provides an intuitive reflection of a pattern's complexity: if energy is concentrated in a few singular values, it indicates a highly regular and repetitive structure; conversely, a dispersed energy distribution suggests the presence of more hierarchical details and complex internal structures.

In summary, SVD not only offers a mathematical representation of energy and structure for geometric patterns but also establishes a foundation for feature extraction in pattern classification and style differentiation.

3. Our method

3.1 Data collection and sample selection

This study selected a total of 43 images of Islamic geometric patterns as analysis samples. The samples were primarily sourced from publicly available databases and relevant research materials, focusing on the presentation of typical geometric structural features rather than achieving comprehensive coverage across different regions or historical periods. The selection criteria included: (1) clear geometric construction of the patterns, allowing for the identification of basic unit forms; (2) inclusion of common repetitive and symmetrical elements found in Islamic decorative art; and (3) high image quality, avoiding obvious damage or blurring distortion. These criteria ensure that even with a limited number of samples, the selected images adequately reflect the main visual characteristics of Islamic geometric patterns, thereby providing a solid foundation for the subsequent quantitative analysis.

3.2 Image Preprocessing and Matrix Representation

To ensure the comparability and consistency of data analysis, all sample images underwent a standardized preprocessing procedure prior to analysis. First, the original images were converted to a uniform format and adjusted to a fixed size of 256×256 pixels through cropping and scaling. Next, color images were transformed into grayscale images to emphasize the geometric structural features and minimize interference caused by color differences. Subsequently, a median filtering method was applied to suppress noise, enhancing the clarity of the pattern edges and structural lines. Upon completing these steps, each image could be represented as a 256×256 grayscale matrix. The entire set of samples thus formed a collection of image matrices, which served as the input data for the subsequent Singular Value Decomposition (SVD) and feature extraction processes.

3.3 SVD Decomposition and Feature Extraction

After completing the image preprocessing, all sample images were uniformly converted into 256×256 grayscale matrices and further flattened into vectors of length 65,536. By sequentially combining the 43 images, a matrix $X \in R^{65536 \times 43}$ was constructed, where each column represents the pixel distribution of an individual image. This matrix served as the core input for the experimental analysis. Using Singular Value Decomposition (SVD), the matrix X was decomposed into three components: the left singular vector matrix, the diagonal matrix of singular values, and the right singular vector matrix. This decomposition allowed the complex image data to be transformed into several quantifiable feature components without losing the overall structural information. Through this approach, significant geometric characteristics of the Islamic patterns could be extracted, facilitating a deeper analysis of their structure and complexity.

In the experimental process, the matrix X was first subjected to Singular Value Decomposition (SVD), yielding a sequence of singular values. These singular values were arranged in descending order, with their decay rates reflecting the energy distribution between the primary structural features and detail components within the images. To ensure the stability of the computations, all singular values underwent normalization prior to extraction, allowing for comparisons of features across different images on the same scale. Subsequently, three core feature metrics were extracted based on the research objectives from the singular values and their corresponding singular vectors. The structural energy contribution was obtained by calculating the proportion of energy contained within the first few singular values, which provides an intuitive representation of the concentration level of the overall structure of the patterns and the significance of their main morphological features. The measure of pattern repetitiveness was derived from the correlation analysis of the right singular vectors, which correspond to the projections of the images in the feature space. The degree of correlation reflects the similarity and repeating patterns among different image units. Finally, the visual complexity was computed based on the spatial distribution characteristics of the images described by the left singular vectors, combined with the measure of information entropy. This metric reflects the richness of the patterns in terms of detailed expression and textural layers.

The entire feature extraction process centers around the gradual reduction of data dimensionality while capturing key geometric information. Through the concentrated distribution of singular values, it becomes evident that some patterns possess highly simplified geometric frameworks, while others exhibit complex and diverse local structures. The correlation of the right singular vectors further reveals potential connections among patterns that share similar structural logics. Additionally, the measurement of visual complexity allows for the differentiation of pattern categories that lean towards either simplicity or intricacy in their decorative styles. The extracted features are ultimately consolidated into a unified numerical matrix, with each image corresponding to a multidimensional feature vector. This matrix not only provides the experimental input for subsequent clustering analysis but also establishes a foundation for comparing the differences and relationships in geometric features among the various patterns. Through this structured approach, the study aims to quantitatively elucidate the characteristics and interconnections of Islamic geometric patterns, contributing to a deeper understanding of their artistic and mathematical values.

3.4 Clustering Analysis

Clustering Analysis After completing the Singular Value Decomposition (SVD) and extracting the feature vectors, the experiment progressed to the clustering analysis phase, aimed at exploring the potential grouping patterns of different geometric designs based on their structural characteristics. Given that each individual image corresponds to a multidimensional feature vector following feature extraction, the sample set was ultimately represented as a feature matrix, where each row represents the numerical description of an image. The goal of clustering is to categorize the patterns based on the similarity of these feature vectors, thereby revealing the intrinsic relationships among different decorative styles. By applying clustering algorithms, such as K-means or hierarchical clustering, the analysis seeks to identify distinct clusters that reflect common geometric and stylistic elements. This approach not only aids in visualizing the similarities and differences among Islamic geometric patterns but also enhances the understanding of how these elements interact within the broader context of Islamic art. The insights gained from this clustering analysis can contribute to a more nuanced interpretation of the aesthetic and mathematical principles that underlie Islamic geometric designs.

In the specific experimental process, the K-means method was first employed to cluster the extracted features. Prior to clustering, the feature matrix underwent standardization to ensure that different feature dimensions contributed to the calculations on the same scale, thereby avoiding the influence of disparities in numerical values on the clustering results. The number of clusters for the K-means algorithm was set to three categories, a choice informed by preliminary observations as well as considerations of several major style types commonly found in Islamic geometric decoration. Once the algorithm was executed, each image was allocated to its corresponding cluster, resulting in an initial classification of styles. This classification provided a framework for further analysis and interpretation of the patterns' geometric and aesthetic characteristics. The outcomes of the K-means clustering were then examined to gain insights into the relationships and distinctions among the various groups, laying the groundwork for a deeper understanding of the structural principles underlying Islamic geometric art.

To further validate the robustness of the clustering results, hierarchical clustering was introduced as a complementary method. By conducting Principal Component Analysis (PCA) on the feature matrix, the high-dimensional feature space was compressed into a two-dimensional or three-dimensional visualization space, allowing for an intuitive presentation of the relative distribution among the samples. Subsequently, Euclidean distance was used as the metric for hierarchical clustering on the reduced-dimensional feature set, generating a dendrogram that illustrates the aggregation relationships among different patterns. By comparing the results of K-means clustering with those from hierarchical clustering, consistency in sample groupings across different classification methods can be assessed, thereby enhancing the reliability of the experimental conclusions. This comprehensive approach not only strengthens the validation of the clustering outcomes but also provides deeper insights into the underlying structural and aesthetic affinities among the Islamic geometric patterns.

4. Analysis of numerical tests

After completing the Singular Value Decomposition (SVD) and feature extraction, this study conducted Principal Component Analysis (PCA) and clustering experiments on the sample patterns, aiming to reveal the intrinsic law of Islamic geometric patterns in terms of their structural characteristics. The experimental results are primarily divided into three parts: the distribution characteristics of the patterns in the principal component space, the clustering results based on the right singular vectors, and the decorative style classifications derived from hierarchical clustering. Each of these aspects will be analyzed in turn.

4.1 The principal component structure of Islamic art patterns

The first four principal components obtained from Singular Value Decomposition (SVD) effectively represent the main differences in geometric features within the sample set. The results indicate that the energy of the first principal component is 2.22×10^5, significantly higher than that of the other principal components, as illustrated in Figure 1. This suggests that most of the structural information of the patterns is concentrated at the global framework level. From the reconstructed images, it is evident that this principal component captures the overall geometric framework, characterized by a blurred yet stable symmetrical structure.

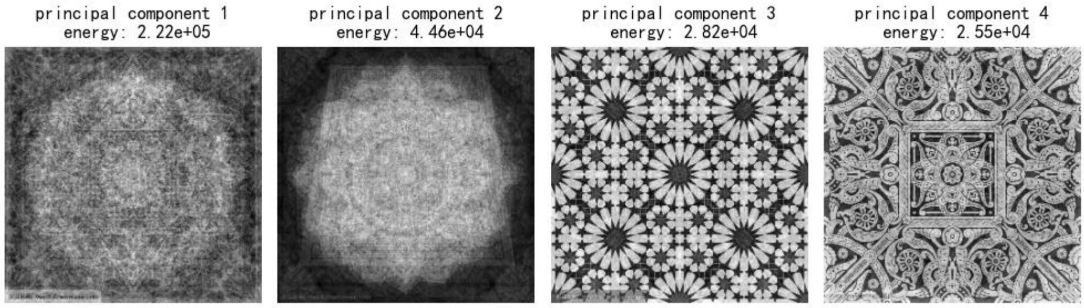

Figure 1 PCA

The energy of the second principal component shows a noticeable decrease, yet it still reveals circular and framework-like hierarchical variations, highlighting local differences built upon the global structure. In contrast, the third and fourth principal components emphasize more specific local elements: the third principal component reinforces the repetitiveness of star-shaped and radial units, while the fourth principal component showcases complex nested textures and variations in detail.

Overall, the Principal Component Analysis reveals the hierarchical structure of Islamic geometric patterns: the first principal component corresponds to global symmetry and order, the second principal component reflects local hierarchical differences, while the subsequent principal components emphasize details and complexity. This progressive feature decomposition, from the global to the local level, provides a solid data foundation for the subsequent clustering analysis.

4.2 Clustering Analysis of Right Singular Vectors

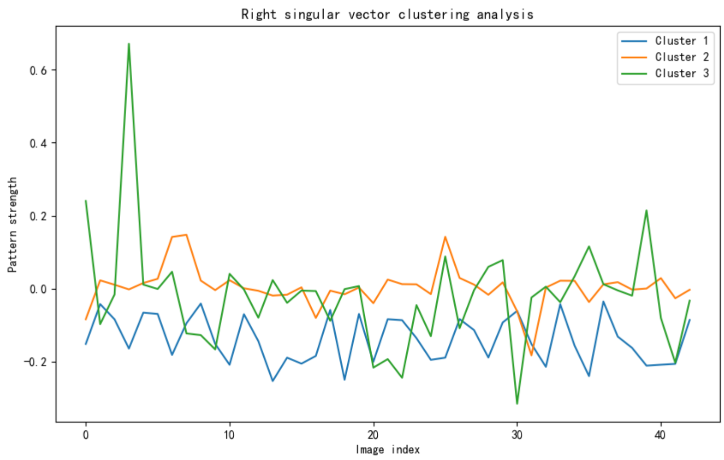

Figure 2 Right singular vector cluster analysis

Based on the results of the principal component analysis, this study further employed the right singular vectors for clustering analysis to reveal the similarities and differences among various patterns in the feature space. Using the K-means method, the samples were categorized into three classes, with the trends of their changes across feature dimensions presented in the form of line charts. The results indicate that the feature curves among different clustering categories exhibit significant differences, demonstrating that the right singular vectors effectively distinguish between different decorative styles, as illustrated in Figure 2.

From the overall trend, the first category of samples exhibits a high degree of fluctuation at several characteristic points, indicating that their geometric framework is prominent and structural energy is concentrated. Patterns in this category typically feature clear symmetrical frameworks and straightforward construction logic. In contrast, the second category of samples shows a relatively smooth distribution of curves, maintaining moderate levels across most dimensions. This reflects a more balanced pattern without significantly prominent local features. Patterns in this category resemble common decorative motifs, emphasizing overall harmony. The third category of samples, on the other hand, displays noticeable abrupt changes at specific characteristic points, with greater fluctuations in the curves. This indicates that their local patterns are complex with strong repetition, although the overall structure is relatively dispersed.

This analysis demonstrates that even with a limited number of samples, the clustering results based on the right singular vectors reveal the diversity of decorative logic within Islamic geometric patterns. The categories identified include a simplistic type that emphasizes overall order, a balanced transitional type, and a highly developed complex type that features significant local complexity. This classification approach provides a quantitative foundation for comparing different patterns and lays the experimental groundwork for subsequent hierarchical clustering analysis.

4.3 Hierarchical cluster analysis

To further validate the clustering results based on the right singular vectors, this study employed hierarchical clustering to categorize the samples and visualized the results by reducing the high-dimensional features to two dimensions using Principal Component Analysis (PCA). As shown in Figure 3, the sample points exhibit a distinct clustering distribution in the reduced feature space, allowing for an overall division into three categories.

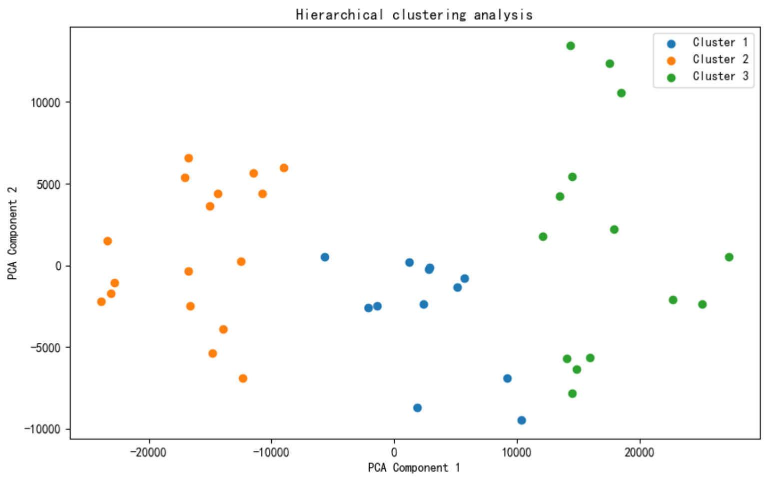

Figure 3 Hierarchical cluster analysis

From the scatter plot, it is evident that the three categories of samples exhibit strong separation along the two-dimensional axes. The first category (blue) is concentrated near the origin, indicating that its feature vectors are generally close to the average level; these patterns often reflect a more balanced geometric structure. The second category (orange) is predominantly distributed in the left region, showing a high degree of aggregation, which suggests that this category of patterns possesses consistency in their local features, potentially corresponding to more regular repetitive patterns. In contrast, the third category (green) is mainly distributed in the right region and shows a greater vertical span, indicating that this category of patterns has more significant variability in complexity and local details, making it the most diverse group in terms of decorative style.

Compared to the previous section's results, the hierarchical clustering maintains consistency with the clustering based on the right singular vectors in the overall classification framework, as both methods categorize the samples into three major classes. However, hierarchical clustering further reveals the relative positional relationships and internal differences between the categories. Notably, the distribution of the third category samples in the two-dimensional space is more dispersed, reflecting significant internal complexity in their decorative styles. This outcome indicates that Islamic geometric patterns not only exhibit unity in their macro structures but also demonstrate a high degree of diversification in local details. This diversification is a crucial source of their artistic expressiveness.

4.4 Comprehensive comparison of classification results

Through a comprehensive examination of the principal component structure of Islamic geometric patterns, clustering based on right singular vectors, and hierarchical clustering analysis, the hierarchies and diversities in geometric features among different patterns become clearly evident. RCA indicates that most of the structural information is concentrated in the first few principal components, particularly the first principal component, which represents the global symmetrical framework. This reflects the characteristic pursuit of order and unity in Islamic art patterns at the macro level. At the same time, the third and fourth principal components reveal a wealth of local patterns and detail complexities, providing important foundations for distinguishing between various decorative styles.

In the clustering analysis, the classification results based on the right singular vectors divided the samples into three categories, revealing three main decorative patterns through the differences in the feature curves. One category emphasizes the clarity of the geometric framework and the concentration of energy, exhibiting a simple and regular style. The second category maintains overall balance, highlighting continuity and a sense of harmony, reflecting common neutral decorative tendencies. Finally, the third category showcases strong local variations and higher complexity, underscoring the diversity and intricacy of decorative styles.

Hierarchical clustering further corroborated these classification results and visually demonstrated the spatial distribution relationships among the three sample categories in the reduced two-dimensional space. The three categories of patterns generally maintain good separation, particularly with a clear distinction between the first and third categories, while the second category exhibits transitional characteristics in certain dimensions. This phenomenon indicates that the decorative styles of Islamic geometric patterns not only have well-defined categories but also encompass interpenetration and transition between styles, reflecting the dynamic balance between unity and diversity in their artistic tradition.

In summary, the experimental results of this study not only validate the effectiveness of Singular Value Decomposition (SVD) and clustering analysis in the study of art patterns from a methodological standpoint, but also reveal the hierarchical structures and stylistic diversity of Islamic geometric patterns from an art historical perspective. This data-driven classification approach provides a new quantitative basis for traditional art research and lays the groundwork for further exploration of its cultural connotations and cross-cultural comparisons.

5. Results and Discussion

This study conducted a quantitative analysis of the structural features of Islamic geometric patterns through the methods of Singular Value Decomposition (SVD) and clustering analysis. The experimental results indicate that the first few principal components effectively encapsulate the main geometric characteristics of the patterns, with the first principal component capturing the overall symmetrical framework, while subsequent components reveal local repetitive units and decorative details. This hierarchical structure illustrates that Islamic art patterns maintain a dynamic balance between overall order and local complexity.

In the clustering experiments, both the K-means clustering based on right singular vectors and the hierarchical clustering based on dimensionality reduction visualizations consistently classified the samples into three categories. These three decorative patterns are represented as structure-concentrated, balanced-transition, and complex-detailed types. This classification not only reflects the aesthetic core of order and unity in Islamic art but also reveals its diversity in decorative logic. The consistency of results obtained from different methods indicates that the feature extraction framework proposed in this study has a high level of stability and interpretability.

Overall, this study showcases the potential of mathematical methods in the field of art research. Through quantitative metrics and clustering analysis, new perspectives can be offered for the classification of styles and cultural interpretations of Islamic art. However, due to limitations in sample size and data sources, the research does have certain shortcomings. Future work could expand the dataset and incorporate additional image processing techniques to achieve more comprehensive conclusions.

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