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A Study on Instructional Design for Geometry Reasoning Ability Based on Van Hiele Theory

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Abstract: Based on the Van Hiele levels of geometric thought theory and taking "Exploring the Conditions for Triangle Similarity" as an example, this article investigates the cultivation pathways for geometric reasoning abilities among junior high school students. The study integrates the core requirements for "geometric intuition" and "reasoning abilities" outlined in the Mathematics Curriculum Standards for Compulsory Education (2022 edition), it explores the connections between the Van Hiele theory and "similar triangles," and the researchers designs tiered instructional activities accordingly. Through a four-phase teaching process" intuitive-perception-exploratory conjecture-reasoning depth-extension enhancement"- teachers guide students to transition gradually from intuitive recognition to formal deduction, which achieving progressive advancement in geometric thinking. The article aims to overcome students' cognitive barrier of "disconnection between intuition and reasoning," it enhance their geometric reasoning abilities and systematic thinking levels, and provide an actionable paradigm reference for junior high school geometry instruction.

1. Introduction

The "Compulsory Education Mathematics Curriculum Standards (2022 Edition)" identifies "geometric intuition" and "reasoning ability" as crucial components within the connotation of core competencies [1]. However, common issues in junior high school geometry teaching include the "disconnection between intuitive perception and logical reasoning" and "students' weak proof abilities." Therefore, this article argues that cultivating students' geometric reasoning ability is essential. The Van Hiele theory is a relatively authoritative and highly representative theory in the study of students' geometric thinking [2].Meanwhile, similar triangles are not only an extension of congruent triangles but also serve as the foundation for learning concepts like the trigonometric ratios of acute angles and trigonometric functions of acute angles. They are also closely related to the proof of certain properties of circles [3]. Consequently, this article takes similar triangles as an example to explore how to enhance students' geometric reasoning ability based on the Van Hiele theory.

2. Teaching Design for "Exploring the Conditions for Triangle Similarity" Under Van Hiele Theory

2.1 Teaching Materials Analysis

The fourth unit "Similarity of Figures" in the BNU Press Edition Mathematics Grade 9 Volume 1 textbook is a key unit expanding junior high geometry from "congruence" to "similarity." As the fourth section of the unit, "Exploring Conditions for Triangle Similarity" both builds upon the previous three sections—deepening the application of the definition of similar triangles (equal corresponding angles and proportional corresponding sides)—and serves as a crucial foundation for subsequent learning topics such as "Application of Properties of Similar Triangles," "Trigonometric Functions of Acute Angles," and "Problems Combining Circles and Similarity." It plays a pivotal "bridging" role within the entire junior high geometry knowledge system [4].

The textbook employs an organizational logic of "perceptual intuition - experimental inquiry - deductive proof - application and extension," aligning with the Van Hiele theory's progression of thinking from the "visual level" to the "formal deduction level"[5]. It first establishes intuitive understanding through real-life examples of similar triangles (such as scaled triangle patterns), then explores the conditions for similarity through measurement experiments, and finally combines the "theorem of proportional segments divided by parallel lines" to complete the deductive proof of some similarity theorems, forming a complete learning chain of "perception - inquiry - proof - application".

2.2 Learning Analysis

Before learning "Exploring the conditions for similarity of triangles" in Grade 9, students have already acquired certain knowledge and ability foundations: in terms of knowledge, students have mastered the definitions of similar polygons and triangles, the theorem of congruent triangles, and the theorem of parallel lines dividing a line segment into proportional segments, and can explore by analogy with congruent determination and using parallel lines to transform into proportional relationships between sides; in terms of ability, after learning geometry in Grade 8, students have intuitive recognition of graphics, experimental operation (measurement, cutting and splicing), and preliminary reasoning and expression abilities, and are in the transition stage from the "analytical level" to the "informal deductive level" of Van Hiele theory. Students can discover patterns through experiments, but still need to strengthen their ability to rigorously reason and prove, and extract similar conditions from complex graphics.

Students also face many potential cognitive difficulties and obstacles: students are prone to confuse the conditions for determining similar and congruent triangles, such as mistakenly equating "two sides are proportional" with "two sides are equal", ignoring "the included angles are equal", or mistakenly taking "the three angles are equal" as an independent condition for determination; when proving the determination theorems of "two sides are proportional and the included angles are equal" and "three sides are proportional", students have difficulty coming up with the auxiliary line method of "constructing similarity or congruence by making parallel lines", and their reasoning steps are prone to skipping steps and lacking basis; when faced with complex graphics, they have difficulty quickly identifying the correspondence between similar triangles; there are significant differences in the thinking levels of students within the class, with a few students with weak foundations staying at the intuitive recognition level, most students at the analysis / informal deduction level, and a few students with surplus ability attempting the formal deduction level.

2.3 Analysis of teaching objectives

Scenario and Problem Objectives:

Teachers and students accurately identify similar triangles and pose the question "How to quickly determine similarity"; they explore inquiries related to "angular relationships," "combination of sides and angles," and "three-side proportions".

Knowledge and Skill Objectives:

Students describe the characteristics of similar triangles, they distinguish between similarity and congruence, and they memorize the definition of similarity (intuitive level); students through experimentation, they generalize and master three similarity criteria (analysis → informal deduction); they combine theorems to prove the criteria for "two angles equal" and "two sides proportional with included angle equal," and they standardize written proofs (informal → formal deduction).

Thinking and Expression Objectives:

Students undergo the "conjecture-verification-proof-summary" process to develop logical thinking; students experience analogical, transformative, and inductive thinking with practical examples; students clearly articulate experimental procedures, reasoning logic, and problem-solving steps in a structured and well-reasoned manner.

Communication and Reflection Objectives:

Students share doubts, students listen to perspectives, and students collaboratively correct errors in group work; students evaluate peers' outcomes, scrutinize the logic of criteria or auxiliary line constructions, and students deepen understanding.

2.4 Analysis of key and difficult points in teaching

Key points: The teaching content's three conditions for judging the similarity of triangles; the teaching focus on exploration and proof of "two triangles with equal angles are similar".

Difficulties: The teaching challenge lies in reasoning and proof of the theorem for determining similar triangles ("two sides are proportional and the included angle is equal" and "three sides are proportional"); students' difficulty in accurately identifying the correspondence between similar triangles in complex graphics, students' difficulty in and flexibly selecting the conditions for determination.

2.5 Teaching process

The teaching model of exploring the conditions for triangle similarity under Van Hiel's theory in the article is a dynamic and cyclic process: (preliminary step) conducting diagnostic assessment → (core cycle) designing hierarchical instructional design → (final step) carrying out formative evaluation and integration.

2.5.1 Pre-class Diagnosis: Establishing the Thinking Starting Point for All Participants

Visual Perception Assessment: Teachers present a mixed set of figures containing both similar and dissimilar triangles, asking students to circle "identical shapes" to evaluate the class's overall visual discrimination ability.

Attribute Perception Assessment: Teachers require students to complete "Similar triangles may relate to ______" (options: side lengths, angle measures, figure position) and teachers explain their reasoning, teachers probing their initial understanding of the essential characteristics of similarity.

Design Intent: The design diagnoses all students' baseline cognitive levels through fundamental tasks to the design informs subsequent differentiated instruction.

2.5.2 Gradient Instructional Activity Design: Cognitive Progression Path for All Students

(1) Intuitive Perception - From Visual Level to Analytical Level

Activity 1: "Graph Deformation Experiment"

Teachers utilize the Geometer's Sketchpad to dynamically demonstrate the process of scaling a triangle. Initially, teachers fix two angles of the triangle. After scaling, have students observe whether the size of the angles changes and how the lengths of the sides change. Teachers guide them to discover that "the angles remain unchanged, the sides change proportionally, and the shape remains unchanged"(as shown in Figure 1);

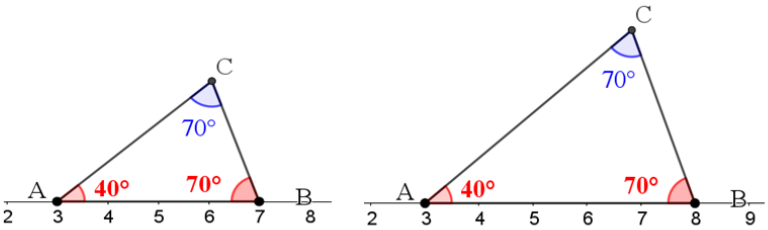

Figure 1: Triangle with two fixed angles

Students "fix another edge and change the size of the angle to show the process of shape change"(as shown in Figure 2), students reinforcing the understanding that the relationship between angles determines the shape.

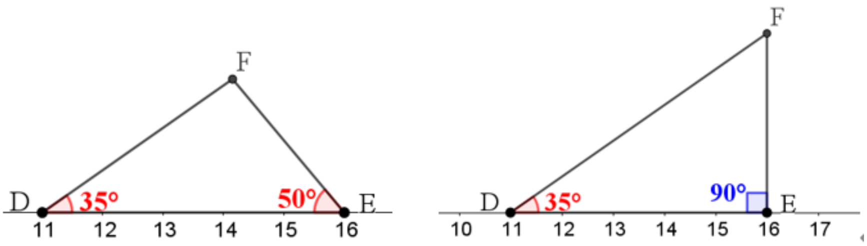

Figure 2: Triangular graph with one fixed edge

Activity 2: "Hands-on Measurement Activity"

Teachers distribute two pre-set similar triangle paper pieces (such as △ABC and △A'B'C', where ∠A=∠A'=50°, ∠B=∠B'=60°, as shown in Figure 3) to each student, teachers guide them to measure the degrees of the three angles and the lengths of the three sides, students record the data, and then teachers and students summarize it in the class.

Teachers fix one edge and change the size of the angle to show the process of shape change, students reinforcing the understanding that the relationship between angles determines the shape.

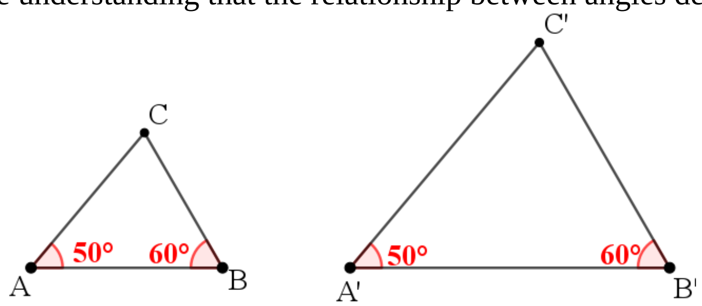

Figure 3: Similar Triangles Paper Cut-out Diagram

Design intention: Teachers help students transition from intuitive cognition of "looking at shapes" to analytical cognition of "analyzing features".

(2) Exploring Conjectures—Transitioning from Analytical Level to Informal Deductive Level

Activity 1: Teachers review the criteria for congruent triangles (SSS, SAS, ASA, etc.), and pose the question: "Congruence is a special case of similarity. Could we there be similarly straightforward conditions for determining triangle similarity without verifying all angles and sides individually".

Activity 2: "Group Exploration Activity"

Teachers divide students into small groups and distribute exploration worksheets centered on three core conjectures:

1) Are two triangles similar if two corresponding angles are equal?

2) Are two triangles similar if all three sides are proportional?

3) Are two triangles similar if two sides are proportional and their included angles are equal?

Teachers provide tools such as rulers, protractors, and compasses. Guide groups through a "draw-measure-compare-reason" process:

For conjecture 1): Students draw △ABC (∠A=60°, ∠B=70°), then draw △DEF (∠D=60°, ∠E=70°). Students measure the third angle and side lengths to verify angle equality and side proportionality.

For conjectures 2) and 3): Students verify through comparative drawings and proportional calculations.

Design Intent: This segment focuses on exploring similarity conditions, guiding students to formulate conjectures based on feature analysis and develop preliminary conclusions through independent verification.

(3) Deepening reasoning - consolidating informal deduction and infiltrating formal thinking

Activity 1: "Error Correction and Analysis"

Teachers presenting incorrect propositions such as "two triangles with equal corresponding angles are similar" and "two triangles with proportional corresponding sides and equal opposite angles are similar", and asking students to illustrate their reasons through drawing and examples, such as drawing an isosceles triangle with a vertex angle of 30° and an isosceles triangle with a base angle of 30°, and students proving their dissimilarity through measurement, this activity deepens students' understanding of "corresponding relationships" and "rigorous conditions".

Activity 2: "Simple Reasoning Application"

The norm proves that "triangles with equal angles are similar"

Given: In the triangles △ABC and △A'B'C', ∠A=∠A', ∠B=∠B,'(as shown in Figure 4)

The statement "△ABC∽△A'B'C'" is true

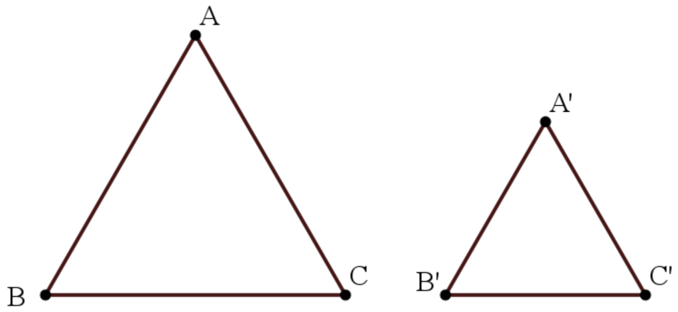

Figure 4: Activity 2 Similar Triangles Diagram

Proof: Cut AD=A'B' on AB, make DE parallel to BC and intersect AC at E, as shown in Figure 5 (auxiliary line and reasoning: demonstrate the method of making auxiliary lines). Lead the whole class to gradually derive:

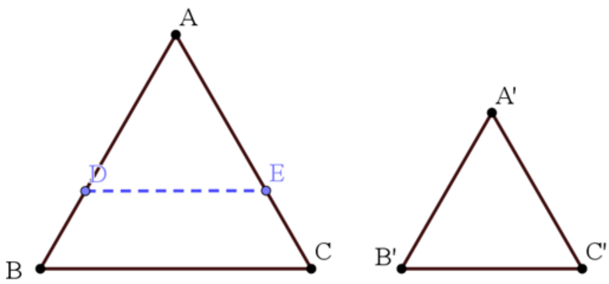

Figure 5: Construction Diagram of Auxiliary Lines for Similar Triangles in Activity 2

$\because DE // BC$

$\therefore \angle ADE = \angle B = \angle B', \angle AED = \angle C$(properties of parallel lines) ;

$\because \angle A = \angle A', AD = A'B'$

$\therefore \triangle ADE \cong \triangle A'B'C'(ASA)$;

$\because DE // BC$

$\therefore \dfrac{AD}{AB} = \dfrac{AE}{AC}$(dividing the segmentsinto proportional by parallel lines) ;

$\because \triangle ADE \cong \triangle A'B'C'$

$\therefore \dfrac{A'B'}{AB} = \dfrac{A'C'}{AC}, \angle C = \angle C'$

$\therefore \triangle ABC \sim \triangle A'B'C'$

Design Intent: This segment aims to strengthen the understanding of similar conditions, guiding students to progress from "experimental verification" to "logical reasoning".

Activity 3: Comprehensive Application of "Using Theorems to Solve Problems"

Given: As shown in Figure 6, in △ABC, DE∥BC, BE and CD intersect at point O.

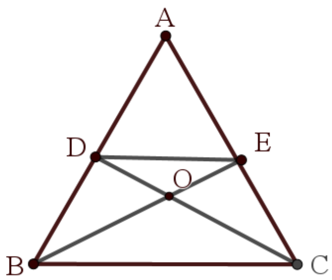

Figure 6: Similar Triangles in Activity 3

Prove that: $\triangle DOE \backsim \triangle COB$, and $\dfrac{DO}{CO} = \dfrac{EO}{BO}$

Analysis of thinking: Guide the whole class to find out the implicit conditions in the figure (DE//BC→∠ODE=∠OCB, ∠OED=∠OBC), and determine to use the judgment theorem of "two angles are equal respectively".

Independent solution: Students independently complete the proof and derivation of proportional segments. The teacher selects homework with different solutions (such as whether to add auxiliary lines, detailed or concise reasoning steps) for class presentation and evaluation, reinforcing the flexibility of theorem application and rigor of reasoning.

(4) Extension and Enhancement Stage (Rigorous Level): Cultivating Systematic Thinking (Elective)

Activity 1: Exploration of the Relationship Between Similarity and Congruence

By posing the question, "When the similarity ratio of similar triangles is 1, what is the relationship between their corresponding sides? What is the relationship between the two triangles at this point?", teachers guide students to conclude that "congruence is a special case of similarity (similarity ratio = 1)". Through comparing the criteria theorems for congruence and similarity, students perceive the progression of geometric concepts.

Activity 2: Geometric meaning of similarity transformation

Teachers utilize a geometric drawing tool to illustrate the process of obtaining a new triangle through scaling (similarity transformation) of an existing triangle. Teachers guide students to observe the characteristics of "unchanged angle sizes and unchanged ratio of sides before and after the transformation". Students initially perceive that similarity transformation is a type of "conformal transformation" in Euclidean geometry, thereby expanding their understanding of geometric transformations.

2.5.3 Evaluation feedback: advanced detection of full-staff thinking

(1) Process Evaluation: Focusing on the Dynamic Development of Thinking

Visual Layer: Teachers assess "accuracy in identifying similar triangles" through classroom observation, such as "whether students can correctly circle similar triangles from mixed shapes".

Analytical Layer: Teachers evaluate "standardization in property exploration" through measurement assignments, such as "whether corresponding angles and sides are accurately measured, and whether proportional patterns can be identified"; teachers assess "depth of property comprehension" through class participation, such as "whether core properties of similar triangles can be clearly articulated".

Deductive Layer: Teachers evaluate "rigor in proof processes" through reasoning assignments, such as "whether reasoning steps are complete and justifications are accurate"; teachers assess "theorem application ability" through problem-solving exercises, such as "whether students can extract judgment conditions from complex diagrams".

(2) Summative Evaluation: Tiered Assessment for All Students (Focusing on Progress Range)

Basic Questions (Visual + Analytical Layer):

1) As shown in Figure 7, students identify triangles of identical shape and state the reasons;

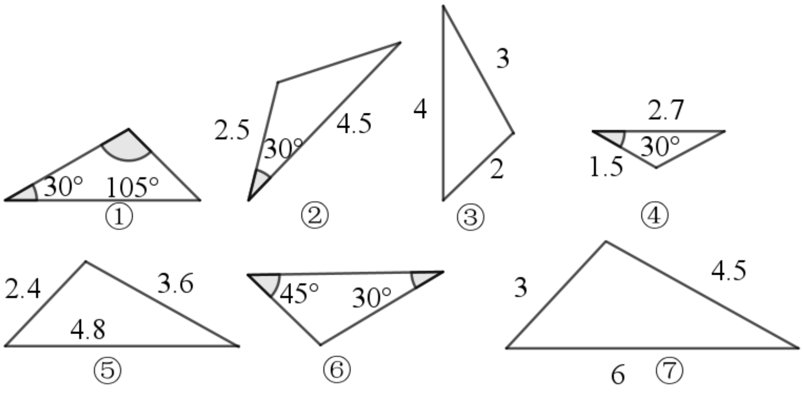

Figure 7: Basic Question Figure

2) Given $\triangle ABC \backsim \triangle A'B'C'$, $\angle A=50°$, $\angle B=70°$, find the measure of $\angle C'$. If AB=2 and A'B'=4, determine the similarity ratio.

Advanced problems (Informal reasoning level):

As shown in Figure 8 $\angle\alpha = \angle\beta = \angle\gamma$, students explain why $\triangle ABC \backsim \triangle ADE$ (no rigorous proof required, briefly state the reasoning).

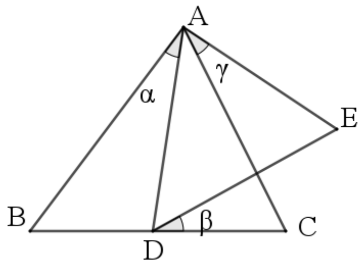

Figure 8: Enhanced Problem Illustration

Extended question (formal deduction layer):

1) Proof: Two triangles are similar if their sides are proportional and their angles are equal;

2) Use the above theorem to prove that if in △ABC, $\frac{AB}{AC}=\frac{AD}{AE}, \angle BAD=\angle CAE$, then $\triangle ABD \backsim \triangle ACE$.

When evaluating, not only should we focus on the accuracy of the answers, but also on the degree of students' attempts from "basic questions" to "extended questions". We should give recognition to students who have made progress and develop follow-up coaching plans for weak links.

Design Intent: Design test papers encompassing different cognitive levels to assess the progression outcomes of all participants, with a specific focus on the magnitude of each student's cognitive improvement.

3. Conclusion

Guided by the Van Hiele Theory, this research implemented systematic instructional design centered on "similar triangles," aiming to foster students' step-by-step development in geometric reasoning. Through layered and progressive activity design, combined with diagnostic assessment and formative evaluation, the instructional design effectively guided students at different cognitive levels to transition from intuitive cognition to formal reasoning. In future teaching, greater attention should be paid to individual student differences, with enhanced emphasis on expressing thought processes and reflective abilities during inquiry. Concurrently, teachers should organically connect learning about similar triangles to subsequent topics such as trigonometric functions and properties of circles, constructing a more comprehensive geometrical knowledge framework. The Van Hiele Theory provides a scientific framework for the development of geometric thinking in teaching, which proving its value for broader exploration and application across more extensive geometric themes.

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Teaching Cases of Secondary School Mathematics Curriculum and Textbook Research Based on the New Curriculum Reform (2025).

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