Curriculum and Teaching Methodology (2025)
Clausius Scientific Press, Canada
DOI: 10.23977/curtm.2025.080506
ISSN 2616-2261 Vol. 8 Num. 5

# *Curriculum and Teaching Methodology: A Study on the Teaching Design of the Fundamental Theorem of Plane Vectors Based on Solo Classification Theory*

**Kangjie Li[1,a,*], Zhongfeng Qu[1,b]**
*[1]School of Mathematical Sciences, University of Jinan, Jinan, China*
*[a]672931553@qq.com, [b]ss_quzf@ujn.edu.cn*
**Corresponding author*

***Keywords:*** Solo Classification Theory; Fundamental Theorem of Plane Vectors; Hierarchical Teaching

***Abstract:*** The Fundamental Theorem of Plane Vectors plays an important role in high school mathematics and serves as an important bridge for students to transition from geometric vector operations to algebraic operations. Based on solo classification theory, this paper classifies students' cognitive development into five levels: pre-structural, uni-structural, multi-structural, associative structural and extended abstract structural, and designs hierarchical teaching activities accordingly. It aims to optimise the quality of teaching and learning, promote students' deep learning, and lay a solid foundation for their mathematical learning and development through instructional design that conforms to the laws of students' cognitive development.

## 1. Introduction

Vectors occupy an important place in mathematics as both an algebraic and geometric object of study. Plane vectors serve as the basis for learning space vectors, and their Fundamental Theorem is even more crucial to understanding vector coordinate representations and implementing algebraic operations. However, students face many difficulties in learning the Fundamental Theorem of Plane Vectors, not only in the conversion of algebraic operations and geometric representations, but also in the fact that the traditional teaching focuses on the conceptual understanding and problem training, ignoring the individual cognitive differences of students, which urgently requires scientific theories to guide the design of teaching and learning [1].

Table 1: Five levels of solo classification theory

| cognitive level | Hierarchical features |
|---|---|
| Pre-structural | Learners who have not developed an effective understanding of the task or topic may not be able to find solutions or grasp the main points. |
| Uni-structural | The learner is able to focus on a relevant aspect of the task or problem, but is limited to a single point in isolation. |
| Multi-structural | Learners are able to focus on multiple aspects of the same topic, acquiring more fragmented points of information that lack connection to each other. |
| Associative structural | The learner takes multiple previously learned points and begins to make connections that integrate into an organic whole. |
| Extended abstract structural | Learners are not only able to integrate what they have learnt, but they are also able to further refine and abstract existing knowledge structures and transfer them to new areas or contexts for application. |

Solo Classification theory is a method of analysing and classifying student learning outcomes developed by John Biggs. The theory classifies learning outcomes into five levels: pre-structural, uni-structural, multi-structural, associative and extended abstract structures, based on students' level of cognitive development [2]. Each tier represents a different level of student understanding and mastery of the content. This hierarchical model of cognitive development helps teachers to better understand their students' learning process and thus design teaching activities that are more in line with students' cognitive development. Table 1 illustrates the five tiers of solo's classification theory and their characteristics, providing educators with a clear framework for more effective planning of instructional strategies and assessment of students' learning progress.

The hierarchical nature of solo classification theory provides a powerful framework for capturing students' cognitive development in planar vector learning and helps teachers to identify students' learning stages and needs. Existing studies have found that Chenhao Wang emphasises the important role of vector teaching in improving students' geometric computational skills and number and shape combination thinking [3]. Accordingly, SOLO classification theory can be used to stratify teaching according to students' level of understanding of vector concepts, operations and applications, thus making teaching more personalised and effective. Liu Na constructs a plane vector unit based on the theory of inverse instructional design, integrating number and geometry knowledge to help students construct a knowledge system [4]. Qin Haijiang et al. revealed the problems in students' planar vector learning through cognitive diagnostic techniques [5]. On this basis, SOLO classification theory can classify students' learning levels in a more detailed way, providing strong support for personalised teaching. Gong Liyuan discusses ways to enhance students' core literacy, and SOLO Classification Theory can help teachers to assess students' level of development in each core literacy indicator, so as to optimise the design of teaching and learning [6]. Although existing studies have provided multidimensional and useful references for the practice of teaching planar vectors from the perspectives of unit integration, contextual design, and technology integration, there is still a lack of systematic research on instructional design that is framed by a single theory. Therefore, this paper attempts to construct a framework for the instructional design of planar vectors supported by solo classification theory. Taking the basic theorem of plane vectors as an example, in the initial teaching stage, guide students to review the vector collinear theorem, so that students can naturally transit from the pre - structural level to the uni - structural level, achieving a smooth shift in thinking from one - dimensional to two - dimensional. With the help of designing a hierarchical and logical chain of questions, students are guided to start from special cases and gradually explore the process of generating theorems, so as to promote the development of students from the level of single structure to the level of multi-structure, and enable them to have a deep understanding of mathematical ideas in the process. In the process of teaching and learning advancement, through organising students to carry out independent investigation and cooperative learning, including hands-on activities and communication and demonstration, students are further promoted to move from the level of multiple structure to the level of correlation structure, and to deepen their understanding of the theorem. Eventually, through example training and extension, students are guided to use the theorems to solve practical problems, which prompts the students' thinking level to move towards the level of expanding the abstract structure, effectively improves the students' mathematical thinking ability and literacy, and provides new perspectives and methods for the practice of teaching mathematics in high school.

## 2. Teaching design of the fundamental theorem of plane vectors based on solo classification theory

### 2.1. Analysis of teaching materials

The Fundamental Theorem of Plane Vectors is the core content of the first lesson of Chapter 6, Section 3, Book 2 of the compulsory high school mathematics of the Humanistic A version, which has a key role in the knowledge system of plane vectors. The theorem is a natural extension of vector operations and their geometrical significance, and at the same time lays the foundation for the coordinate representation of plane vectors and their applications. Students need to understand the formation process of the theorem algebraically and grasp its essence geometrically in conjunction with the algorithm of vectors. In the teaching process, teachers should guide students to sort out the overall knowledge framework and clarify the intrinsic connection between various knowledge points. Prior to studying the theorem, students primarily explored vector problems from a geometric perspective, focusing on the magnitude and direction of vectors; And through the study of this theorem, students will be able to use coordinate representations to perform operations and investigate properties of vectors from an algebraic perspective. Therefore, in the teaching design, we should start from geometric intuition and gradually guide students to transition to algebraic representation, so as to fully understand the connotation of the Fundamental Theorem of Plane Vectors.

### 2.2. Analysis of Students' Conditions

In Mathematics students have mastered the concepts related to plane vectors and the linear operations of plane vectors and the plane vector covariance theorem. In Physics students have mastered the decomposition and synthesis of forces, which provides an intuitive physical background for them to understand the decomposition and synthesis of vectors from a geometric point of view. However, students may face difficulties when transitioning from geometric vector operations to algebraic operations. Therefore, in the teaching process, teachers need to design a hierarchical and logical chain of problems to guide students from the ‘one-dimensional’ situation to the ‘two-dimensional’ situation, gradually explore the fundamental theorem of plane vectors, help students build a complete knowledge system and improve their mathematical thinking skills.

### 2.3. Analysis of Students' Conditions

According to the stage characteristics of students' cognitive development, combined with the theory of solo classification, the teaching objectives correspond to the level of students' thinking development, aiming at gradually guiding students to make the transition from low-level thinking to high-level thinking and realising in-depth learning through hierarchical teaching activities. The specific pedagogical objectives are set out below:

1) Review the conditions for vector covariance to stimulate thinking about the ‘two-dimensional’ case and inspire students to make the transition from the pre-structural level to the single-structural level.

2) Explore the problem of representing arbitrary vectors in the plane, conjecture to be represented by two incoherent vectors, organise operations and demonstrations to promote the development of thinking to the level of multiple structures.

3) through the question, drawing exploration, classification and discussion of vector position, experience the process of proving the theorem, deepen the understanding of the theorem ‘arbitrariness’, consolidate the level of thinking of multi-structural.

4) Discuss special cases, deepen the knowledge of the scope of application of the theorem, construct a complete theorem system, improve the analytical ability, and promote the transition of thinking to the level of correlation structure.

5) Explore the uniqueness of linear representation of vectors, use theorems to solve geometric problems, synthesise methods, establish knowledge links and enhance logical reasoning and arithmetic skills.

## 2.4. Teaching Procedures

(1)Pre-structural level

Question 1: As shown in Fig.1, *u*,*v* are a set of co-linear vectors. Answer: What is the sufficient condition for two vectors to be co-linear?

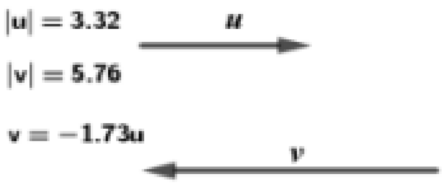


Figure 1: Schematic diagram of vector covariance

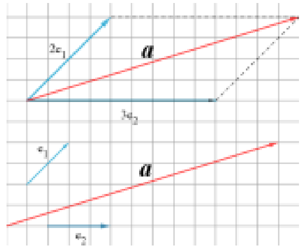


Figure 2: Schematic diagram of vector decomposition

[Intention Behind the Design]

Present covariant vectors visually with the help of GeoGebra software to help students recall the conditions under which vectors are covariant. This process uses students' prior knowledge of one-dimensional vectors to build a bridge to the two-dimensional case, guiding students from the pre-structural level to the single-structural level.

(2)Uni-structural level

Question 2:Suppose there is a non-zero vector *u* in the plane, can any vector in the plane be represented by *u*? If a vector *u* cannot be represented, how many more vectors are needed? Why?

[Intention Behind the Design]

This question is designed to guide students through the transition from one to two dimensions. Based on their observations and imagination, students can easily realise that it is difficult to represent any vector in the plane with only a single vector *u*, and then associate this with the need to increase the number of vectors. Most students made an initial guess that two vectors could represent all vectors in the plane, but had difficulty articulating why. At this point, the transition to the single structure level is aided by successive follow-up questions that lead students to the conjecture that two noncollinear vectors are needed.

Question 3:As shown in Figure 2, let $\vec{e_1}$ and $\vec{e_2}$ be two non-collinear vectors in the same plane. Let $\vec{a}$ be a vector in this plane that is not collinear with either $\vec{e_1}$ or $\vec{e_1}$ Choose any point $O$ in the plane and construct $\overrightarrow{OA}=\vec{e_1},\overrightarrow{OB}=\vec{e_2},\overrightarrow{OC}=\vec{a}$ . Decompose $\vec{a}$ in the direction of $\vec{e_1}$ and $\vec{e_2}$ , what do you discover?

Activity 1: Students work on their own, the teacher guides them and invites them to present their work. (Students complete the graph on the grid paper with three vectors drawn on it. The teacher projects the students' results onto the screen and then the students exchange and present them)

After students have completed the decomposition of this particular vector, ask them to think about whether the method used in this decomposition process applies only to this one vector. Could the same idea be used if we were to decompose other vectors in the plane that have different positions, magnitudes, and directions? The plane is divided into different regions by these two noncollinear vectors; would the decomposition change in any way for vectors in these different regions? Derive $\vec{a}=2\vec{e_1}+3\vec{e_2}$ extend to $\vec{a}=\lambda_1\vec{e_1}+\lambda_2\vec{e_2}$

[Intention Behind the Design]

In Activity 1, students delve into the relationships between plane vectors by thinking, manipulating,and communicating. With a series of well-designed, step-by-step investigative questions, students' thinking can be gradually expanded, so that the research problem is more and more clear, so that students experience the discovery of the basic theorem of plane vectors, making students from a single structural level to the level of multi-structural thinking direction.

(3)Multi-structural level

Activity 2: Keep the vectors $\overrightarrow{e_1},\overrightarrow{e_2}$ unchanged as shown in the diagram above. Students, please take turns posing questions to each other, then illustrate another non-zero vector $\vec{b}$ .Can it be expressed in the same way using $\overrightarrow{e_1},\overrightarrow{e_2}$ ?

To explore the arbitrariness of $\vec{b}$ , it can be classified according to the four regions created by dividing the plane with the lines containing the non-collinear vectors $\overrightarrow{e_1}$ and $\overrightarrow{e_2}$ (as shown in Figure 3). Discuss the position of vector $\vec{b}$ in these regions (as shown in Figure 4): when $\vec{b}$ is located in Region I, $\vec{b}$ can be directly represented using $\overrightarrow{e_1}$ and $\overrightarrow{e_2}$ based on the parallelogram rule; when $\vec{b}$ is in Region II, first find the opposite vector $-\overrightarrow{e_2}$ , and use $\overrightarrow{e_1}$ and $-\overrightarrow{e_2}$ to represent $\vec{b}$ ; similarly, when $\vec{b}$ is in Region III, use $-\overrightarrow{e_1}$ and $-\overrightarrow{e_2}$ to represent $\vec{b}$ ; when $\vec{b}$ is in Region IV, use $-\overrightarrow{e_2}$ and $\overrightarrow{e_2}$ to represent $\vec{b}$ . Additionally, special consideration must be given to the case where vector $\vec{b}$ lies on a line.

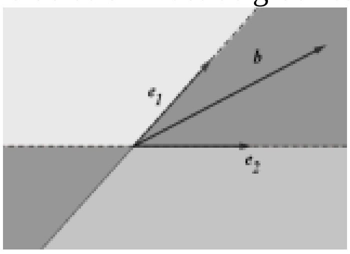


Figure 3: Division of the plane by vectors

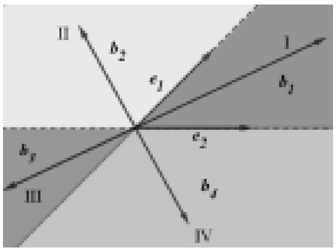


Figure 4: Decomposition of vectors in different quadrants

[Intention Behind the Design]

Let students experience the process of proving the theorem, so that students can understand the association, analogy, abstraction, generalisation and other important ways of mathematical learning, to guide students to perceive the theorem of ‘arbitrariness’, thinking from the particular to the general, and to provoke the theorem of thinking. The activity of visualising ‘arbitrariness’ through IT displays is designed to fit the cognitive development of solo's multiple structural levels and to develop students' analytical and inductive skills.

Question 4: As shown in Figure 5, when $\vec{a}$ is the zero vector, how can $\vec{a}$ be expressed using $\overrightarrow{e_1}$ and $\overrightarrow{e_2}$ ? When $\vec{a}$ is collinear with $\overrightarrow{e_1}$ or $\overrightarrow{e_2}$ , how can $\vec{a}$ be represented using $\overrightarrow{e_1}$ and $\overrightarrow{e_2}$ ?

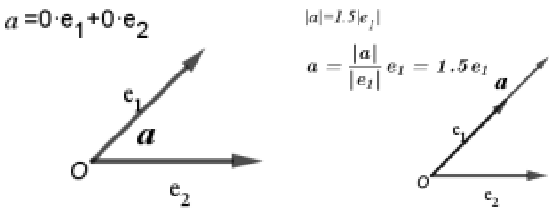


Figure 5: Schematic representation of the decomposition of vectors in special cases

[Intention Behind the Design]

Discuss special cases to deepen students' understanding of the conclusions drawn. If $\vec{a}$ is the zero vector, let $\lambda_1 = \lambda_2 = 0$ , then $\vec{a} = 0 \cdot \overrightarrow{e_1} + 0 \cdot \overrightarrow{e_2}$ . If a non-zero vector $\vec{a}$ is in the same direction as $\overrightarrow{e_1}$ , let

$\lambda_1 = \frac{|\vec{a}|}{|\vec{e_1}|}, \lambda_2 = 0,$ then $a = \frac{|\vec{a}|}{|\vec{e_1}|}\vec{e_1} + 0 \cdot \vec{e_2}.$ Similarly, if a non-zero vector $\vec{a}$ is in the opposite direction to $\vec{e_1}$, then let $\lambda_1 = -\frac{|\vec{a}|}{|\vec{e_1}|}, \lambda_2 = 0,$ hence $a = -\frac{|\vec{a}|}{|\vec{e_1}|}\vec{e_1} + 0 \cdot \vec{e_2}.$ Guide students to extend from general cases to special cases, and through categorized discussions, confirm the consistency of vector representation with base vectors in different shapes, avoiding a one-sided understanding of any vector in the theorem.By exploring and proving with problem chains, students' thinking gradually transitions from the original single-structure level to a multi-structure level that is closer to the level of related structures.

(4)Associative structural level

Question 5: From the investigation above, it is known that if $\vec{e_1}, \vec{e_2}$ are two non-collinear vectors in a plane, for any vector $\vec{a}$ in this plane, there exist real numbers $\lambda_1, \lambda_2$ such that $\vec{a} = \lambda_1\vec{e_1} + \lambda_2\vec{e_2}$ Are $\lambda_1, \lambda_2$ unique? Why?

[Intention Behind the Design]

This question is mainly for students to transition from ‘can we represent vectors’ to ‘is there only one way to represent them’. In the process of thinking, students need to combine the linear operations of vectors, the intuition of geometric decomposition, and the logic of algebraic proofs, and slowly appreciate that Vectors have both the arithmetic characteristics of numbers and the intuitive properties of shapes. Help students' level of thinking hierarchically move from a multi-point horizontal structure to an associative structural level.Instead of looking at knowledge points in isolation when analysing them, students find connections between different pieces of knowledge, learn to reason and argue in a variety of ways, and build a more systematic understanding of knowledge.

Question 6: As shown in Figure 6, in parallelogram *ABCD*, points *M* and *N* are the midpoints of *BC* and *DC* respectively. $\overrightarrow{AB} = \vec{a}, \overrightarrow{AD} = \vec{b},$ Express $\overrightarrow{BN}$ and $\overrightarrow{DM}$ using $\vec{a}$ and $\vec{b}$.

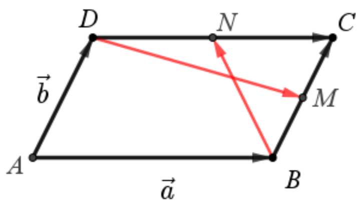


Figure 6: Schematic representation of vectors in a parallelogram.

[Intention Behind the Design]

Using the parallelogram as a vehicle, this question is designed to guide students in applying their knowledge of linear operations with vectors to translate vector relationships in geometric shapes into algebraic operations. By solving this problem, students are encouraged to sort out their thinking structure, transition from the use of a single point of knowledge to the integrated use of multiple knowledge, and develop their logical reasoning and mathematical calculation skills, so that the level of students' correlation structure tends to stabilise.

(5)Extended abstract structural level

Question 7: As shown in Figure 7, in triangle $\Delta ABC$, the midpoint of side *BC* is *O*. A line *EF* is drawn through point *O*, intersecting the extension of *AB* at point *F*. If $\overrightarrow{AB} = m\overrightarrow{AF}, \overrightarrow{AC} = n\overrightarrow{AE},$ what is the value of $m + n$?

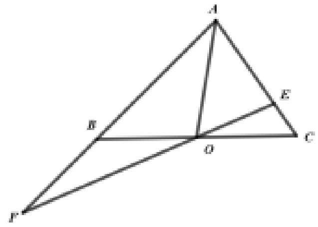


Figure 7: Schematic linear representation of vectors in triangles

Solution 1: Choose two different sets of bases to describe the same vector, which means using bases $\{\overrightarrow{AB}, \overrightarrow{AC}\}$ or $\{\overrightarrow{AM}, \overrightarrow{AN}\}$ to express $\overrightarrow{AO}$, and combine the "uniqueness" of the base to solve, highlighting the importance of equation thinking and transformation in the problem-solving process. Solution 2: Draw *BM* through point *B* parallel to *AC*, and intersect *EF* at point *M*. Based on the condition that point *O* is the midpoint of *BC*, we know *MB* = *EC*. Since $\frac{BM}{EA}=\frac{BF}{FA}$, it follows that $\frac{EC}{EA}=\frac{BF}{FA}$ Given $\overrightarrow{AB}=m\overrightarrow{AF}$ and $\overrightarrow{AC}=n\overrightarrow{AE}$ we find $n-1=1-m$, hence $m+n=2$.

[Intention Behind the Design]

The main purpose of designing this example is to consolidate students' choice of bases and operations, so that students can deepen their understanding of the Fundamental Theorem of Plane Vectors by solving practical problems. By comparing two different solution methods, we can distill the ideas of equations and transformation, allowing students to elevate their problem-solving experiences to a level of methodological transfer. This enables them to appreciate the fundamental role of theorems in complex problems, and to cultivate their abilities in abstract generalization and flexible application, thereby meeting the requirements of the abstract structural level.

## 3. Conclusion

This study explores a systematic instructional design for the Fundamental Theorem of Plane Vectors based on SOLO Classification Theory, aiming to promote deeper student learning through tiered teaching activities. The theory classifies students' cognitive development into five levels: pre-structural, uni-structural, multi-structural, associative structural and extended abstract structural, which provides a clear idea of hierarchical design for teaching and learning activities. By designing chains of questions that are hierarchical and logical, the study guides students through a gradual transition from lower to higher levels of thinking. At the same time, the theory of solo categorisation helps teachers to grasp the learning stages and needs of students, and to design more personalised and targeted teaching activities to enhance teaching effectiveness. The application of this theory makes the content more systematic and hierarchical, making it easier for teachers to organise and for students to understand. SOLO classification theory not only guides students to explore the process of theorem generation from concrete examples, but also helps students to classify and discuss vectors in different regions to deepen their understanding of the theorem. Through example training and extension, students are able to apply theorems to solve practical problems and enhance their mathematical thinking skills and literacy. Therefore, SOLO classification theory shows a good potential for application in the teaching of the Fundamental Theorem of Plane Vectors, which provides new perspectives and methods for high school mathematics teaching practice, and has a high value of promotion. This systematic teaching and learning design helps students to gradually construct a complete knowledge system under the guidance of the teacher, laying a solid foundation for future mathematics learning.

## References

*[1] Wu Lina, Chen Yujuan. Example of the construction and application of 'structural patterns' in 'plane vectors'--Thinking triggered by a 2019 college entrance examination question[J]. Mathematics Bulletin,2020,59(06):32-36.*
*[2] JOHN B.BIGGS, KEVIN F. COLLIS. Evaluating the Quality of Learning——The SOLO Taxonomy [M].Singapore: Academic Press,1982.*
*[3] Chenhao Wang. The study of vectors in high school mathematics[J]. China High-Tech Zone,2018,(02):106.*
*[4] Liu Na, Wu Xiaohong. The design of unit teaching based on big concepts--Taking the unit of 'plane vectors' as an example[J]. Asia-Pacific Education,2022,(20):100-102.*
*[5] Qin HJ, Huo XC, Guo L. A cognitive diagnostic study of plane vectors in high school[J]. Journal of Mathematics Education, 2024,33(02):1-7.*
*[6] Gong Liyuan. The teaching design of plane vectors unit based on the enhancement of core mathematics literacy[J]. Science and Technology Perspectives,2021,(13):109-110.*