

Satellite Clock Bias Prediction Method for BeiDou-3 Satellites Based on Entropy Weight Method

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Abstract: In order to improve the accuracy and stability of satellite clock bias prediction, a combined satellite clock bias prediction method based on the entropy weight method is proposed. Firstly, the method adopts a quadratic polynomial model and a gray model to make a single prediction of satellite clock bias and generate two sets of prediction results. Then, by calculating the entropy of error information of the two sets of prediction results, it determines the weights of each model and realizes the optimal fusion of the models. Finally, the entropy weight combination method is used to obtain a higher precision prediction result. Four different types of BeiDou-3 satellites were randomly selected for the prediction test by using the precision satellite clock bias products released by the GNSS Analysis Center of Wuhan University. The results show that the method can provide high-precision short- and medium-term predictions of BeiDou-3 satellite clock bias, and its 6-h average prediction accuracy and stability are 0.22ns and 0.46ns, respectively, which are 72.15% and 48.84% higher than the average prediction accuracy of quadratic polynomial and gray models, and the stability is 70.00% and 20.69% higher, respectively.

1. Introduction

Global Navigation Satellite System (GNSS) is a time-based system whose Positioning Navigation Timing (PNT) accuracy depends heavily on the accuracy of time measurements [1-2]. In Precise Point Positioning (PPP) technique, in order to obtain centimeter-level positioning accuracy, it is necessary to use the predictive satellite clock bias (SCB) as a known quantity to be substituted into the equation for positioning solution. Therefore, the prediction of high-precision SCB is an important prerequisite for realizing centimeter-level PPP technology, and its accuracy will directly affect the positioning performance of PPP technology [3-4].

Many scholars at home and abroad have conducted extensive and in-depth research and achieved a series of research results. For example, quadratic polynomial model (QPM), gray model (GM (1,1)), auto-regressive moving average (ARMA), and spectrum analysis (SA), etc. [5-10]. These methods are applicable to the prediction of the atomic clock bias of navigation satellites under different conditions, but each method has its scope of application and limitations. For example, the QPM fits the historical clock bias data with time as the independent variable, which is easy to

compute and has a high short-term prediction accuracy. However, its medium- and long-term prediction accuracy decreases significantly with the increase of time. The GM (1,1) is suitable for small samples and uncertainty data, and has a strong anti-interference ability, but the adaptability is different on different on-board clocks and the dependence on the data volume is strong, which may lead to a large error. The ARMA can effectively capture the linear dependence structure of the time series data, and is suitable for prediction the clock bias of smooth series, but it is less adaptable to non-smooth series, and the selection of the model order is difficult to determine. The SA is able to deal with bellwether series characterized by periodicity, but it may not perform well on non-periodic data and the parameter estimation is more complicated.

In order to improve the accuracy of SCB prediction and make full use of the respective advantages of the models, this paper proposes a combined SCB prediction method based on the entropy weight method. The method firstly adopts the QPM and the GM (1,1) to make a single prediction of the SCB, and generates two sets of prediction results; then, by calculating the entropy of the error information of the two sets of prediction results, it determines the weights of the models and realizes the optimization of model fusion; finally, it obtains the higher precision prediction results by using the weighted combination method. In addition, four different types of BeiDou-3 satellites were randomly selected for prediction experiments by using the precision SCB data released by the GNSS Analysis Center of Wuhan University. The superiority and effectiveness of the method are verified by comparing and analyzing the prediction results with those of the QPM and the gray prediction model, which are commonly used in SCB prediction.

2. Establishment of SCB Combination Prediction Model Based on Entropy Weight Method

The entropy weight method measures the uncertainty of an indicator by calculating the entropy value of each assessment indicator. Indicators with high entropy values imply less information and higher uncertainty, while indicators with low entropy values indicate rich information and lower uncertainty. Based on the entropy value, the weight of each indicator can be further derived to highlight those indicators that have a greater impact on the decision outcome [11-13]. The steps for solving the weights of each model are as follows:

(1) Assuming that there are α SCB prediction models and β error indicators, there is an error matrix Z' :

$$Z' = \begin{bmatrix} z'_{11} & z'_{12} & \cdots & z'_{1\beta} \\ z'_{21} & z'_{22} & \cdots & z'_{2\beta} \\ \vdots & \vdots & \cdots & \vdots \\ z'_{\alpha 1} & z'_{\alpha 2} & \cdots & z'_{\alpha\beta} \end{bmatrix} \quad (1)$$

(2) Standardize each data in the error matrix Z' using the departure standardization method, and finally get the matrix Z , the departure standardization formula is as follows:

$$z_{ij} = \frac{(z'_{ij} - \min z'_{ij})}{(\max z'_{ij} - \min z'_{ij})} \quad (2)$$

Where $\max z'_{ij}$ and $\min z'_{ij}$ are the maximum and minimum values of z'_{ij} , respectively.

(3) Calculate the information entropy of the j th data e_j for:

$$e_j = -K \sum_{i=1}^{\beta} \left(\frac{z_{ij}}{\sum_{i=1}^{\beta} z_{ij}} \ln \frac{z_{ij}}{\sum_{i=1}^{\beta} z_{ij}} \right) \quad (3)$$

Where K is a coefficient and satisfies $K \cdot \ln(\beta) = 1$.

(4) Calculate the weight coefficient of the j th attribute component as:

$$k_j = \frac{h_j}{\sum_{j=1}^{\beta} h_j} \quad (4)$$

Where k_j is the weight of the single model and h_j is the coefficient of variation of the j th attribute component and satisfies $h_j = 1 - e_j$.

In this paper, the QPM and the GM (1,1) are used to construct the combined prediction model based on the entropy weighting method as:

$$C(q) = k_1 \partial_1(q) + k_2 \partial_2(q) \quad (5)$$

Where k_1 is the weight of the QPM, k_2 is the weight of the GM (1,1), and $\partial_1(q), \partial_2(q)$ are the SCB prediction values of the QPM and the GM (1,1), respectively. The flow of this combined prediction model is shown in Figure 1:

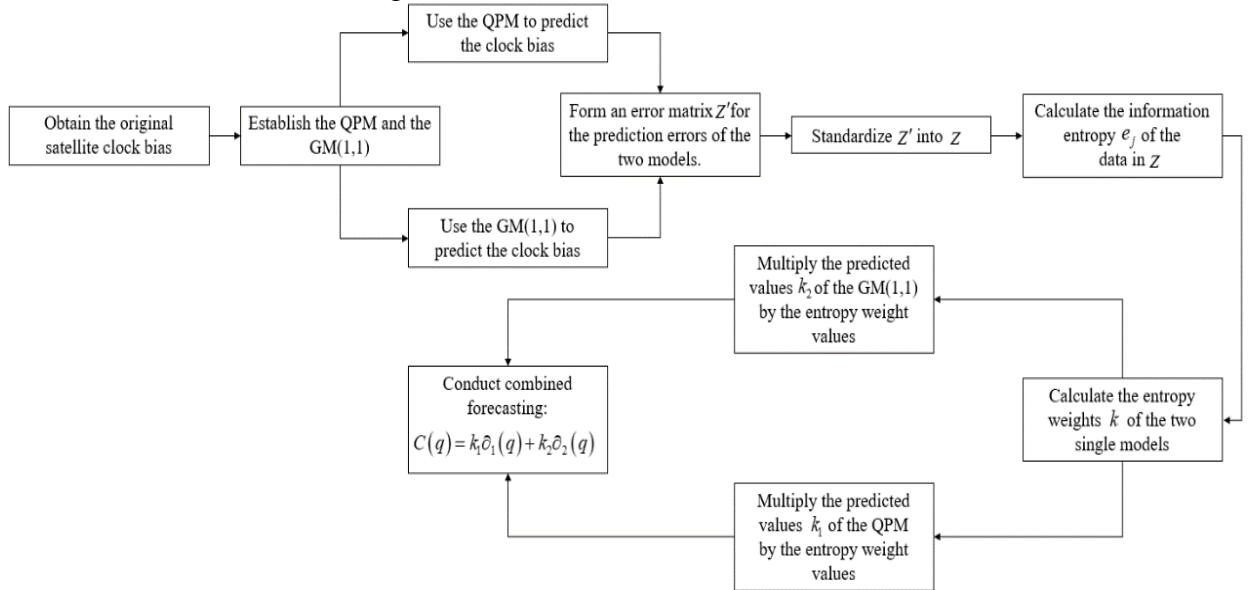


Figure 1: The prediction process of SCB combination based on the entropy weight method

3. Tests and Analysis

3.1 Sources of experimental data

In order to verify the validity and feasibility of the combined model, the BDS after-the-fact precision SCB product released by the GNSS Analysis Center of Wuhan University on August 12, 2024 is used as the experimental data, with a sampling interval of 5 min. more than 40 BDS

satellites are in orbit, and the BeiDou 3 on-board clocks mainly consist of the following three types: BLOCK IGSO-H clocks, BLOCK MEO-Rb clock and BLOCK MEO-H clock. In order to make the research results provide valuable references for China's BeiDou-3 satellite navigation system in terms of clock difference prediction, the clock bias of four satellites, namely, MEO-Rb PRN 22, IGSO-H PRN 40, MEO-Rb PRN 42, and MEO-H PRN 44, are randomly selected for the prediction test. Their relevant information is shown in Table 1.

Table 1: Selected satellite related information.

Satellite number	Clock type	Launch time	Trends in clock bias
PRN 22	MEO-Rb	2018.07.29	monotonically increasing
PRN 40	IGSO-H	2019.11.05	monotonically decreasing
PRN 42	MEO-Rb	2017.11.05	monotonically decreasing
PRN 44	MEO-H	2019.11.23	monotonically increasing

The variation of the precision SCB time series of these four satellites 6h before August 11, 2024 is shown in Figure 2, in which the clock bias time series of satellites PRN 40 and PRN 42 show a monotonically decreasing trend, and that of satellites PRN 22 and PRN 44 show a monotonically increasing trend, and all the satellites of BeiDou 3 were launched in 2017-2019, and the selected four satellites were launched in each of these three years, which is fully representative.

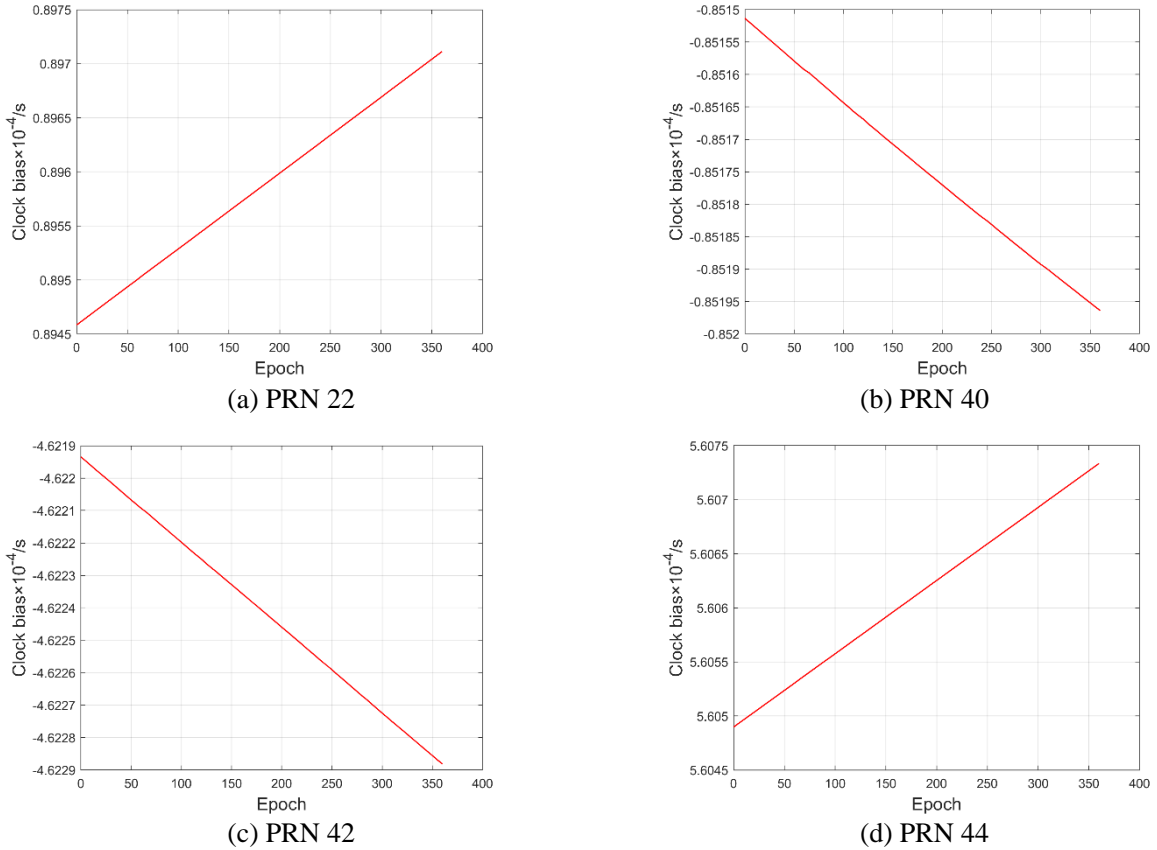


Figure 2: Chart of clock bias variation of the PRN 22, PRN 40, PRN 42 and PRN 44 satellites.

3.2 Prediction results and analysis

In order to fully analyze the prediction performance of the method in this paper, the SCB data of 6h before August 12, 2024 of BDS are used to establish a QPM, a GM (1,1), and a combined prediction model based on entropy weighting method (shortened as: Combined model), respectively, to prediction the SCB of the next 6h. The prediction error of each model can be obtained by subtracting the final precision SCB of the next 6h released by the GNSS analysis center of Wuhan University from the corresponding SCB prediction by each model. Since this paper adopts the precision SCB products released by the GNSS analysis center of Wuhan University, the error of the clock bias itself is less than 0.1ns, so it can be taken as the “true value”, and the maximum error value is subtracted from the minimum error value by using the Root Mean Square error (RMS) and the extreme range (Range), i.e., the maximum error value is subtracted from the minimum error value. RMS and Range, i.e., the maximum error value minus the minimum error value, are used to evaluate and compare the prediction accuracy and stability of the models. The RMS and Range are calculated as follows:

$$RMS = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (6)$$

$$Range = \max(y_i - \hat{y}_i) - \min(y_i - \hat{y}_i) \quad (7)$$

The prediction error variations and error statistics for each model are shown in Figure 3 and in Table 2:

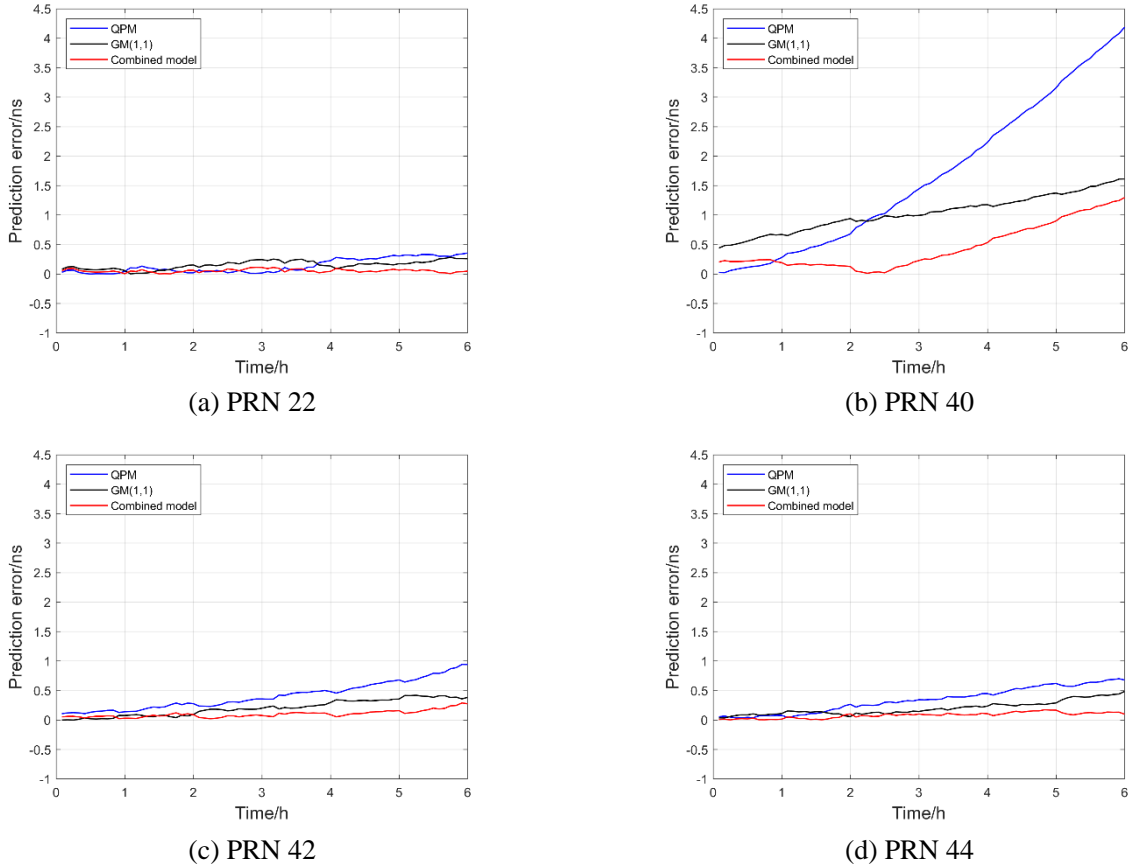


Figure 3: Prediction error variation chart of 6h SCB.

Table 2: Statistical results of SCB prediction error (unit: ns).

Method	Evaluation indicator	PRN 22	PRN 40	PRN 42	PRN 44	Average value
QPM	RMS	0.19	2.10	0.47	0.40	0.79
	Range	0.35	4.16	0.83	0.66	1.50
GM (1,1)	RMS	0.17	1.08	0.25	0.23	0.43
	Range	0.28	1.17	0.42	0.45	0.58
Combined model	RMS	0.06	0.59	0.12	0.09	0.22
	Range	0.11	1.29	0.27	0.17	0.46

Table 3: Average prediction accuracy and improvement rate of each model in 6 hours.

Method	RMS	Range
Combined model	0.22	0.46
QPM	0.79	1.50
improvement rate(%)	72.15%	70.00%
GM (1,1)	0.43	0.58
improvement rate(%)	48.84%	20.69%

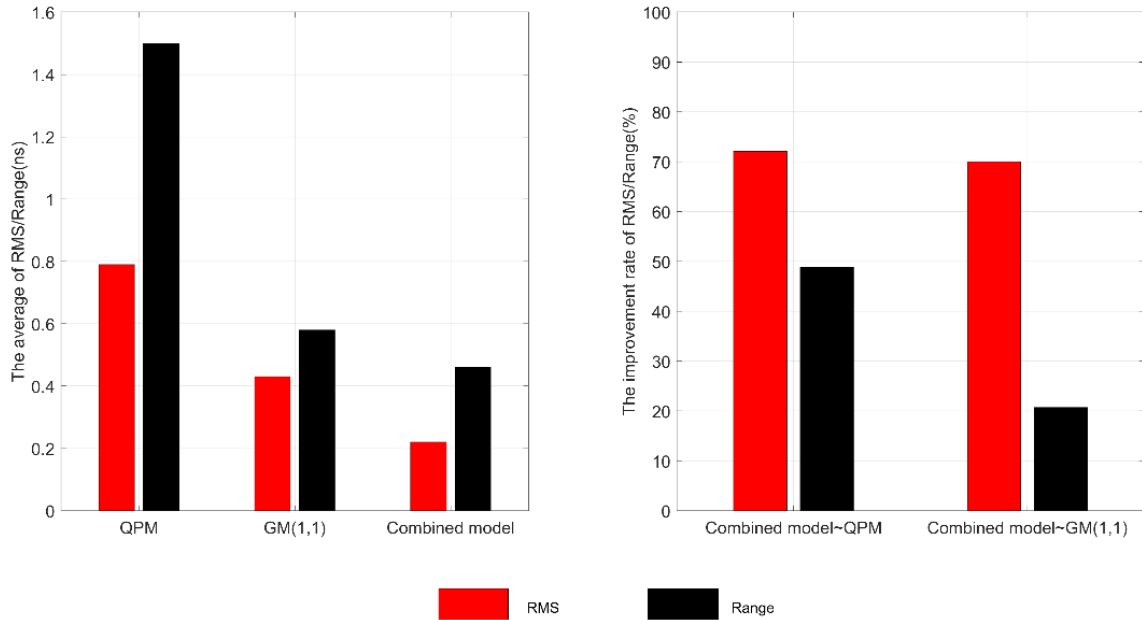


Figure 4: 6h average prediction accuracy, stability and improvement rate.

These can be seen in conjunction with Figure 3-4 and Table 2-3, in the 6-hour short-term prediction, the average prediction accuracy and stability of the QPM are 0.79ns and 1.50ns, respectively; the average prediction accuracy and stability of the GM (1,1) are 0.43ns and 0.58ns, respectively; and the average prediction accuracy and stability of the combined model based on entropy weighting are 0.22ns and 0.46ns, respectively, which are 72.15% and 70.00% higher compared to the average prediction accuracy and stability of the QPM, while the average prediction

accuracy and stability of the combined model are 48.84% and 20.69% higher than those of the GM (1,1). The errors of the combined model in predicting clock bias of IGSO-H satellites are significantly larger than those of the other three clock types. In addition, the prediction accuracy of the combined model for satellites with monotonically increasing clock bias is generally higher than that for satellites with monotonically decreasing clock bias, indicating that the combined model is more effective in predicting clock bias for satellites with monotonically increasing clock bias.

4. Conclusions

In order to further improve the accuracy and stability of SCB prediction, a combined SCB prediction method based on the entropy weight method is proposed. The method firstly adopts the QPM and the GM (1,1) to make a single prediction of the SCB, and obtains two sets of prediction results; then, by calculating the entropy of the error information of the two sets of prediction results, it determines the weights of both model and realizes the optimal fusion of the models; finally, it obtains a higher accuracy of the prediction results by using the weighted combination method. This combination model can combine the prediction advantages of the two models and further improve the prediction performance of the model, thus improving the accuracy of SCB prediction. The results of the theoretical and prediction experiments also show the effectiveness and feasibility of the combined model, which provides a new method and idea for the high-precision prediction of SCB. In addition, the prediction accuracy of the combined model for satellites with monotonically increasing clock bias is generally higher than that for satellites with monotonically decreasing clock bias.

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