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Research on Tennis Match Momentum Based on Dynamic Quantitative Model

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Abstract: In tennis matches, the psychological and physiological state of the players in the match is gradually considered to be the key factor affecting the result of the match, and "momentum" is considered to be one of the important factors determining the trend of the match. However, how to quantify and accurately assess the impact of momentum on the outcome of a match remains a challenge. To solve this problem, this paper proposes a dynamic quantitative model based on tennis match momentum analysis method. Taking the 2023 Wimbledon men's singles final as an example, this paper combines statistical analysis, entropy weight method, T-test, binary logistic regression analysis and other methods, and uses SPSS, Matlab, Excel and other tools to deeply explore the impact of momentum-related factors on tennis matches, and quantifies the effect of momentum on match results. The results show that the dynamic momentum quantitative model has strong explanatory power and practical guiding significance, and can provide optimization strategies and decision-making suggestions for coaches and athletes.

1. Introduction

Tennis is a combination of high-intensity confrontation and psychological game, and the "momentum" phenomenon in the game refers to the psychological and competitive advantages accumulated by players through continuous scores or excellent performances, which usually have a significant impact on the direction and outcome of the match at the key moment of the match. For example, in the 2023 Wimbledon men's singles final, Carlos Alcala successfully defeated Novak Djokovic with consecutive points at crucial moments, demonstrating the power of momentum[1]. The phenomenon of momentum is not only reflected in the players' technical and psychological advantages, but also provides an important basis for the coach's tactical adjustment. However, although the influence of momentum on the outcome of a game is widely recognized, current research has limitations in terms of quantitative and dynamic analysis. Many studies measure the influence of momentum through simple binary variables (such as winning and losing), but ignore complex factors such as score margin and key points [2]. In recent years, with the development of data science, more and more dynamic quantitative models [3] and machine learning methods [4] (such as time series analysis, logistic regression, CatBoost regression, etc.) have been used in momentum analysis to improve the accuracy of momentum turning point prediction. However, these models face the contradiction between model complexity and operability in practical application, and existing studies have not systematically explored the influence of different venue types, competition stages and players' technical characteristics on momentum change [5].

In order to make up for the shortcomings of the existing researches, a new dynamic quantitative model is proposed in this paper, which is combined with logistic regression analysis to deeply investigate the evolution law of momentum in tennis matches and its influence on match results. Different from the traditional static analysis method, this paper combines the dynamic change of momentum with different stages of the game process (such as opening, key points, tiebreakers), and analyzes the performance of momentum in different stages of the game and its specific impact on the technical performance of players. In addition, this paper also integrates a variety of quantitative analysis methods (such as entropy weight method, t test, etc.), in-depth analysis of the driving factors of momentum generation, and discusses its relationship with the outcome of the game, players' psychological state.

Through these innovative analyses, this paper not only makes up for the shortcomings of existing studies, but also provides a scientific basis for strategic decision-making in tennis matches. It is expected to further optimize the applicability and practicability of the model through more extensive data verification.

2. A study of player performance in a tennis match

2.1 Player performance characteristics statistics in tennis matches

In tennis matches, seven performance characteristics are selected for evaluation, which are AEC rate, double error rate, first serve success rate, first serve score rate, second serve score rate, second serve score rate and second serve score rate [6]. ACE rate is usually a measure of a player's ability to score directly when serving (i.e. the opponent does not even touch the ball); The Double Fault Rate is a measure of how often a player commits double faults (i.e., two consecutive service failures) when serving. First Serve Percentage is a measure of the frequency of a player's first serve (successful first serve); First Serve Return Points Won. Percentage is a measure of how often a player wins points on an opponent's first serve, etc [7].

Take a randomly selected match as an example: for the NTH point of a player, calculate the ACE rate of the first n points, Seven characteristics such as Double Fault Rate. The selection of n should be the score after the number of shots, and the number of shots in the game as the time point to measure the progress of the game. Because positive indicators refer to indicators used to measure and evaluate positive development or progress in a certain field or aspect, the formula for calculating positive indicators in the calculated probability is as follows:

$$a_i=1*\frac{x_k}{y_k}\text{ , }k=1,2,3,4,5,6 \tag{1}$$

Negative indicators are often used to indicate adverse changes or outcomes in order to assess and monitor non-achievement of objectives. Let the negative indicator be calculated as follows:

$$b_j=1*\frac{x_q}{y_q}\text{ ,}q=7 \tag{2}$$

Where x_k,x_q represents the number of positive or negative indicators occurring (k= 1,2,3,4,5,6, q=7); y_k,y_q represents the total number of positive or negative indicators obtained (k= 1,2,3,4,5,6, q=7); $\frac{x_k}{y_k}$ represents the incidence of positive indicators, that is, in a specific period of time, The frequency or proportion of positive indicators; $\frac{x_q}{y_q}$ represents the incidence of negative indicators. For positive indicators, the higher the variable, the better, and the opposite is true for negative indicators.

2.2 Use entropy weight method to assign weight to the performance characteristics of each player

Entropy weight method (EWM) is a commonly used weighting method that measures value dispersion in decision-making. The greater the degree of dispersion, the greater the degree of differentiation, and more information can be derived. Meanwhile, higher weight should be given to the index, and vice versa[8].

In some decision-making problems, the weight of the index is often uncertain. By calculating the information entropy and weight of the index, the entropy weight method can reasonably reflect the weight distribution in the comprehensive evaluation of the index, and help the decision maker determine the weight of the index more objectively and reduce subjectivity and uncertainty.

Firstly, 0-1 normalization is carried out on the data after data inspection, and the data of different indicators are converted to the same dimension and range for comprehensive evaluation. Secondly, according to the standardized data, the information entropy of each index is calculated. Information entropy is used to measure the diversity and uncertainty of indicators, and the higher the entropy value, the higher the diversity of indicators. Then the weight of each index is calculated according to the information entropy of the index. In the entropy weight method, the weight represents the importance degree of each evaluation index in the overall evaluation system. The weight can be determined by the ratio of entropy to the sum of entropy. The smaller the entropy, the greater the weight, and the weight is represented by the symbols W_k (k=1,2,3,4,5,6) and W_q (q=7). Seven statistics are obtained for the variation of weight W in 281-300. The average value of the probability weights of the indicators was obtained, and the indicators were sorted according to their sizes, as shown in Table 1.

Table 1 Player 1 and Player 2 weight ranking

Player1			Player2		
Feature	Average	Ranking	Feature	Average	Ranking
ACE	0.17722434	1	ACE	0.154973067	3
DF	0.117299046	7	DF	0.134413611	4
FSP	0.17248558	2	FSP	0.16195444	2
FSPWP	0.17096159	3	FSPWP	0.12822772	6
SSP	0.126141495	5	SSP	0.114603519	7
FSRPWP	0.124576208	6	FSRPWP	0.175970486	1
SSRPWP	0.111307642	4	SSRPWP	0.129867161	5

In the current context, the weight represents the contribution degree of decision information. The higher the weight of an indicator, the stronger the ability to distinguish evaluation objects, which means that this indicator plays a more critical role in comprehensive evaluation or decision analysis. The ranking according to the weight size also more clearly reflects the impact of this indicator on athletes, as shown in Figure 1 and Figure 2.

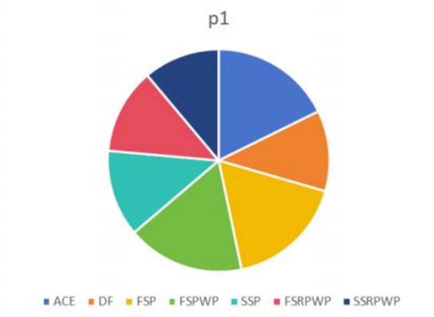

Figure 1 Player 1

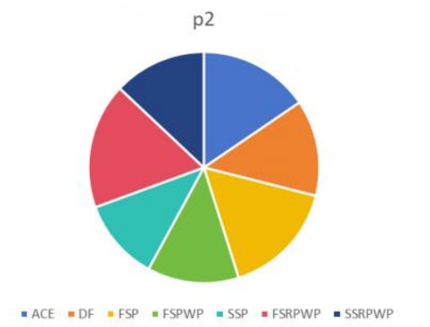

Figure 2 Player 2

At the beginning of the competition, the indicators have not yet fully appeared, so the table from i to the third digit is calculated. In summary, the changes of weights of 7 indicators in the 281-300 marks are obtained. The figure shows the occurrence probabilities of player 1 and player 2 at 281-300 minutes. After averaging, pie charts are used to visualize the data. The results show that for player 1, the occurrence probabilities of ACE and FSP are higher than other indicators, while DF, as a negative indicator, has the lowest occurrence probability. This indicates that the double error rate of the player is very low, and the service wheel has a greater advantage; On the contrary, for the other player, the probability of FSRPWP and FSP is the least, and the probability of negative indicator DF is in the middle of the measured indicators, indicating that the player is good at receiving and has a more obvious advantage in the receiving round than the opponent.

2.3 Quantifying player performance

Finally, the change of seven statistics in 281-300 points, the change of weight ω and the final score are obtained. Based on the weight of the index, the standardized value and weight of each index are multiplied and summed to get the final comprehensive evaluation result.

In addition to the seven characteristics we calculated, there are other factors such as Break Point Save Percentage in tennis matches. However, due to the high level of Wimbledon competition, the probability of these indicators appearing in the competition of high-level athletes is very low and the data amount is not large enough, which is not suitable for the analysis of entropy weight method. Let η_{k-q} be the overall status of the player, and the final formula is as follows:

$$\eta=\sum_{k=1}^{6} \omega_k \frac{x_k}{y_k}-\omega_q \frac{x_q}{y_q} \text{ ,k=1,2,3,4,5,6 ,q=7} \tag{3}$$

According to formula (3), the original score of the comprehensive state score of two players in the case of 281-300 points is calculated, and the probability sum is 1 after the data is standardized. According to the results, data visualization is performed to get a line chart to facilitate the observation of data rules, as shown in Figure 3.

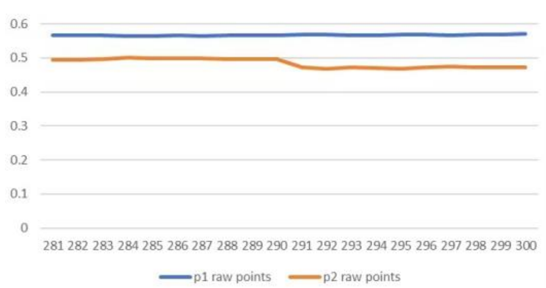

Figure 3 Comprehensive status scores of players

For player 1, it is found that the state curve is stable during the game, indicating that the play is stable; For player 2, the initial state remained relatively stable, and the state declined after the 290th point, indicating that the psychological tension may fluctuate greatly due to the pressure of the score.

Comparing the size of real-time state scores of Player 1 and Player 2, it is found that the overall strength of Player 1 is better than that of Player 2. Let Δηl denote the difference between the overall state scores of player 1 and player 2 at a certain moment, by comparing the size of the difference at different moments, we can determine the difference in the state of the players.The final formula is as follows:

$$\Delta\eta_1 = |\eta_1 - \eta_2| \tag{4}$$

As shown in Figure 4, the normalized data is used to predict the probability of scoring the next ball for both players, and it is found that the probability of winning for Player 1 increases significantly after the 290th ball, which is consistent with the pattern in Figure 3.

Figure 4 Probability of winning

Test: The results of the prediction and evaluation model are consistent with the actual situation, and Player 1 wins in the end, as shown in Figure 5:

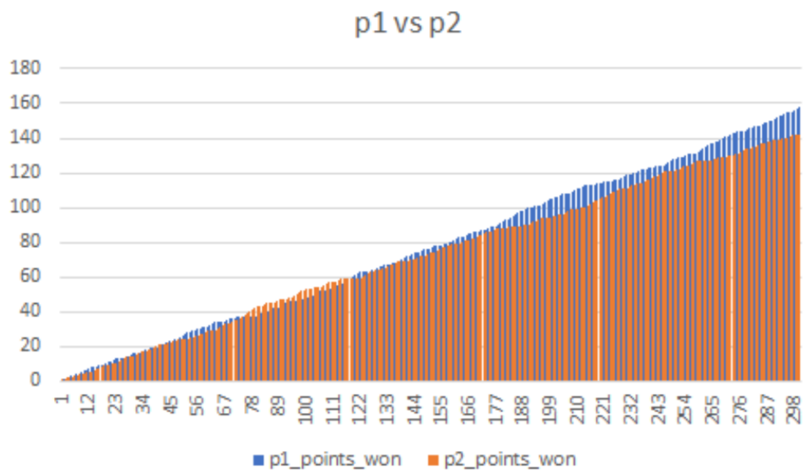

Figure 5 p1 VS p2

2.4 Statistical analysis of key performances of players

First of all, the required data will be selected. In order to make sure that each individual has equal chances of being drawn so that the sample is more representative, 7200 points are randomly selected from a total of 7280 points in the game after the top 32, and 300 points are divided into a group of 24 cases to facilitate the data statistics. Secondly, when the number of consecutive wins of the players is more than or equal to 3 or when the players win two or more games in a row, it will be labeled as the players' small "momentum" or big momentum, and the small momentum will be the main one, and the big momentum will be accompanied by the big momentum for the statistics only in the previous study[9].

Among the 7200 points scored, the relationship between the ten indicators that affect the match in the event of a “momentum” occurs. In addition to the indicators selected in the first question, we have also added the indicators that better reflect the psychological condition of the players during the match, so the indicators are: win rate, double fault rate, unforced error rate, first serve percentage, successful break rate (break points are the match points in the opponent's serve), serve retention rate, match point percentage (match point is the points won by active attacking during the match), net point percentage, long ball (the ball that is pressed to the baseline), point percentage (see AS and AT columns), and point conversion rate. Points percentage (look at the AS vs. AT columns), Point conversion rate. For example, Winning Percentage is a measure of the percentage of points won by a player in a match compared to the total number of points won and is used as an indicator of a player's overall performance; Serve Hold Percentage is the percentage of points that a player successfully holds his or her own serve; Number of Unforced Errors is the number of times a player loses a point on his or her own due to a technical error or poor decision-making, which occurs when there is no direct pressure from the opponent. The number of unforced errors is the number of points a player loses on his own due to technical errors or poor decision-making that occur without direct pressure from the opponent, etc. The relationship between each ratio and momentum is shown in Table 2

Table2 Relationship between ratios and momentum

Group	Momentum Score Rate	None Momentum Score Rate	……	Momentum Double Error Rate	None Momentum Scoring Rate
1	0.710526316	0.464285714	……	0.0434782609	0.0463576159
2	0.674418605	0.387850467	……	0	0.0141843972
……	……	……	……	……	……
24	0.676470588	0.414141414	……	0.037037037	0.0546875

3. A study of the impact of momentum on the game

3.1 To find out if “momentum” has any effect on the game

According to the information, ANOVA and t-test are both commonly used statistical methods, but ANOVA is suitable for comparing the difference in means between multiple samples, while t-test is suitable for comparing the difference in means between two samples. The t-test was chosen to solve the problem because the situations were categorized into momentum and random scores[10].

For the table of characteristic statistics, let 1 represent the period of momentum, and 0 represent the situation of random score. Comparing the mean values of the two different situations, it is found that except for Winners Ratio, Unnecessary Error Rate and Double Fault Rate, which are smaller than the random situation during the momentum period, the mean values of all the other indicators are larger during the momentum period. Based on the difference between the mean values when the momentum is at 1 and 0, it is found that the difference between Long Ball Scoring Rate, Winners Ratio and Double Fault Rate is small, while the difference between the other indicators is large, and the difference between Break Point Conversion Rate is the largest. Point Conversion Rate has the largest difference.

Through the independent samples test, comparing the significance value with 0.05, it can be found that the significance of the three indicators of scoring rate, match point scoring rate, and breakage rate is less than 0.05, and the null hypothesis is rejected, and the momentum has a significant relationship with them.

3.2 Logistic regression modeling - quantifying specific relationships

In order to facilitate putting the data into SPSS for binary logistic regression analysis, the table was reorganized by grouping the data for the same indicator of gaining or losing momentum into one column and adding another column of data to determine what period of time the data was in.

The variables were screened, the correlations between the characteristics were calculated and passed the correlation test and the variance (inflation factor) test. In order to avoid the problem of multicollinearity, which would have an impact on the model's effectiveness, a feature was considered to pass the test when the correlation coefficient, r, was in the range of 0-0.8 and the VIF was in the range of 0-10. After screening the variables, the serve percentage was removed.

Binary logistic regression modeling using SPSS. Binary logistic regression is a statistical learning method for solving binary classification problems. It is based on a linear regression model that models probability by applying a logistic transformation to the output of a linear function that restricts the output to between 0 and 1. The model converts the predictions of the linear regression model into probabilities by means of a logistic function, usually a Sigmoid function, which is the most commonly used logistic function in logistic regression and which maps any real number to the interval (0, 1) so that it can be interpreted as a probability. The function formula is：

$$\sigma(z)=\frac{1}{1+e^{-z}}(z) \tag{5}$$

where z here is a linear combination:$z = \beta_0 + \beta_1 d_1 + \beta_2 d_2 + \cdots + \beta_n d_n$.

According to the results, the data presented in the summary of the model Cox-Snell R2 is 0.750 and Negolko R2 is 1.000, when it is closer to 1, the model has better explanatory power, which indicates that the model has better explanatory power. For the Hosmer-Lemeshaw test, which is used to assess the fit of the model, the difference between the predicted and actual values of the model is tested for significance.

A high p-value (significance) (>0.05) usually indicates a good model fit, and the graphical results are significant at 1.000, indicating a good model fit, as shown in Table 3, where B is the coefficient of the equation, and the coefficients indicate the effect of the independent variable on the log odds of the probability of occurrence of the dependent variable.

Table 3 Variables in the equation

	B	Standard Error	Wald	P-value
Scoring Rate	110.143	103034.235	0.000	0.999
Match Point Scoring Rate	3.299	45113.198	0.000	1.000
Rally Scoring Rate	-57.935	185145.332	0.000	1.000
Winning Shots Ratio	-29.931	137300.536	0.000	1.000
Net Play Scoring Rate	20.219	96678.455	0.000	1.000
Break Point Conversion	30.264	23742.094	0.000	0.999
First Serve Scoring Rate	61.222	82181.319	0.000	0.999
Unforced Error Rate	3.116	186066.044	0.000	1.000
Double Fault Rate	-228.277	483633.684	0.000	1.000
Constant	-84.029	106195.270	0.000	0.999

3.3 Specific quantitative rules for momentum

In the above discussion, the subjective definition of "momentum" is that after winning two consecutive sets or three consecutive balls, according to the significance level of T-test, it is known that this "momentum" is significantly correlated with the three characteristics of scoring rate, match point scoring rate and break rate, and the influence of momentum on the average break rate is more obvious, and the value should be higher. Using points, match points and breaks to redefine "momentum" and quantify "momentum".

The quantified rule is: for scoring events, considering that continuous scoring will make the "momentum" higher, so after winning the S-ball in a row, the momentum of the next ball is s, and the "momentum" is recalculated when the score is lost; For match point events, after the match point is scored, the next ball "momentum" +1; For the break event, since the previous analysis has known that the influence of "momentum" is more obvious in numerical terms, after the break, the next shot "momentum" +2; Based on this rule, we collected all match information of Carlos Alcaraz and Novak Djokovic in the data except the final, and screened the number of balls whose "momentum" is s, as shown in Table 4:

Table 4: Momentum corresponds to number

Momentum	0	1	2	3	4	5	6	7	8	9	10	11
Novak Djokovic	392	188	106	69	53	27	10	6	2	2	1	1
Carlos Alcaraz	400	199	108	71	47	31	9	3	2	2	1	0

Through the tabulated data, it is found that when the "momentum" is 3 or higher, the sample number is small, and the "momentum" is not obvious in this area, so we all believe that "momentum" =3.

3.4 The score rate fluctuates over time to predict the final result

By calculating the scoring rate of the sample corresponding to each "momentum", the scoring rate of two people is obtained - Figure 6 and Figure 7 of "Momentum". The meaning of this figure is that a person's scoring rate is different at different levels of "momentum", and different people's "momentum" has different effects on the scoring rate. Take this data to the finals to verify that fluctuations in "momentum" over the course of the match predict the outcome of the match. For the S-ball, calculate the "momentum" of the two people respectively, compare the scoring rate corresponding to the "momentum" of the two people at this time, predict that the person with a high scoring rate will win the ball. In the prediction of the final, the accuracy of the model is 52.55%, and the stability is greater than 50%, which indicates that the prediction is effective.

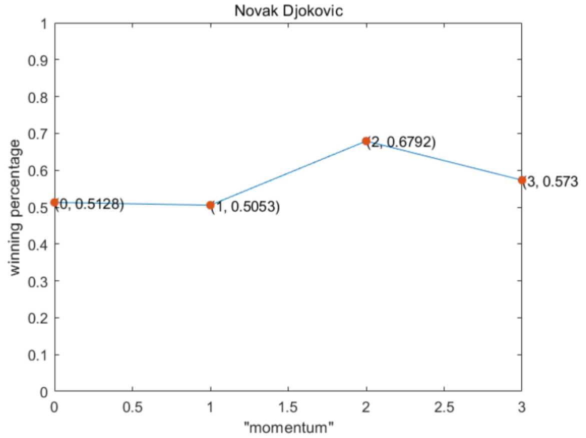

Figure 6 Scoring rate of Novak Djokovic under different momentum

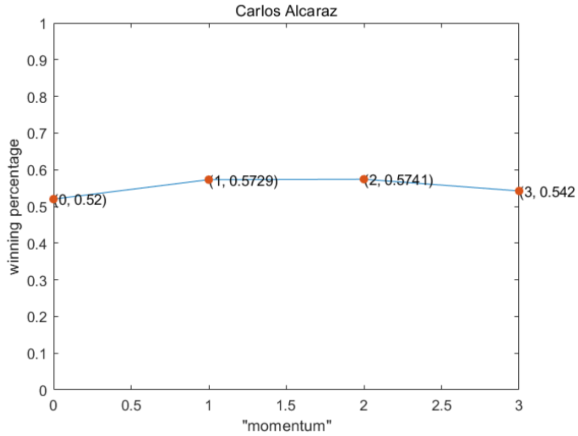

Figure 7 Scoring rate of Carlos Alcaraz under different momentum

Since the difference in "momentum" fluctuations in past races is taken into account and the possible scores of two players are added and normalized, the difference in momentum fluctuations of two players in a race can be significantly seen through the fluctuation figure 8. This wave curve provides the following advice for a player who is about to start a new match with another player: According to the scoring rate-Momentum graph of the next opponent, the sensitivity of the opponent to momentum can be known. If the scoring rate of the opponent increases significantly when the momentum increases, it is suggested that the player slow down the pace of the game or call a timeout when the momentum of the opponent rises. At the same time, we should also know how sensitive the players are to "momentum" and take a more favorable plan for themselves.

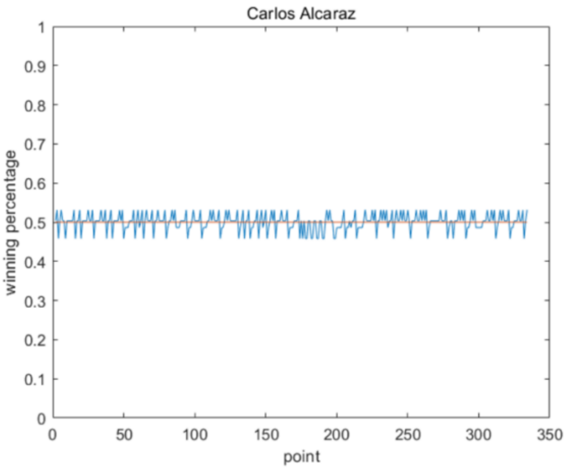

Figure 8. Momentum fluctuations throughout the round

3.5 Comparison based on logistic regression model

The first statistic is from 1 to $(300+c_m)$,(m=1,2,……,33,34) For the real-time situation of the nine features, the accuracy was predicted by the binary logistic regression model, and the accuracy of the prediction was compared with that of the previous model. The results of binary logistic regression model analysis by SPSS software are as follows:

Omnibus Tests of Model Coefficients were tests to check whether the coefficients of the entire model were significant. The results showed that the coefficients were 0.000 and less than 0.05, indicating that all variables in the model as a whole contributed significantly to the predictor variables. Model Abstract: Central measurement provides a quantitative index of model fit degree, in which R Square value is 0.324, the closer it is to 1, the stronger the explanatory ability of the model, and the explanatory ability of the model is average [11]. Hosmer and Lemeshow tests are used to evaluate the fit of the model and to check whether the difference between the predicted and actual values of the model is significant. A high p-value (> 0.05) usually indicates a good model fit, and our significance value is 0.426.

It is found that the fitting degree of the binary logistic regression model is 72.1%, which cannot predict the new match in real time, which is inferior to the previous model, but it can evaluate which indicator is the main factor affecting the momentum. According to the results of the significance of variables in the equation, the significance of first-shot scoring rate is less than 0.01, which is the most significant factor affecting the momentum fluctuation.

4. Conclusion

In this paper, we found that scoring rate, match point scoring rate, and break rate have a significant effect on momentum, and combined with the binary logistic regression analysis method, we established a dynamic momentum quantitative model. The results show that the model's real-time prediction of the next point result is better than that of the comparison model, which has strong explanatory power and practical guidance significance. However, the quantitative law of momentum in this paper is not very rigorous, and needs to be verified by using more game data; and the study is limited to the tennis game, and in the future, we can look for the commonality of general ball sports under the effect of momentum. Overall, this paper innovatively proposes a way to quantify momentum and verifies its feasibility, which can be used to provide optimization strategies and decision-making suggestions.

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