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Extreme risk spillovers between Chinese crude oil and stock markets: A CoVaR-Copula approach

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Keywords: Shanghai crude oil futures marke, Stock market, Extreme risk spillovers, asymmetry, Upside and downside risk, Copula-CoVaR

Abstract: In portfolio construction and risk management, the extreme risk spillover effect between markets should be fully considered. In this paper, the AR-GARCH-Copula with skewed t-distribution is used to characterize the dependence between China's crude oil futures market and the stock market, and then the CoVaR method is used to measure the size and intensity of upside and downside extreme risk spillovers between China's crude oil futures market and China's stock market, and to analyze the asymmetry in the extreme risk spillovers effect. The results of the study show that: (1) there is an obvious positive correlation tail dependence between China's crude oil futures market and the stock market; (2) there is indeed a bidirectional asymmetric extreme risk spillover effect between China's crude oil futures market and the stock market, which is manifested in the asymmetry of market-to-market extreme risk spillover as well as asymmetry of upside and downside risk; and (3) there is obvious variability between the extreme risk of China's crude oil futures market and Shanghai and Shenzhen stock markets in terms of extreme risk spillover intensity in China's crude oil futures market and the Shanghai and Shenzhen stock markets are significantly different, with the former having a greater risk spillover intensity than the latter.

1. Introduction

The monitoring and control of financial market risk have long been a shared concern for investors, policymakers, and academics. With the intensification of economic and financial globalization, risk spillovers among financial assets have become increasingly evident. Fluctuations in one market are often rapidly transmitted to others, triggering chain reactions across markets (Yang et al., 2020[1]). Extreme risks, though low in probability, demand particular attention due to their potential to cause severe capital losses and systemic financial crises, posing significant threats to economic stability. Accurate identification and measurement of such risks are essential for reducing market uncertainty and safeguarding the financial system. As a strategically significant commodity, crude oil combines energy, commodity, and financial characteristics. Its price volatility profoundly impacts the global economy, politics, and financial systems. Extreme fluctuations in crude oil prices can disrupt capital markets, leading to sharp declines in the prices of financial assets such as stocks, bonds, and foreign exchange, thereby intensifying financial risks and threatening economic growth. Against the backdrop of China's growing dependence on imported crude oil, ensuring domestic financial market stability and energy security has become increasingly urgent. The launch of the Shanghai crude oil futures in 2018, now the world's third-largest crude oil futures market, provides market participants with an essential risk management tool.

From both theoretical and practical perspectives, potential risk transmission mechanisms exist between the crude oil and stock markets. Crude oil price fluctuations influence stock returns through channels such as corporate production costs, investment profitability, and macroeconomic expectations, while stock market volatility can affect investor preferences and price expectations for crude oil. Additionally, market speculation, investor sentiment, and news effects further intensify the interconnections between these markets. However, most existing research focuses on conditional mean and variance, which fail to capture tail dependencies under extreme conditions, particularly the asymmetry between upside and downside risks. Tail dependencies, which reflect extreme risks during upward (upside risk) and downward (downside risk) market movements, often exhibit distinct transmission characteristics under different scenarios. Neglecting tail regions has led to significant errors in predicting and managing cross-market risk contagion, especially during sharp declines or extreme volatility. To address this gap, this study employs an AR-GARCH-Copula model with a skewed t-distribution to capture nonlinear and tail dependencies between China's crude oil futures and stock markets. The CoVaR (Conditional Value at Risk) method is further applied to quantify the magnitude and direction of risk spillovers under extreme scenarios. By identifying asymmetric relationships under extreme conditions, this study provides new insights into the interaction mechanisms between these markets during periods of turbulence and offers valuable guidance for regulators, investors, and corporate managers. The findings also contribute theoretical and empirical support for safeguarding China's energy and financial security.

2. Literature Review

In recent years, significant theoretical and empirical research has been conducted on the relationship between the stock and crude oil markets, both domestically and internationally. However, due to the relatively late introduction of China's crude oil futures market, most studies have focused on the relationship between domestic and international stock markets and global crude oil prices. The current research on the crude oil market and stock market can be broadly classified into two main categories: one examining the correlation between the two markets, and the other exploring the spillover effects between them. Although there is a considerable amount of research on the correlation between crude oil futures and stock markets, no consensus has been reached regarding their relationship.

2.1. Correlation between Crude Oil Market and Stock Market

Some scholars argue that there is a positive relationship between the crude oil futures market and the stock market. For instance, Nadal, Szklo, and Lucena (2012) [2], using the DCC-GARCH model, analyzed the time-varying impact of supply and demand oil shocks on the correlation between oil price changes and stock market returns. Their findings suggested that during periods of peak financial market volatility (e.g., late 2007 to 2008), demand shocks had a positive effect on the correlation between oil prices and stock market returns. Similarly, Chen and Huang (2015) [3], employing the quantile regression model, found a positive linkage between the international crude oil market and global stock markets, with the relationship becoming more pronounced under extreme stock market conditions.

On the other hand, some scholars believe there is a negative relationship between the crude oil futures market and the stock market. Antonakakis and Filis (2012) [4], using the DCC-GARCH model, studied the time-varying correlation between oil prices and the stock markets of oil-importing countries (the United States, the United Kingdom, and Germany) and oil-exporting economies (Canada and Norway). Their results suggested that in the United States, falling oil prices could have a positive effect on stock prices, as lower production costs and improved profitability benefit companies. Raza et al. (2017) [5] investigated the asymmetric effects of gold, oil prices, and their volatility on emerging market stock prices, concluding that oil price fluctuations had a negative impact on all emerging markets.

2.2. Risk Spillovers between the Crude Oil Market and the Stock Market

Numerous studies have focused on the risk spillover effects between the crude oil market and stock market, employing models such as Granger causality, GARCH, Copula, quantile regression, expected shortfall (ES), value at risk (VaR), and CoVaR, often in combination. For example, Ji et al. (2014) [6], using a time-varying Copula-GARCH-CoVaR model, examined the dynamic interdependencies between different types of oil shocks and stock returns in BRICS countries. Their study found that stock returns in BRICS countries exhibited a dynamic relationship with oil shocks, which varied over time and differed by type of shock. Caporale et al. (2013) [7] constructed a VAR-GARCH model to analyze the dynamic relationship between international oil prices and stock market returns in China, finding a significant dynamic impact of oil prices on Chinese stock markets. Almaadid et al. (2018) [8] used the GARCH-BEKK model to empirically analyze the relationship between oil price volatility and stock market returns in GCC countries. Huang Shupei et al. (2016) [9] applied wavelet analysis and vector autoregressive models to study the effects of supply- and demand-driven oil price fluctuations on stock markets at different time scales. Chen et al. (2015) [10] used the VaR model to analyze the correlation between China's crude oil futures prices and stock price indices. Wang and Han (2017) [11] employed a GARCH-time-varying Copula-CoVaR model to investigate risk spillovers between the international oil market and stock markets, finding bidirectional risk spillovers. Zhong and Li (2019) [12] used time-varying POT models and the DY spillover index to examine the tail risk spillovers between international oil prices, macroeconomic variables, and the Chinese stock market. Kou et al. (2020) [13] employed the DCC-MGARCH model to investigate the dynamic relationship between the global oil market and stock markets, the impact of major shocks on their correlation, and the quantification of risk spillovers using CoVaR. Furthermore, a significant body of research has focused on the asymmetric nature of risk spillovers between the crude oil market and stock market. Researchers such as Reboredo et al. (2014) [14], Bittlingmayer (2017) [15], Sim et al. (2014) [16], Park and Ratti (2008) [17], and Mohanty et al. (2017) [18] have employed nonlinear models to study the asymmetric effects of oil price volatility on stock market returns. Gao and Gao (2016) [19] used the STR model to examine the nonlinear and asymmetric impacts of China’s crude oil futures prices on the stock market.

This body of research highlights the complexity of the relationship between the crude oil market and stock markets, with differing views on whether their relationship is positive or negative, and emphasizes the importance of understanding the dynamic and asymmetric nature of risk spillovers in financial markets. Based on the comprehensive and comparative analysis of existing literatures, the innovation of this paper is mainly reflected in the following aspects: (1) This paper not only analyzes the Shanghai stock market, but also covers the Shenzhen stock market, which makes the research on the Chinese stock market more comprehensive and in-depth; (2) In the fitting process of Copula function, this paper includes a variety of time-varying Copula functions into the research framework, and finally selects the optimal time-varying Copula function to fully describe the possible time-varying, nonlinear and tail dependence relationships between markets; (3) By using CoVaR, △CoVaR and %CoVaR methods, this paper systematically measures the asymmetric extreme risk spillover effect between China's crude oil futures and the stock market, fully considers the upside and downside risks, and makes a detailed quantitative analysis of both the upside and downside risks. To sum up, this paper is significantly innovative in conducting a comprehensive quantitative measurement of the asymmetric extreme risk spillover effect and its characteristics in China's crude oil futures market and stock market, and provides a more detailed and comprehensive analysis than existing studies.

3. Methodology

3.1. Marginal distribution modelling

Financial variables commonly exhibit characteristics such as autocorrelation, conditional heteroscedasticity, and leptokurtosis. To address these issues and investigate asymmetries, this study primarily constructs an AR(p)-GARCH(m,n) model to obtain the respective marginal distributions, as expressed in the following equation:

$$r_t = \phi_0 + \sum_{j=1}^{p} \phi_j r_{t-j} + \varepsilon_t \tag{1}$$

$$\varepsilon_t = \sigma_t z_t \tag{2}$$

$$\sigma_t^2 = \omega + \sum_{h=1}^{m} \alpha_h \varepsilon_{t-h}^2 + \sum_{k=1}^{n} \beta_k \sigma_{t-k}^2 \tag{3}$$

Where z_t denotes the standardized residual which follows the skewed Student-t distribution with ε_t and σ_t^2 refer to the error term and the conditional variance, respectively.

3.2. Copula modelling

Before applying the Copula model, it is necessary to perform a probability integral transformation on the residuals obtained in the previous step, resulting in transformed residuals that follow a uniform distribution on the interval [0, 1]. According to Sklar (1959) [20], there exists a Copula function such that the following equation holds:

$$F_{XY}(x, y) = C(F_X(x), F_Y(y)) \tag{4}$$

When $F_X(x)$ and $F_Y(y)$ are continuous functions, let u= $F_X(x)$ and v = $F_Y(y)$. The Copula function C(u,v) uniquely determines a bivariate joint distribution function, where u and v follow a uniform distribution on the interval [0, 1]. Denoting the Copula density function as c(u,v); the joint density function $f_{XY}(x, y)$ can be expressed as:

$$f_{XY}(x, y) = c(u, v) f_X(x) f_Y(y) \tag{5}$$

$$c(u, v) = \frac{\partial^2 C(u,v)}{\partial u \partial v} \tag{6}$$

Where $f_X(x)$ and $f_Y(y)$ represent the marginal density functions of variables X and Y, respectively, with $f_X(x) = F_X'(x)$ and $f_Y(y) = F_Y'(y)$. There are various types of Copula functions, each exhibiting distinct characteristics of dependence structure. In this study, we will consider the principles of minimizing the AIC value, minimizing the BIC value, and maximizing the likelihood function to select the optimal Copula model for describing the tail dependence between the sequences.

3.3. CoVaR modelling

VAR and CoVaR are two commonly used risk metrics to measure the level of risk in a financial asset or portfolio. VaR is calculated using the formula:

$$\Pr\{R_t \leq VaR_{\alpha,t}\} = \alpha \tag{7}$$

Where 1-α is the confidence interval and R_t is the return of the financial asset or portfolio in period t. However, VaR only provides the maximum loss at a given confidence level and does not take into account losses in excess of VaR, which in reality may have a much greater impact.The CoVaR approach proposed by Adrian and Brunnermeier (2016) [21] measures the average loss in excess of VaR, provides a more comprehensive measure of risk, and more accurately reflects the extreme risk spillovers under risk. CoVaR is the maximum loss faced by other financial institutions or portfolios when a financial institution or portfolio suffers an extreme loss for a given holding period and confidence level, and is calculated as:

$$Pr\{R_t^j \leq CoVaR_{\alpha,t}^j | R_t^i \leq VaR_{\alpha,t}^i\} = \alpha \tag{8}$$

The difference between the value at risk to which financial market j is exposed when financial market i is in an extreme risk state and when it is risk-free is the degree of risk contribution of market i to market j, which expresses the risk spillover of market i to market j, and is calculated as follows:

$$\Delta CoVaR_{\alpha,t}^j = CoVaR_{\alpha,t}^{j|i} - VaR_{\alpha,t}^{j|i} \tag{9}$$

For the phenomenon that the unconditional value at risk of different financial markets varies too much, it will become a standardized scale by de-measurement, which will be calculated as follows:

$$\%CoVaR_{\alpha,t}^{j|i} = \frac{\Delta CoVaR_{\alpha,t}^{j|i}}{VaR_{\alpha,t}^j} \times 100\% \tag{10}$$

With standardization, risk spillovers between financial markets can be measured more accurately.

4. Empirical Results and Analysis

4.1. Data

In this paper, Shanghai crude oil futures (SC) is used as a proxy variable for China's crude oil futures market; considering the existence of Shanghai and Shenzhen stock markets, the Shanghai Composite Index (SH) and the Shenzhen Component Index (SZ) are used as proxies for China's stock market, respectively. The sample interval of this paper is from March 27, 2018 to December 13, 2024, and the daily closing prices are selected, with a total of 1,633 sets of data. The daily closing price data are obtained from Wind database. In order to obtain the smooth series, the above three sets of series are subjected to the logarithmic difference process, which is calculated as:

$$R_{i,t} = 100 \times (\ln P_t - \ln P_{t-1}) \tag{11}$$

$R_{i,t}$ represent the daily logarithmic returns of the market at moment t. P_{t-1} and P_t are the daily closing prices of market i at moment t-1 and moment t, respectively. The obtained logarithmic returns of Shanghai crude oil futures, Shanghai Composite Index and Shenzhen Composite Index are denoted as RSC, RSH and RSZ, respectively.

4.2. Descriptive Statistics

A descriptive statistical analysis and correlation tests were performed on the three sets of return series obtained from the aforementioned processing. The results are summarized in Table 1.

Table 1: Descriptive Statistics.

	RSC	RSH	RSZ
Mean	0.015	0.005	0.001
Max	-14.321	-8.039	-8.825
Min	15.864	7.755	10.135
Std. dev.	2.588	1.120	1.481
Skewness	-0.235	-0.289	-0.021
Kurtosis	6.708	9.161	7.806
J-B	949.960(0.000)	2603.609(0.000)	1571.722(0.000)
ADF	-42.025(0.000)	-40.025(0.000)	-38.736(0.000)
LB	24.630(0.216)	28.062 (0.108)	20.752(0.412)
ARCH-LM	123.792(0.000)	136.638(0.000)	137.343(0.000)

Note: J–B is the Jarque–Bera test for normality.LB is the Ljung–Box test of serial correlation. *, **, and *** indicate significant at the 10%, 5%, and 1% levels, respectively. Values in () represent p-values.

As shown in Table 1, the mean return of the Shanghai crude oil futures market (RSC) is higher than those of the Shanghai stock market (RSH) and Shenzhen stock market (RSZ), while its standard deviation is also larger. This indicates that the Shanghai crude oil futures market exhibits greater volatility compared to the stock markets. The skewness of RSC (-0.235), RSH (-0.289), and RSZ (-0.021) are all negative, suggesting left-skewed distributions. However, the skewness values are relatively small, indicating that the distributions exhibit a degree of symmetry. The kurtosis values for RSC (6.708), RSH (9.161), and RSZ (7.806) are all greater than 3, indicating that the distributions are characterized by heavy tails and a higher probability of extreme values. The Jarque-Bera (JB) test results confirm that, at the 1% significance level, the null hypothesis of normality is rejected for all three return series, indicating that they do not follow a normal distribution. The Augmented Dickey-Fuller (ADF) test further verifies that all three return series are stationary. Additionally, the ARCH-LM test results show that, at the 1% significance level, the null hypothesis of no ARCH effects is rejected for all three return series. Based on these findings, a GARCH model will be employed to construct the corresponding marginal distribution model.

4.3. Marginal distribution

In this study, the AR(1)-GARCH(1,1)-skewed-t model is employed to model the return series of the Shanghai Composite Index, the Shenzhen Component Index, and the Shanghai Crude Oil Futures, respectively. The AR-GARCH model effectively addresses the autocorrelation and conditional heteroskedasticity commonly observed in financial time series, while the skewed-t distribution is well-suited to capturing the heavy-tailed and asymmetric features of financial data. By applying these models, the corresponding marginal distributions are obtained, and the parameter estimates are presented in Table 2.

Table 2: Estimates for the marginal distribution models.

	RSC	RSH	RSZ
Mean equation			
φ_0	0.049	-0.006	-0.011
φ_1	-0.041	-0.023	-0.005
Variance equation			
ω	0.227**	0.049***	0.078***

Continued Table 2: Estimates for the marginal distribution models.

	RSC	RSH	RSZ
α	0.098***	0.073***	0.077***
β	0.875***	0.887***	0.886***
ν	4.528***	5.203***	7.115***
LB	18.507(0.554)	31.182(0.053)	18.523(0.553)
ARCH-LM	10.285(0.416)	10.758(0.377)	13.377(0.203)
K-S	0.021(0.492)	0.015(0.829)	0.012(0.972)

Table 2 reveals that the GARCH component coefficients (β) for all series are statistically significant, indicating that the volatility of China's crude oil futures market and stock markets is strongly influenced by the volatility of the previous period, demonstrating clear volatility clustering. Furthermore, the degree of freedom parameter (υ) is significant at the 1% level, confirming the presence of skewness and fat-tailed characteristics in all series. The p-values of the Ljung-Box (LB) test and the ARCH-LM test exceed 0.05, indicating that the residual series obtained after GARCH modeling exhibit no autocorrelation or conditional heteroskedasticity. Additionally, the Kolmogorov-Smirnov (K-S) test confirms that the transformed series follow a uniform (0, 1) distribution.

In summary, the AR(1)-GARCH(1,1)-skewed-t model proves to be a suitable framework for modeling the return series of the crude oil futures and stock markets. This provides a solid foundation for the subsequent application of the Copula model to analyze the interdependence between these markets.

4.4. Copula estimation results and tail dependence analysis

Compared to the static Copula function, the time-varying Copula function is better suited to capturing the inter-market linkage information. Therefore, this paper selects the time-varying Gaussian Copule, time-varying t-Copula, time-varying Clayton Copule, and time-varying SJC Copula functions to fit the series obtained from the above treatments respectively. The fitting estimation results are presented in Table 3.

Table 3: Results of the evaluation of each time-varying Copula model.

Copula	RSC-RSH			RSC-RSZ		
	AIC	BIC	LogL	AIC	BIC	LogL
Gaussian-Copula	-60.512	-49.717	32.256	-24.193	-13.397	14.096
t-Copula	**-68.247**	**-52.054**	**37.123**	**-34.522**	**-18.329**	**20.261**
Clayton-Copula	-47.173	-30.980	26.586	-21.085	-4.892	13.543
SJC-Copula	-55.849	-23.464	33.925	-17.264	15.122	14.632

Note: AIC is the Akaike Information Criterion, BIC is the Bayesian Information Criterion, and LogL denotes the Great Likelihood.

In the process of Copula model fitting, the fitting effect of the model is generally judged by observing the values of AIC and BIC together with the great likelihood value, the smaller the values of AIC and BIC, and the larger the great likelihood value, the better the fitting effect is indicated. Observing the above fitting results, it can be seen that when the time-varying t-Copula function is used, the corresponding AIC and BIC values are the smallest, and the great likelihood values are the largest, which indicates that the time-varying t-Copula function is the optimal Copula function, so this paper adopts the time-varying t-Copula function for the Copula modeling of the three groups of sequences.

Table 4: Time-varying t-Copula parameter estimation results.

	RSC-RSH	RSC-RSZ
n	12.981***	11.679***
β_1	0.033	0.034
β_2	0.705***	0.702***

As shown in Table 4 above, except for β_1, which is not significant, all the other parameters are significant at the 1% significant level, indicating that the time-varying t-Copula model can better reflect the linkage between the Shanghai crude oil futures market and the stock market. In order to better analyze the linkage between the Shanghai crude oil futures market and the stock market, this paper uses the time-varying correlation coefficient graph to visualize the tail dependence between the two markets.

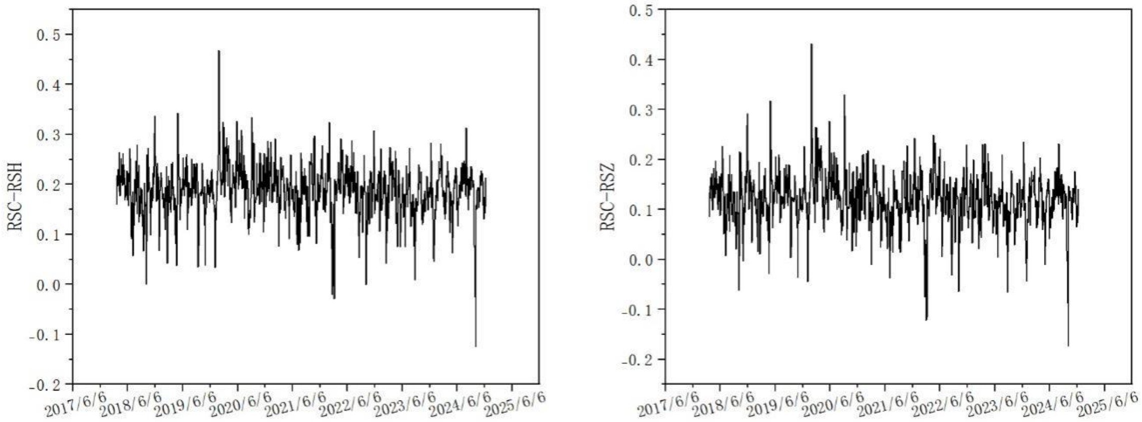

Figure 1: Time-varying correlation coefficients.

Figure 1 shows that the time-varying correlation coefficients between the Shanghai crude oil futures market and the Shanghai and Shenzhen stock markets are generally similar. In most cases, the Shanghai crude oil futures market and the stock markets (Shanghai and Shenzhen) exhibit a positive correlation, except for a brief period of negative dependence. This indicates that the two markets are prone to the phenomenon of "rising and falling together" in prices. Comparing the two graphs, it is evident that the positive correlation between the Shanghai crude oil futures market and the Shanghai stock market is significantly stronger than that with the Shenzhen stock market. Specifically, the correlation coefficient between the Shanghai crude oil futures market and the Shanghai stock market fluctuates around 0.2, while the correlation coefficient with the Shenzhen stock market fluctuates around 0.1. An analysis of the time-varying correlation coefficient plot reveals several periods of drastic changes in correlation, occurring near the end of 2019, late March 2022, and October 2024. These changes correspond to key events such as the COVID-19 outbreak, the Russia-Ukraine conflict, and the Federal Reserve's interest rate cuts. These findings suggest that extreme events can significantly influence the linkage between the two markets, heightening tail risks and generating extreme risk spillovers.

4.5. VaR estimation

This study measures the individual risk of the Shanghai crude oil futures market and the stock market using Value at Risk (VaR) and presents the corresponding upside and downside VaR charts for each market, as shown in Figure 2.

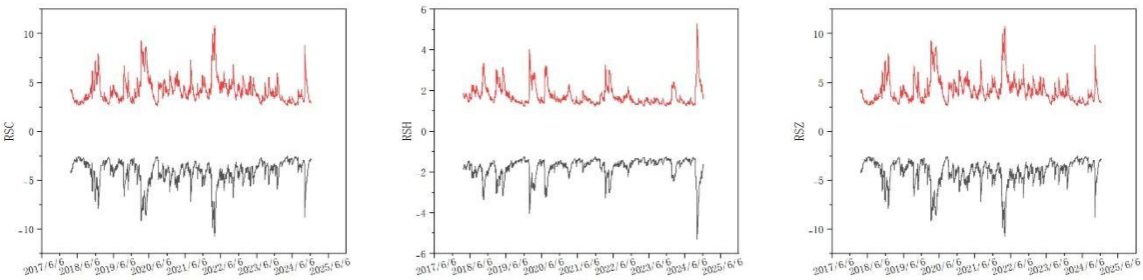

Figure 2: Upside and downside VaR.

A comparison of the three graphs in Figure 2 reveals that the upside VaR of the Shanghai crude oil futures market is higher than that of the Shanghai and Shenzhen stock markets, while its downside VaR is lower. This indicates that the inherent risks of the Shanghai crude oil futures market are greater than those of the stock markets. The higher self-risk in the Shanghai crude oil futures market can be attributed to differences in market maturity. Since China's crude oil futures market was established relatively recently, it is less mature compared to the stock market, which has a longer history. The trading and regulatory mechanisms in the crude oil futures market are not yet as developed or robust as those in the stock market. Additionally, the crude oil futures market offers relatively limited hedging instruments, whereas the stock market provides a variety of risk management tools, such as options, exchange-traded funds (ETFs), and other derivatives. Consequently, the limited risk management mechanisms in the crude oil futures market contribute to its higher self-risk compared to the stock market.

4.6. Empirical results of risk spillovers

In order to measure the extreme risk spillover effect between the Shanghai crude oil futures market and the stock market, this paper calculates the upside and downside risk spillover values CoVaR, ΔCoVaR, and %CoVaR respectively, and the results are shown in Table 5:

Table 5: Risk spillover between China’s crude oil futures market and stock market.

Direction	Upside			Downside		
	CoVaR	ΔCoVaR	%CoVaR	CoVaR	ΔCoVaR	%CoVaR
RSC→RSH	2.519	0.798	46.98%	-2.512	-0.780	45.57%
RSC→RSZ	2.814	0.516	22.76%	-2.823	-0.505	22.01%
RSH→RSC	4.487	0.331	8.20%	-4.393	-0.333	8.47%
RSZ→RSC	4.485	0.285	7.02%	-4.390	-0.288	7.27%

First, vertically, there is a two-way risk spillover effect between the Shanghai crude oil futures market and the stock market, and this effect shows obvious asymmetry. Specifically, the Shanghai crude oil futures market's upside and downside risk premiums (%CoVaR) to the Shanghai market are both about 46%, while the Shanghai crude oil futures market's upside and downside risk premiums to the Shenzhen market are about 22%. In contrast, the upside and downside risk premiums of Shanghai market and Shenzhen market to Shanghai crude oil futures market are around 7%~8% respectively. This suggests that there is an asymmetric extreme risk spillover effect between the

Shanghai crude oil futures market and the stock market, as evidenced by the fact that the extreme risk spillover intensity of the Shanghai crude oil futures market to the stock market is significantly higher than that of the stock market to the Shanghai crude oil futures market. In addition, the two-way extreme risk spillover intensity between Shanghai market and Shanghai crude oil futures market is also significantly higher than the two-way extreme risk spillover intensity between Shenzhen market and Shanghai crude oil futures market.

The reasons for this phenomenon include the following. First, volatility differences and leverage effects. The crude oil futures market is usually characterized by higher volatility and tends to fluctuate more than the stock market due to changes in supply and demand, geopolitical and macroeconomic factors. As a result, the volatility of the crude oil futures market has a more significant impact on the stock market. In addition, the leverage effect is prevalent in futures trading, i.e., investors only need to pay margin to participate in the transaction, and this leverage effect further amplifies the price volatility of the crude oil market and enhances its risk-transferring ability to the stock market. Second, the global economic impact. Crude oil, as an important economic indicator, is widely regarded as a barometer of global economic activity. Fluctuations in crude oil prices reflect changes in global economic activity, directly affecting the production costs and profit expectations of enterprises, which in turn have a significant impact on the stock market. In particular, rising crude oil prices may trigger rising inflationary expectations, affecting the monetary policies of central banks and, in turn, the performance of the stock market. Finally, investor behavior. Speculators in the crude oil market react sensitively to price fluctuations, leading to further amplification of the impact of crude oil market volatility on the stock market.

Secondly, horizontally, the risk spillover from Shanghai crude oil futures market to stock market and stock market to Shanghai crude oil futures market are also asymmetric, which is specifically reflected in the different performance of upside risk and downside risk. Specifically, the upside %CoVaR and downside %CoVaR of the Shanghai crude oil futures market to the Shanghai stock market are 46.98% and 45.57%, respectively; and the upside %CoVaR and downside %CoVaR to the Shenzhen stock market are 22.76% and 22.01%, respectively, which indicates that the spillover effect of the Shanghai crude oil futures market on the upside risk of the Shanghai crude oil futures market is larger than the downside risk of the Shenzhen and Shanghai stock markets, respectively. That is, the extreme rise in crude oil prices triggers a sharp rise in stock market prices than the extreme fall in crude oil prices triggers a sharp fall in stock market prices. Meanwhile, the upside %CoVaR and downside %CoVaR of Shanghai market to Shanghai crude oil futures market are 8.20% and 8.47%, while the upside %CoVaR and downside %CoVaR of Shanghai market to Shanghai crude oil futures market are 7.02% and 7.27%, which indicate that the downside risk of Shenzhen and Shanghai markets to the Shanghai crude oil futures market is slightly larger than the upside risk.

The Shanghai crude oil futures market poses more upside than downside risks to the stock market, which may stem from the following reasons. First, rising crude oil prices are usually associated with economic growth, especially during periods of economic recovery or expansion, and high oil prices reflect increased demand, which is positive for stocks in related industries (e.g., energy, transportation, etc.), which in turn drives up stock prices. In times of high market sentiment, investors are more likely to be motivated by upside risks. On the contrary, while a fall in crude oil prices may reduce the production costs of the relevant companies, its negative expectations on economic growth may make investors more cautious in the face of downside risks, leading to a relatively mild reaction from the stock market. As a result, the spillover effect of upside risks to crude oil prices on the stock market is more pronounced.

5. Conclusions

Taking Shanghai crude oil futures market and Shanghai and Shenzhen stock markets as research objects, this paper explores the extreme risk spillover effect between China's crude oil futures market and stock market. The AR-GARCH-Copula model obeying the skewed t-distribution is used to characterize the volatility between the two markets and capture the tail dynamic dependence, and then the CoVaR model is constructed to empirically analyze the extreme risk spillover effect. The results of the study show that: firstly, there is a significant positive tail dependence between China's crude oil futures market and the stock market, which is reflected by the dynamic dependence coefficients of the two markets being greater than 0; secondly, there is an asymmetric bi-directional extreme risk spillover effect between China's crude oil futures market and the stock market. Specifically, on the one hand, the overall risk spillover intensity of the crude oil futures market to the stock market is significantly higher than that of the stock market to the crude oil futures market; on the other hand, the upside risk spillover value of the crude oil futures market to the stock market is higher than the downside risk spillover value, while the downside risk spillover value of the stock market to the crude oil futures market is slightly higher than the upside risk spillover value. Finally, there is a significant difference in the intensity of risk spillover between China's crude oil futures market and Shanghai and Shenzhen markets, which is manifested in the fact that the bidirectional risk spillover effect between the crude oil futures market and Shanghai market are both significantly higher than the bidirectional risk spillover effect with Shenzhen market.

Based on the above conclusions, this paper puts forward the following suggestions: first, given that China's crude oil futures market, as an emerging market, has relatively low market maturity and high risk volatility, the government should accelerate the improvement of market trading mechanism and system construction, formulate effective risk prevention measures in advance, and build a sound risk prevention system to prevent the further proliferation of extreme risks in the crude oil futures market. Secondly, for investors, the two-way asymmetric extreme risk spillover effect between China's crude oil futures market and the stock market can provide reference for the construction of their investment portfolio. However, given the asymmetric nature of upside and downside risks, investors should adjust their investment strategies in a timely manner according to the risk conditions between the markets and respond flexibly to minimize the investment losses that may be caused by extreme risks. Finally, policy-making authorities and financial regulators should comprehensively consider the linkage between the crude oil market and the stock market and their extreme risk spillovers when conducting macro-control and risk management. In order to maintain the smooth operation of the two markets and avoid a single policy leading to sharp fluctuations in one of them, the Government and regulators should adopt diversified policy measures to minimize risk contagion, maintain the overall stability of the financial market and reduce the complexity of financial risk management.

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