Geoscience and Remote Sensing (2025)
Clausius Scientific Press, Canada

DOI: 10.23977/geors.2025.080102
ISSN 2523-2592 Vol. 8 Num. 1

Comparison and Accuracy Analysis of Several Satellite Clock Bias Interpolation Methods for Precise Point Positioning

Chaopan Yang[1], Ye Yu[1,*], Weimin Jia[1], Guodong Jin[1], Yihong Li[1], Wei Jin[1], Jianwei Zhao[1]

[1]*Rocket Force University of Engineering, Xi'an, 710025, Shaanxi, China*
yuye115@mails.ucas.edu.cn
[*]*Corresponding author*

Keywords: Satellite clock bias (SCB); linear interpolation; cubic spline interpolation; Lagrange interpolation; interpolation accuracy

Abstract: The satellite clock bias (SCB) with high sampling rate is needed in precise point positioning, and the sampling interval of SCB provided by the analysis center is relatively large, so SCB with large interval must be interpolated and encrypted by interpolation methods. Three interpolation methods are used to interpolate SCB from 5-minute sampling intervals to 30-second sampling intervals, and the interpolated results are compared and analyzed with the published SCB for 30-second sampling intervals. The results show that the accuracy of linear interpolation method is the highest, followed by cubic spline interpolation method and then Lagrange interpolation method. In addition, the interpolation accuracy of the three interpolation methods is the highest for GPS IIF and IIIA satellites launched in recent years, followed by IIR-M satellites launched in the middle of the period, and then IIR satellites launched in the early period.

1. Introduction

Precise Point Positioning (PPP) technology is a high-precision positioning technology in the field of satellite navigation and positioning, which takes the data collected by a single receiver as the main observation value, and uses post-processing precision ephemeris as well as precision satellite clock bias (SCB) to carry out PPP, which has the advantages of low cost of construction, not restricted by the distance of action, flexible operational mobility, and many advantages [1-3]. To achieve centimeter-level positioning accuracy in PPP technology, precise satellite orbits and clock bias must be incorporated into the equations as known parameters for position determination. At present, the GPS ultra-fast orbit products provided by the International GNSS Service (IGS) and its affiliated analysis centers are able to meet the requirements of PPP technology [4, 5]. The sampling rate of the receiver in actual positioning is generally 30s, 15s or even denser, but the sampling interval of the SCB provided by IGS is 5 min. therefore, certain interpolation methods must be adopted to encrypt or fit the precision interpolation of SCB at larger intervals, so as to obtain the precision SCB at any moment, which is crucial for improving the accuracy of the PPP technology [6-8].

Currently, the commonly used SCB interpolation methods include linear interpolation, Lagrange interpolation, generalized extended approximation, Newton interpolation, sliding polynomial interpolation, Hermite interpolation, spline function interpolation, and Chebyshev's fitting and least squares curve fitting [9-16]. Literature [9] introduced the generalized delay interpolation method into SCB interpolation for the first time, and compared and analyzed the accuracy with Lagrange interpolation and cubic spline interpolation, and got the conclusion that the method has high interpolation accuracy. Literature [10] used Lagrange interpolation and Chebyshev's fitting method respectively to interpolate the precision SCB of GPS satellites from 15 min to 5 min and compared the interpolation accuracy, and found that the interpolation result of Chebyshev's fitting method has higher accuracy. Literature [11] utilized linear interpolation, Lagrange interpolation and quadratic Hermite interpolation to interpolate the precision SCB of GPS satellites from 5min to 30s and compared the interpolation accuracy, and found that the quadratic Hermite interpolation method has the highest interpolation accuracy considering the clock speed. These studies only analyzed the interpolation accuracy of 15min and 5min precision SCB products released by IGS analysis center, and did not analyze the interpolation accuracy of 5min precision SCB products released by GNSS analysis center of Wuhan university in China.

In order to fully analyze the interpolation accuracy and practical effect of linear interpolation, cubic spline interpolation and Lagrange interpolation, the after-the-fact precision SCB product with a sampling interval of 5min released by the GNSS analysis center of Wuhan University is used, and the three kinds of interpolation methods are applied to interpolate the precision SCB with a sampling interval of 5min to the SCB with a sampling interval of 30s, and some useful conclusions are obtained by comparing and analyzing the results of the interpolation and that of the precision SCB with a sampling interval of 30s released by the GNSS analysis center of Wuhan University. The interpolation results are compared and analyzed with the precision SCB of 30s sampling interval released by Wuhan university GNSS analysis center, and some useful conclusions are drawn.

2. Principle of Satellite Clock Bias Interpolation Algorithm

2.1 Linear Interpolation

Linear interpolation [11] is an interpolation method for one-dimensional data, which allows numerical estimation based on the two data adjacent to the left and right of the point to be interpolated in a one-dimensional data sequence. The most important feature of the linear interpolation method is that its formula is simple and easy to program and implement. Assuming that the SCB corresponding to the moments t_0 and t_1 are $f(t_0)$ and $f(t_1)$ respectively, the interpolation polynomials through the points $(t_0, f(t_0))$ and $(t_1, f(t_1))$ can be expressed as follows:

$$L_1(t) = l_0(t) f(t_0) + l_1(t) f(t_1) \quad (1)$$

where $l_0(t)$, $l_1(t)$ are linear interpolation basis functions with respect to the moments t_0 and t_1, which can be expressed as $l_0(t) = \frac{t-t_1}{t_0-t_1}, l_1(t) = \frac{t-t_0}{t_1-t_0}$.

The interpolating basis functions $l_0(t)$ and $l_1(t)$ are obtained by substituting them into Eq. (1):

$$L_1(t)=\frac{t-t_1}{t_0-t_1}f(t_0)+\frac{t-t_0}{t_1-t_0}f(t_1) \tag{2}$$

The SCB data can be linearly interpolated using equation (2).

2.2 Cubic Spline Interpolation

Assuming that cubic spline interpolation [12-13] is carried out for the SCB corresponding to $t_0,t_1,\cdots,t_{n-1}$ time is $f(t_0),f(t_1),\cdots,f(t_{n-1})$,respectively, it is necessary to construct the spline function $s_k(t)$ firstly, and let the expression of this spline function on the interval $[t_k,t_{k+1}]$ can be expressed as follows:

$$s_k(t)=s_{k1}(t-t_k)^3+s_{k2}(t-t_k)^2+s_{k3}(t-t_k)+s_{k4} \tag{3}$$

where $k=0,1,\ldots,n-1$.

The function $s(t)$ needs to fulfill the following two conditions:

(1) Interpolation condition: $s(t_k)=f(t_k),k=0,1,\cdots,n-1$. This ensures that the spline function passes through the given data point (x_k,y_k).

(2) Continuity condition: at the inner node, the spline function $s_k(t)$ and its first-order and second-order derivatives $s'_k(t)$、$s''_k(t)$ should be kept continuous, i.e., $s_{k-1}(t_k)=s_k(t_k)$、$s'_{k-1}(t_k)=s'_k(t_k)$、$s''_{k-1}(t_k)=s''_k(t_k)$, where $k=0,1,\ldots,n-1$. The above conditions ensure the smoothness and continuity of the spline function, so that the curve obtained by interpolating between the SCB data points is both smooth and can meet the interpolation requirements.

2.3 Lagrange Interpolation

Lagrange interpolation [10, 14-16] is a classical polynomial interpolation method that allows a polynomial to be constructed to approximate a certain function based on a given set of data points. Assuming that for a given $n+1$ different interpolating nodes $t_0,t_1,\cdots,t_n$ whose corresponding SCB are $f(t_0),f(t_1),\cdots,f(t_n)$, a n sub-interpolating polynomial can be constructed as:

$$L_n(t)=\sum_{k=0}^{n}f(t_k)l_k(t) \tag{4}$$

where $l_k(t),k=0,1,\cdots n$ are all polynomials of the n order and are called Lagrange basis function or interpolating basis function.

The interpolating basis functions can be simplified as:

$$P(t_i)=f(t_i),\quad i=0,1,\cdots,n \tag{5}$$

$$\sum_{k=0}^{n}f(t_k)l_k(t_i)=f(t_i),i=0,1,\cdots,n \tag{6}$$

$l_k(t_i)$ need to be met:

$$l_k(t_i)=\begin{cases}1, & i=k\\ 0, & i\neq k\end{cases} \tag{7}$$

This follows from the fact that all n nodes $t_i(i\neq k)$ are zeros of a polynomial $l_k(t)$ of the n order:

$$l_k(t)=A_k\prod_{\substack{i=0\\ i\neq k}}^{n}(t-t_i) \tag{8}$$

where, A_k is the coefficient to be determined, which is then obtained from equation (7):

$$A_k=\frac{1}{\prod_{\substack{i=0\\ i\neq k}}^{n}(t_k-t_i)} \tag{9}$$

From this, the final result can be obtained as:

$$L_n(t)=\sum_{\substack{i=0\\ i\neq k}}^{n}\frac{(t-t_i)}{(t_k-t_i)},k=0,1,\cdots,n \tag{10}$$

The SCB data can be Lagrange interpolated using equation (10).

3. Experiment and Analysis

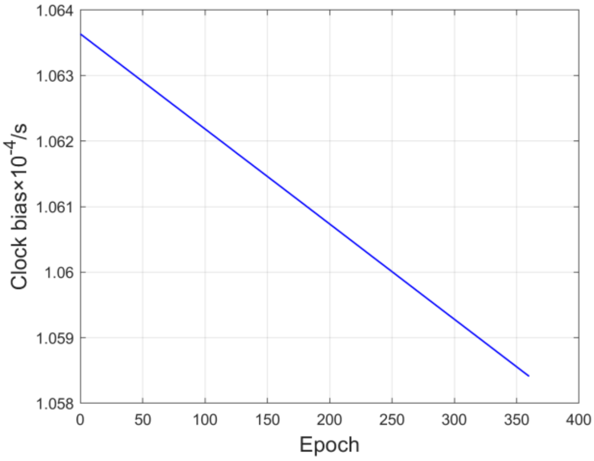
3.1 Experimental Data Sources

In order to fully analyze the accuracy and practical effect of the three SCB interpolation methods in this paper, the after-the-fact precision SCB data with sampling intervals of 5min and 30s on day 0 of week 2023 released by GNSS analysis center of Wuhan university are used to conduct simulation experiments. There are more than 30 GPS satellites in orbit in this time period, and their on-board clocks are of the following five types: BLOCK IIR-Rb clock, BLOCK IIR-M-Rb clock, BLOCK IIF-Rb clock, BLOCK III-A-Rb clock and BLOCK IIF-Cs clock. Since the on-board clocks of the BeiDou system are basically the same as those of the GPS system, in order to make the results of the study provide some references for the BeiDou satellite navigation system of our country in the research of SCB interpolation and encryption, the satellites of GPS IIF-Rb PRN06, GPS IIR-M-Rb PRN07, GPS III-A-Rb PRN11, and GPS IIR-Rb PRN19 are randomly selected for the experiments. The clock bias data of these satellites are experimented. Their related information is shown in Table 1.

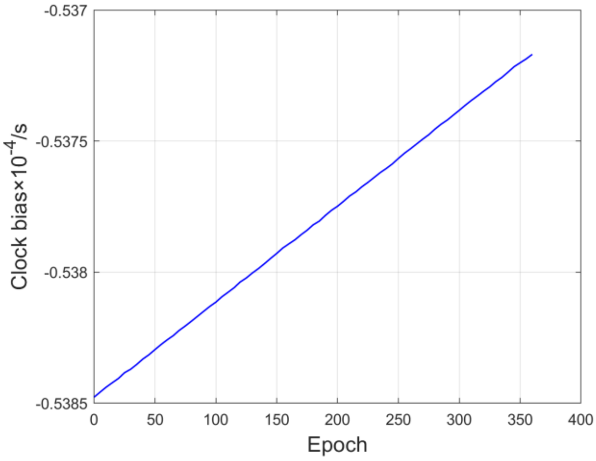
Table 1: Selected satellite related information.

Satellite number	Clock type	SVN	Orbital plane	NORAD	Launch time	Trends in clock bias
PRN 06	IIF-Rb	67	D4	39741	2014.5.17	positive monotonically decreasing
PRN 07	IIR-M-Rb	48	A4	32711	2008.3.15	monotonically increasing negative value
PRN 11	III-A-Rb	78	D5	48859	2021.6.17	negative value monotonically decreasing
PRN 19	IIR-Rb	59	C5	28190	2004.3.20	positive monotonically increasing

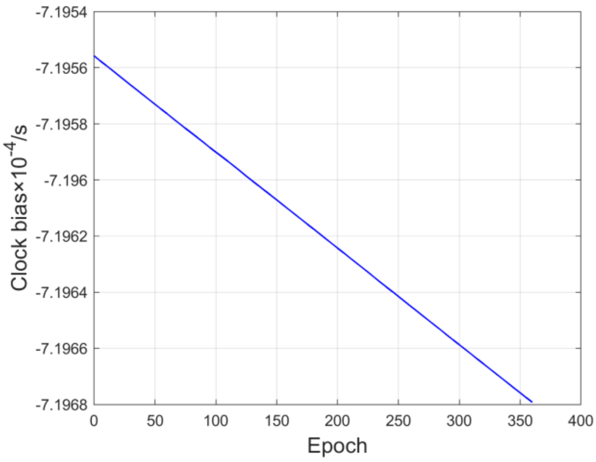
The variations of the precision SCB time series of these four satellites at the 6h 5min sampling interval before day 0 of GPS week 2023 are shown in Figure 1, with a positive monotonically decreasing trend in the clock bias time series of PRN01, a negative monotonically increasing trend in the clock bias time series of PRN07, and a negative monotonically decreasing trend in the clock bias time series of PRN11, and a negative monotonically decreasing trend in the clock bias time series of PRN19. The clock bias time series of PRN19 has a positive monotonically increasing trend. In addition, the selected satellites are fully representative of the GPS IIR and IIR-M satellites launched in the early period, the GPS IIF satellites launched in the middle period, and the GPS III-A satellites launched in the recent years.



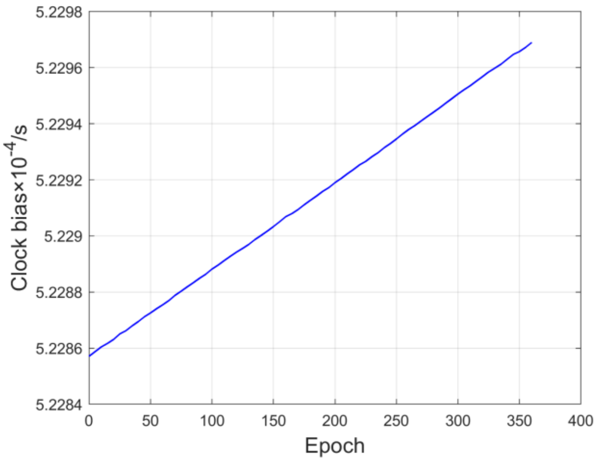
(a) PRN06



(b) PRN07



(c) PRN11



(d) PRN19

Figure 1: Chart of clock bias variation of PRN06, PRN07, PRN11 and PRN19 satellites.

3.2 Experimental Results and Analysis

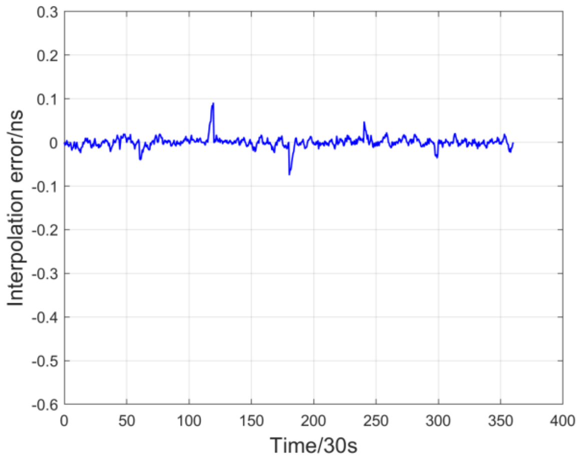
At present, the sampling intervals of the precision SCB products released by the GNSS analysis center of Wuhan university are 5min and 30s. In order to fully compare and analyze the accuracy of the three kinds of SCB interpolation models in this paper, the precision SCB with a sampling interval of 5min is used as the data to be interpolated at 6h before the 0th day of the week of GPS 2023, and the linear interpolation model, the cubic spline interpolation model, and the Lagrange interpolation model are established respectively to interpolate the 5min sampling interval to the 30s sampling interval SCB, the precision SCB of 5min sampling interval is interpolated to the SCB of

30s sampling interval. As this experiment uses the precision SCB products released by Wuhan university GNSS analysis center, its own error is less than 0.1ns, so it can be used as the true value, the SCB obtained by the interpolation model of the three SCB interpolation model is used as the calculated value, and the difference between the true value and the calculated value can be obtained as the error of the interpolation model. The Mean Squared Error (MSE) and Mean Absolute Error (MAE) (the specific calculation formula is shown in equations (11) and (12)) are used as the evaluation indexes to test the degree of interpolation effect of the three interpolation models. Among them, MSE is more sensitive to large errors because the square of the error amplifies the large error, and the smaller the value of MSE, the interpolation result of the model is more accurate. While MAE treats all errors equally and does not amplify or minimize any deviation, the smaller the value of MAE, the closer the interpolation result of the model is to the true value. If the MSE value of the interpolated model is large but the MAE value is relatively small, this may indicate that the model has some large anomalous errors. On the contrary, if the MAE value is large but the MSE value is relatively small, this may indicate that the model has a more uniform error distribution. The simultaneous use of these two-evaluation metrics provides a more complete picture of the model's performance on different types of errors as a way to compare the interpolation accuracy of these three interpolation models. See Figure 2-Figure 4 and Table 2 for details.

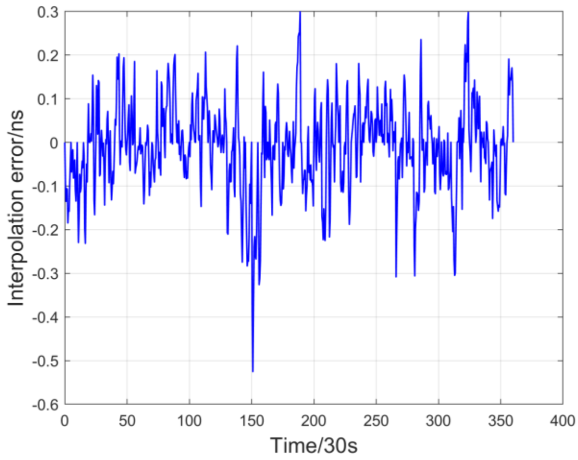
$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2 \tag{11}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|y_i - \hat{y}_i\right| \tag{12}$$

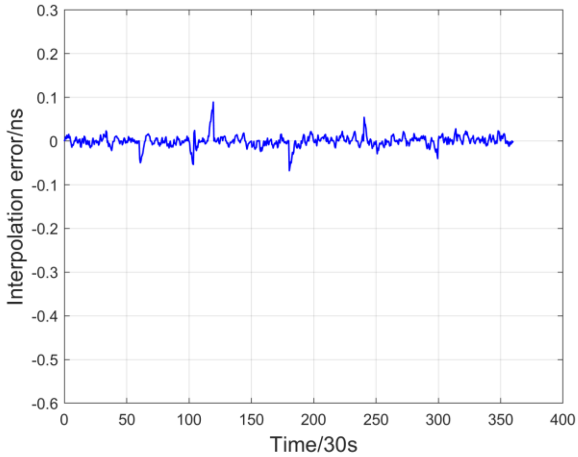
Figure 2 shows the variation of linear interpolation error for satellites PRN06, PRN07, PRN11 and PRN19, Figure 3 shows the variation of cubic spline interpolation error for satellites PRN06, PRN07, PRN11 and PRN19, Figure 4 shows the variation of Lagrange interpolation error for satellites PRN06, PRN07, PRN11 and PRN19, and Table 2 shows the satellite statistical results of the clock bias interpolation errors.



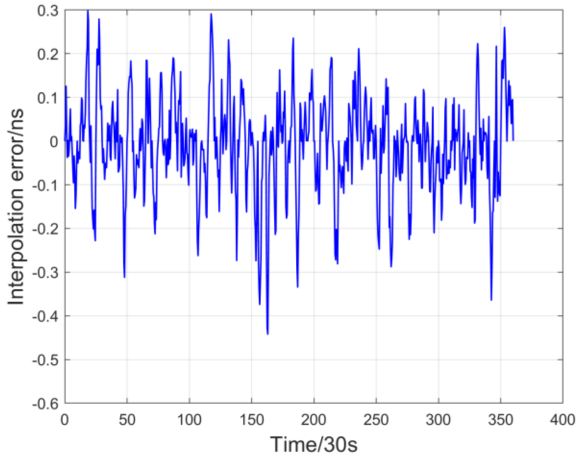
(a) PRN06



(b) PRN07

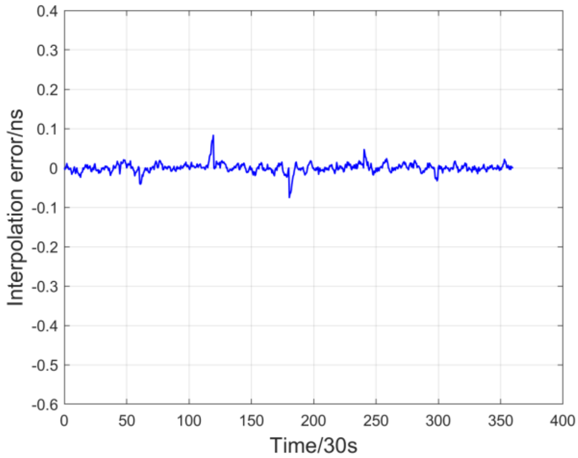


(c) PRN11

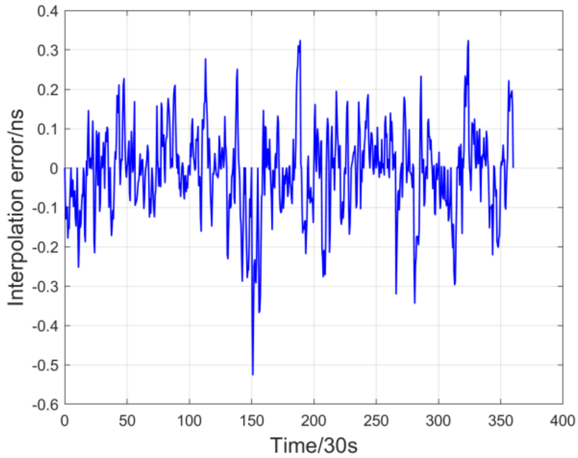


(d) PRN19

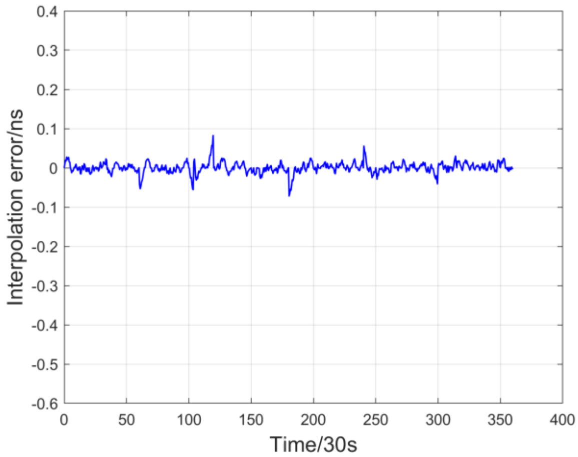
Figure 2: Linear interpolation errors of PRN06, PRN07, PRN11 and PRN19 satellites.



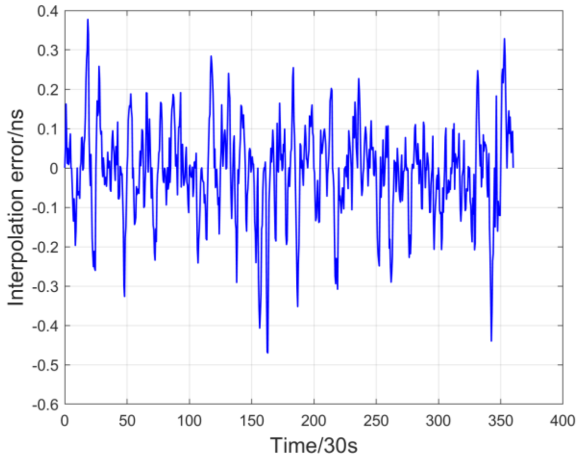
(a) PRN06



(b) PRN07

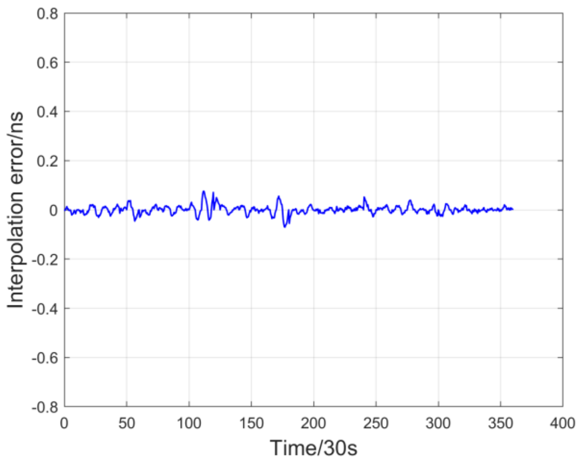


(c) PRN11

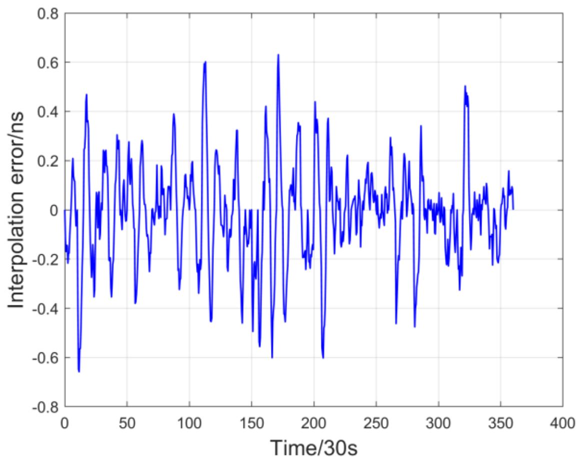


(d) PRN19

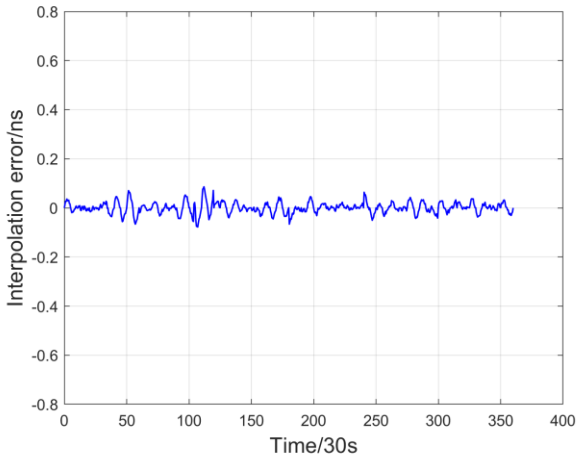
Figure 3: Cubic spline interpolation errors of PRN06, PRN07, PRN11 and PRN19 Satellites.



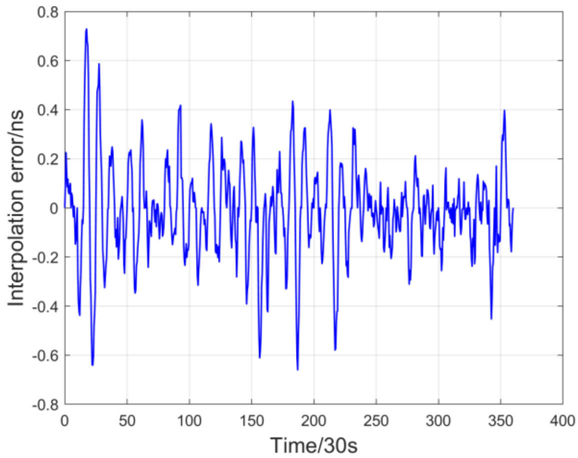
(a) PRN06



(b) PRN07



(c) PRN11



(d) PRN19

Figure 4: Lagrange interpolation errors of PRN06, PRN07, PRN11 and PRN19 Satellites.

Table 2: Comparison of accuracy of different interpolation models for satellite clock bias.

(unit: MSE/ns^2, MAE/ns)

Method	Evaluation indicators	PRN06	PRN07	PRN11	PRN19	Average
Linear interpolation	MSE	0.0002	0.0108	0.0002	0.0124	0.0059
	MAE	0.0075	0.0753	0.0085	0.0808	0.0430
Cubic spline interpolation	MSE	0.0001	0.0129	0.0002	0.0145	0.0069
	MAE	0.0077	0.0825	0.0092	0.0871	0.0466
Lagrange interpolation	MSE	0.0003	0.0418	0.0005	0.0386	0.0203
	MAE	0.0119	0.1488	0.0169	0.1397	0.0793

This can be seen by combining Figure 2-Figure 4 and analyzing Table 2:

Overall, for the interpolation accuracy of the SCB, the linear interpolation method has the highest interpolation accuracy, and its average mean square error can reach 0.0059ns, and the average absolute error can reach 0.0430ns; followed by the accuracy of cubic spline interpolation, and its average mean square error can reach 0.0069ns, and the average absolute error can reach 0.0466ns; Lagrange interpolation method, due to the influence of the Runge phenomenon, its interpolation accuracy is reduced, the average mean square error reaches 0.0203ns, and the average absolute error reaches 0.0793ns. The average mean square error and average absolute error of the linear interpolation method are improved by 14.49% and 7.73%, respectively. Compared to the cubic spline interpolation, the average mean square error and average absolute error are improved by 70.94% and 45.78%. Compared to the Lagrange interpolation are improved by 45.78%. The interpolation accuracy of the linear interpolation method is basically comparable to that of the cubic spline interpolation method, and their average mean square errors can be controlled within 0.0069ns, and the average absolute errors can be controlled within 0.0466ns.

4. Conclusion

In order to fully analyze the accuracy and practical effect of the three SCB interpolation methods in this paper, the precision SCB products with sampling intervals of 5min and 30s released by the GNSS analysis center of Wuhan university are used as the base data, and the interpolation accuracy of the precision SCB products with 5min sampling interval is analyzed by using linear interpolation, cubic spline interpolation and Lagrange interpolation methods. In this study, the precision SCB of 5min sampling interval is interpolated to the precision SCB of 30s sampling interval, and the precision SCB of known 30s sampling interval is compared and analyzed. From the interpolation results, the three interpolation methods can be used to encrypt and interpolate the SCB products, in which the linear interpolation method offers the highest accuracy, followed by cubic spline interpolation, and then Lagrange interpolation. In addition, the three interpolation methods have the highest interpolation accuracies for GPS IIF-type and GPS IIIA-type satellites launched in the last few years, followed by GPS IIR-M-type satellites launched in the middle of the period, and again by GPS IIR-type satellites launched in the early period.

Acknowledgements

This study is supported by the National Natural Science Foundation of China (No. 12403080), China Postdoctoral Science Foundation Funded Project (No.2024M764304), Postdoctoral Fellowship Program of CPSF (No. GZC20233565), Natural Science Basic Research Program in Shaanxi Province of China (No.2023-JC-QN-0027) and the Youth Fund of PLA Rocket Force University of Engineering (No.2021QN-B007).

References

[1] Aghajany H S, Rohm W, Hadas T, et al. Machine learning-based tropospheric delay prediction for real-time precise point positioning under extreme weather conditions[J]. GPS Solutions, 2024, 29(1):36-37.

[2] Aginiparthi S A, Vankadara K R, Mokkapati K R, et al. Evaluating the single-frequency static precise point positioning accuracies from multi- constellation GNSS observations at an Indian low-latitude station[J]. Journal of Applied Geodesy,2024,18(4):699-707.

[3] Li M, Huang T, Li W, et al. Precise point positioning with mixed single- and dual-frequency GNSS observations from Android smartphones considering code-carrier inconsistency[J]. Advances in Space Research, 2024, 74(6):2664-2679.

[4] Tan C B, Gao M, Meng Z H, et al. GPS/BDS/Galileo/GLONASS real-time precision point positioning performance evaluation[J]. Journal of Navigation and Positioning, 2024, 12(4): 90-98.

[5] Xu X Z, He X F, Zhou F, et al. Performance assessment of precise point positioning for products from different analysis centers[J]. Journal of Navigation and Positioning, 2024, 12(5): 9-18.

[6] Geng J H, Wen Q, Chen G, et al. All-frequency IGS phase clock/bias product combination to improve PPP

ambiguity resolution[J]. Journal of Geodesy,2024,98(6):1-15.
[7] Chen P X, Zheng D Y, Nie W F, et al. Precise point positioning (PPP) based on the machine learning-based ionospheric tomography[J]. Advances in Space Research,2024,74(10): 4835-4848.
[8] Yi Z H. Effect mitigation of satellite clock bias interpolation error on PPP by using time-differenced positioning technique[J]. Journal of Navigation and Positioning, 2024, 12(6): 47-53.
[9] Chen P, Chen Z Y, Shen J H, et al. Application of generalized extended interpolation method in precise clock bias interpolation[J]. Application of generalized extended interpolation method in precise clock bias interpolation[J].
[10] Li D Z, Gu H H, Li Y Y, et al. Study on the GPS precise clock error interpolation[J]. GPS Solutions, 2023, 2(1):36-38
[11] Wang J, Fang S S. The three methods and the analysis of GPS precision satellite clock offset interpolation[J]. GNSS World of China,2012,37(04):49-52.
[12] Yang L, Zhang S B, Wan Y H, et al. Comparison of four interpolation of satellite clock bias[J]. Journal of Geomatics,2011,36(03):8-10.
[13] Hua X R. Application of sliding generalized extension interpolation method in GLONASS precise clock correction[J]. GNSS World of China,2022,47(02):38-43.
[14] Duan Q C, Xu B C. Analysis of IGS precise ephemeris and clock error interpolation based on lagrange [J]. Water Science and Technology & Economy,2016,22(09):50-53.
[15] Lei Y, Zhao D N. Reading and encryption methods for IGS precise clock offset files [J]. Geospatial Information, 2013, 11(3):32-33+42+4.
[16] Kuo R X, Yang S W. Accuracy analysis of generalized extension interpolation method in QZSS satellite clock bias interpolation[J] GNSS World of China, vol. 2022,47(04):73-78.