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Research on the Stability of Nonlinear Oscillator Systems Based on Time Delay Coupling

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Abstract: In the field of contemporary physics and engineering, the study of nonlinear oscillator systems has important theoretical significance and broad application prospects. Among them, nonlinear oscillator systems with time delay effects have received widespread attention due to their unique multi-steady state, chaos, and complex periodic motion. This paper aims to conduct in-depth research on nonlinear oscillator systems with delay effects, revealing their complex motion patterns and the essence of their mutual influence. This paper aims to model nonlinear systems with time delay effects. Using a combination of numerical simulation and theoretical analysis, its dynamic characteristics under different working conditions are explored. The dynamic behavior of the system is studied using phase diagrams and Lyapunov exponents. On this basis, representative periodic and chaotic movements are identified by systematically scanning its parameter space. In the case of the Lyapunov exponent ($\lambda<0$), the system was stable; when λ approached 0, the system reached a critical steady state. The research results of the paper provide new ideas and experimental methods for the study of delay coupling systems, as well as new ideas and methods for the design and optimization of such systems.

1. Introduction

In recent years, nonlinear dynamic systems, especially nonlinear oscillator systems with time delay effects, have become a hot topic of concern in disciplines such as physics, engineering, biology, and computer science. Its uniqueness lies in its ability to simulate synchronization of neural networks, mismatch of myocardial tissue, and collaborative control of various robots. In terms of research methods, quantitative analysis and numerical simulation are organically combined to establish accurate mathematical models and conduct effective numerical simulations to explore their dynamic characteristics and stability. Therefore, conducting in-depth research can greatly promote the development of relevant theories and provide important references for its application in engineering.

Existing literature indicates that delay effects cannot be ignored in many practical systems, such as material delay effects and friction delay in mechanical systems. These studies provide important theoretical foundations and analytical methods, but there is still limited research on nonlinear oscillator systems with coupled delay effects. The dynamic behavior under different parameter conditions is systematically studied by establishing a typical delay coupled nonlinear oscillator model and using numerical integration and dynamic analysis tools. By analyzing the phase diagram, Poincare surface of section, and Lyapunov exponent of the system, the influence of delay coupling on the multi steady state, chaos, and transition processes of the system is revealed.

The research of this paper mainly includes the following parts. Firstly, a nonlinear oscillator model with delay coupling is established and described in detail, and its mathematical expression and physical significance are clarified. Secondly, numerical simulation methods are used to solve and analyze the system's dynamic behavior under different parameter conditions, focusing on the system's steady-state and transitional phenomena. Finally, starting from the dynamic characteristics of delay coupling oscillator systems, the aim is to reveal their inherent laws and predictability, thereby guiding intelligent decision-making.

2. Related Work

Nonlinear oscillator systems and artificial equipment are widely present in nature, such as neural networks, ecosystems, power networks, mechanical systems, etc. The mutual influence between the components usually determines the dynamic behavior of such systems. Zhang Qichang explored the complex frequency method for multi-dimensional strongly nonlinear vibration systems [1]. Liu Jun conducted a study on the nonlinear vibration model of the longitudinal, transverse torsional coupling of the drill string in deep-water non riser drilling [2]. Han Guang explored the multi field coupling nonlinear vibration analysis technique for cross-shaped micro resonant beams [3]. Hou Xiangyu studied the nonlinear vibration scheme of drill bit longitudinal torsional coupling under complex variable time delay effects [4]. Jin Dongping proposed the polynomial vector method for nonlinear vibration systems [5]. However, current research mainly focuses on the analysis of single nonlinear effects, and there is little discussion on the complex dynamic behavior caused by delay coupling, failing to fully reveal its underlying mechanism.

In recent years, the study of the delay effect in nonlinear oscillator systems has gradually received attention, and its existence in practical systems has given this research broad application prospects. Thoroughly studying the system's dynamic behavior not only enriches the theory of nonlinear dynamics but also provides new ideas and methods for engineering applications. Qin Yupeng conducted a segmented homotopy analysis of the nonlinear coupled vibration system of marine risers [6]. Zhu Xifeng explored the low-frequency dynamic characteristics of mechanical vibration systems under nonlinear constraints [7]. Gong Jiabei studied the nonlinear vibration scheme of an industrial robot gear transmission system [8]. Ahamed R studied the dynamic analysis of nonlinear oscillation systems based on magnetic springs [9]. Chen J studied the non reciprocal characteristics of energy transfer in nonlinear asymmetric oscillator systems with different vibrational states [10]. However, existing research mainly relies on simplified models and fails to fully consider the complexity of delay coupling, lacking systematic and comprehensive analysis, resulting in limited guidance in practical applications.

3. Methods

3.1 Construction of Delay Coupled Nonlinear Oscillator Systems

(1) Establishment of system model

To study the dynamic behavior of delay-coupled nonlinear oscillator systems, it is necessary to first construct an appropriate mathematical model. The model should be able to accurately describe the motion status and delay coupling relationship of each oscillator in the system. Usually, this model can be represented by a set of nonlinear differential equations. When constructing the model, the following key factors need to be considered: the number of oscillators, the motion equation of each oscillator, the coupling mode between oscillators, and the delay time. The number of oscillators determines the system's scale and complexity; each oscillator's motion equation describes its variation over time; the coupling mode between oscillators determines the form of their interaction; the delay time reflects the delay in information transmission in the system.

(2) Determination and adjustment of parameters

This paper further investigates parameters such as the natural frequency, damping coefficient, coupling strength, and delay time of the oscillator. The size of each parameter has a significant impact on the dynamic characteristics of the system. Appropriate parameter values are chosen through a combination of experimental testing, numerical simulation, and theoretical analysis. In the experiment, parameter estimation can be obtained by detecting the motion state of the oscillator in the real system; in the process of numerical simulation, the trial and error method or optimization method is used to optimize the parameters; in theory, stability theory, bifurcation theory, and other methods are applied to analyze the dynamic characteristics of the model and make predictions.

3.2 Nonlinear Oscillator System Dynamics Mechanism

(1) Stability

Stability is a very important issue in vibration systems. The existence of delay time can seriously affect the stability of delay-coupled systems [11]. Therefore, it is necessary to study its stability further. The system's stability is studied using linearization methods, Lyapunov exponent methods, and Floquet methods. Using this method, the stability conditions of the system near the equilibrium point can be determined, and various parameter changes can be predicted.

(2) Bifurcation and chaos phenomena

Bifurcation and chaos are two common complex phenomena in nonlinear dynamical systems. A new nonlinear dynamic model is proposed and experimentally studied on this basis. Chaos refers to the system being in an uncertain state for a long time, and its high sensitivity to initial values. For nonlinear vibration systems with time delay effects, the time delay effect can induce more complex bifurcation and chaotic behavior. Therefore, it is necessary to conduct more in-depth research on it. This paper intends to use a combination of numerical simulation and experiments to observe and analyze the bifurcation and chaos phenomena, and explore their impact on the system's dynamic characteristics [12].

4. Results and Discussion

4.1 Experimental Setup

Without loss of generality, we assume a Stuart-Landau model based on dual-channel diffusively coupled nonlinear oscillators, where each oscillator is characterized by a pair of variables, $\dot{x}_i(t)$ and $y_i(t)$, representing the state of oscillator i at time t. The coupling between oscillators is governed by both the positions and velocities of the oscillators, with the strength of the coupling modulated by the parameters k.

$$\dot{x}_i(t) = P_i x_i(t) - \omega_i y_i(t) + k\left(x_j(t-\tau) - x_i(t)\right) \tag{1}$$

$$\dot{y}_i(t) = P_i y_i(t) + \omega_i x_i(t) + k\left(y_j(t-\tau) - y_i(t)\right) \tag{2}$$

$P_i = 1 - (x_i^2 + y_i^2)$ represents the nonlinear feedback term that limits the amplitude of the oscillations, ω_i is the natural frequency of oscillator, dictating the speed of the oscillation, k is the coupling strength, representing the coupling correlation strength between systems, τ is time delay in the coupling, reflecting the time it takes for the state of one oscillator to influence the other.

After in-depth research on the dynamic mechanism of the hysteretic coupling nonlinear oscillator system and model construction, it is decided to verify the actual effect of this model through a series of experiments. Next, a detailed introduction is provided to the environment, parameter settings, evaluation criteria, and observed results of these experiments.

(1) Experimental environment and parameter adjustment

The experimental platform uses high-performance computing servers with powerful processing capabilities and sufficient memory to ensure smooth experimentation. Regarding software, professional tools such as Origin are used to accurately simulate delay coupled nonlinear oscillator systems. At the same time, it is also equipped with high-precision data acquisition cards and sensors to capture various data in real-time during the experiment.

In terms of parameter settings, different oscillators are selected to simulate the dynamic behavior of multi-oscillator systems. The natural frequencies of each oscillator are carefully selected to simulate the diversity of oscillator frequencies in actual systems. In addition, appropriate damping coefficients are set according to experimental requirements, and coupling strength and delay time are adjusted to observe how these factors affect the system's dynamic characteristics.

Although specific nonlinear dynamic equations may vary from system to system, a typical nonlinear oscillator equation can be expressed as:

$$x + \omega_0^2 x + \gamma x + f(x) = 0 \tag{3}$$

Among them, x is the displacement of the oscillator; ω_0 is the natural frequency; γ is the damping coefficient; $f(x)$ is a nonlinear term (which may include delay effects).

(2) Evaluation criteria and calculation methods

In order to comprehensively evaluate the experimental results, the following key indicators are set:

Firstly, the stability of the system is evaluated by calculating the Lyapunov exponent. This index can visually display whether the system is stable near equilibrium. If the index is regular, it represents system instability; if it is negative, it represents system stability.

Secondly, a bifurcation diagram of the system response with parameter changes is drawn. This graph demonstrates the bifurcation phenomenon of the system under different parameter conditions. By the number and type of bifurcation points, the system's complexity can be evaluated.

Two methods are usually used to judge chaotic phenomena: maximum Lyapunov exponent and power spectral density analysis. The former can directly indicate whether the system has entered a chaotic state, while the latter observes the existence of chaotic broadband noise and continuous spectrum by analyzing the spectral characteristics of the system output.

4.2 Results

(1) Stability testing of delay coupled nonlinear oscillator systems

The coupling strength of numbers 1-4 is 0.1; the coupling strength of numbers 5-8 is 0.3; the coupling strength of numbers 9-12 is 0.5; the coupling strength of numbers 13-16 is 0.8. The delay time is represented by *t_delay*; the coupling strength is set to k(that is *Serial number* in following figures); the Lyapunov exponent is λ. The stability test results of the delay-coupled nonlinear oscillator system are shown in Figure 1. *Serial number*=10, Lyapunov exponent refers to the relationship between the λ number and the time delay, as shown in Figure 2.

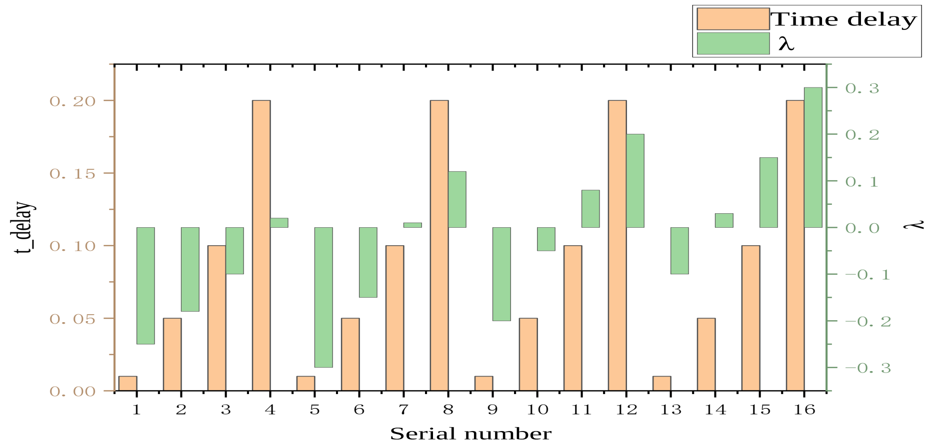

Figure 1. Stability test results of delay coupled nonlinear oscillator system

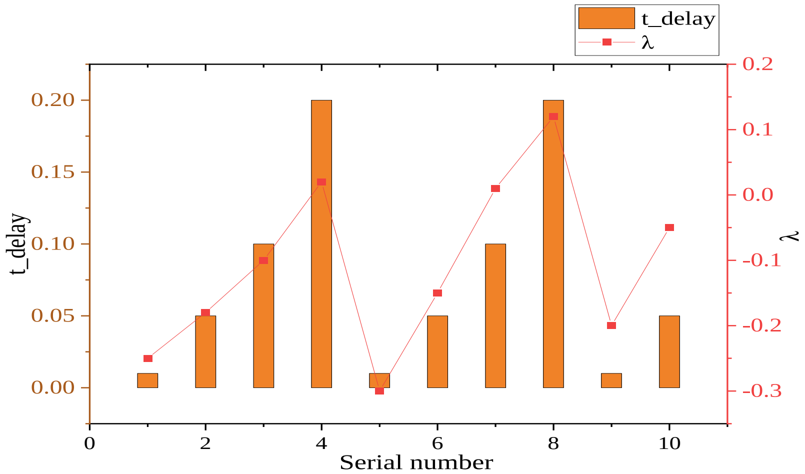

Figure 2. Serial number=10, Lyapunov refers to the relationship between the λ number and the time delay

The coupling strength *k* and delay time (*t_delay*) are important factors affecting the stability of delay coupling systems. In this paper, the system stability gradually decreases as the coupling strength increases. When the coupling strength is not significant (e.g. *k*=0.1), the system still exhibits good stability performance (λ<0). However, as the coupling strength increases (k=0.5, *k*=0.8), in some cases (λ>0), it can lead to system instability. The system stability continuously decreases as the delay time (t_delay) increases. On this basis, it is derived that the system is stable when the Lyapunov exponent (λ)<0; when λ approaches 0, the system reaches a critical steady state; in the case of λ>0, the system is unstable. In engineering practice, stability evaluation can be conducted based on this criterion to propose measures to improve or reduce instability risks.

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(2) System bifurcation phenomenon testing

The test results of the system bifurcation phenomenon are shown in Table 1.

Table 1. Test results of system bifurcation phenomenon

Coupling strength (k)	Number of bifurcation points	Description of dynamic characteristics
0.1	0	No bifurcation, stable periodic motion
0.2	1	The first bifurcation point appears, and the periodic motion becomes more complex
0.3	2	As the number of bifurcation points increases, the dynamic behavior of the system becomes more complex
0.4	3	The bifurcation point continues to increase, and the system enters a multi period coexistence state
0.5	4	The number of bifurcation points further increases, and the dynamic characteristics of the system are enriched
0.6	5	The number of bifurcation points has significantly increased, and the dynamic behavior of the system is complex and variable
0.7	6	The number of bifurcation points continues to increase, and the system may approach a chaotic state
0.8	7	The number of bifurcation points reaches its peak, and the dynamic behavior of the system is extremely complex
0.9	8	The number of bifurcation points remains stable, and the dynamic characteristics of the system are complex but relatively stable
1	9	The last observation point is that the system may be at the edge of chaos or have entered a chaotic state

As the coupling strength gradually increases, the number of bifurcations also increases, which reflects the system's evolution process from simple to complex. There are fewer bifurcations at the beginning, and the system mainly consists of periodic motion. However, as the number of nodes increases, the system's motion becomes more complex. Once the coupling degree reaches a certain level, the system becomes abnormally chaotic and makes some unpredictable actions.

The research findings of this paper help better understand the generation of bifurcation and its impact on overall dynamic behavior. In reality, the strength of this coupling can be adjusted according to needs, thereby changing the performance of a system, making it more stable, and also making it more complex.

(3) Actual system output and predicted output

The actual system output and predicted output at different time points are shown in Figure 3.

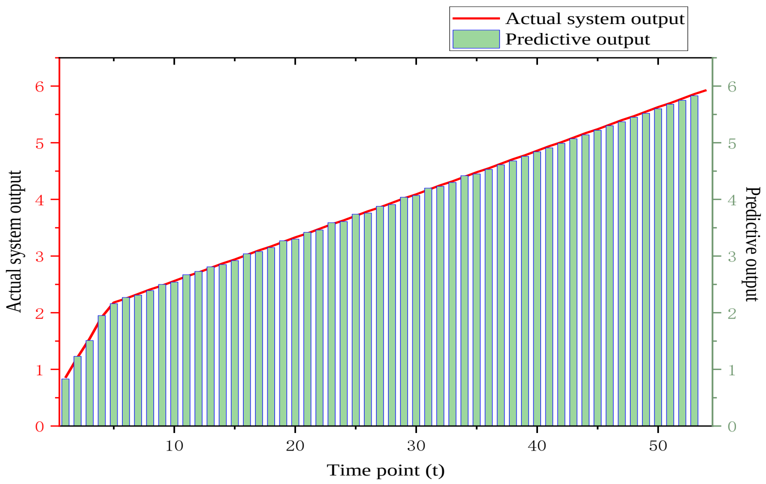

Figure 3. Actual system output and predicted output at different time points

From this, it can be seen that the dynamic model of the nonlinear oscillator system established by utilizing the delay coupling effect can better predict the true dynamic characteristics. Overall, there is not much difference between the predicted output and the actual system output, indicating that the model has high accuracy.

In the initial analysis stage, the difference between the predicted output and the actual output is small, which may be due to the system not yet entering a complex dynamic state. However, over time, especially after time point 40, although there are still slight differences between the predicted output and the actual output, these differences do not significantly affect the overall predictive performance of the model.

Overall, this data validates the effectiveness of the nonlinear oscillator system dynamics model based on delay coupling in practical applications. By continuously monitoring and adjusting model parameters, its prediction accuracy can be further improved, thereby better understanding and controlling the dynamic behavior of actual systems.

(4) Robustness testing

The robustness test results are shown in Figure 4.

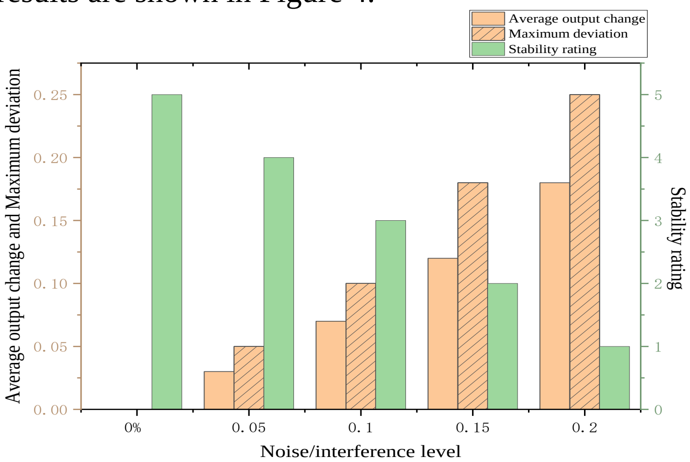

Figure 4. Robustness test results

Firstly, when the system is in an ideal state without noise and interference (0%), its average output change is 0, and the stability rating reaches the highest level of 5 (very stable), with a maximum deviation of 0. The experimental results show that the system can maintain stability and accuracy without external disturbances.

However, as the level of noise and disturbance gradually increases, the system performance changes. In the case of 5% noise and disturbance, the stability of the system has decreased to 4 (steady state), and the maximum error has slightly increased, but the average output of the system has not changed significantly. This represents that while maintaining stability, external forces also gradually influence this system.

As the level of noise and disturbance increases, the stability of the system decreases, and its average and maximum deviation also increase. When the level of noise and disturbance is 20%, its stability has decreased to the minimum (unstable) level 1, while its average output deviation and maximum deviation are both relatively high. This indicates that under strong noise and disturbances, the stability and accuracy of the signal have been greatly reduced.

5. Conclusions

This paper intends to study nonlinear oscillator systems with delay coupling effects. The main research content includes establishing a nonlinear oscillator model with delay coupling characteristics and using numerical simulation to study its dynamic response under different parameters. This paper intends to use the numerical integration method to study the system's dynamic behavior using methods such as Poincare surface of section and Lyapunov exponent. This paper focuses on multi stability phenomena, chaotic behavior, and transformation processes to reveal the delay coupling mechanism in nonlinear oscillators. Research has shown that delay effects significantly impact the dynamic characteristics of nonlinear oscillator systems. Under certain coupling effects, the system exhibits multi-steady state characteristics. In addition, when the parameters of the system change, the system transitions from an ordered state to a chaotic state, exhibiting a typical nonlinear dynamic characteristic. On this basis, the Lyapunov exponent method is used for experimental research. The research findings of this paper contribute to a deeper understanding of the impact of delay coupling effects on the dynamic characteristics of nonlinear systems and provide new ideas for related theoretical research.

Although there have been some important results, there are also some shortcomings. Firstly, this study focuses mainly on numerical calculations and do not fully test the universality of theoretical results. Secondly, the model construction and parameter selection are too simplistic and cannot fully reflect the complex situations in reality. In addition, due to factors such as computational resources and time constraints, the mining of model parameters is limited to a certain extent, thus ignoring some important dynamic characteristics in the system, their dynamic characteristics maybe more complex in engineering practice. Therefore, this paper can be improved in future research. By combining experiments, the results of numerical simulation and theoretical analysis are verified in order to enhance the reliability and applicability of research results, and in-depth research on the impact mechanism of time delay coupling effects is conducted on the dynamic characteristics of vibration systems, this paper can enhance people's understanding and control level of time-delay systems, and provide new research ideas and methods for the development of related disciplines.

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