Curriculum and Teaching Methodology (2024)
Clausius Scientific Press, Canada
DOI: 10.23977/curtm.2024.070824
ISSN 2616-2261 Vol. 7 Num. 8

Analysis of a Kind of Electromagnetic Damped Motion Process

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Keywords: Rectangular coil; Circular coil; Triangular coil; Electromagnetic damping; Integral form

Abstract: The scenario of coils moving in and out of the magnetic field boundary is a combination of magnetism, electricity, mechanics, motion and geometry problems. It is the main way to test the understanding of electromagnetic induction in high school physics. Students are required to have a comprehensive ability to analyze problems. Nevertheless, it is a variable motion with a changing acceleration. To discover its kinematical equations, integration is needed, which is difficult for high school students to analyze this kind of motion in detail. They can only consider qualitatively. But sometimes, qualitative analysis alone is not enough for such problems. Even high school teachers may feel troubled. Besides, there are few literatures that specifically study the kinematical equations of electromagnetic damped motion. In this paper, specific examples will be given to study the kinematical equations about rectangular metal coils frictionlessly moving in and out of a large enough area with uniform magnetic filed. Common relationship between the speed and the position of circular metal coils and triangular metal coils moving in and out of the boundary of uniform magnetic field is described. The general method for solving such problems is summarized, so that high school students can learn this part of content better.

1. Introduction

There is such a type of problem in high school physics: studying the kinematical rule and dynamic performance of metal coils under the action of Ampere force as they pass through an area with magnetic field. Since the Ampere force on metal coils in motion is related to the speed of metal coils, and in turn, the Ampere force affects the motion speed of metal coils, the metal coils are in a deceleration motion with a changing acceleration. This kind of deceleration motion is like the metal coils being subjected to a friction resistance, so it is called electromagnetic damped motion. Although the kinematical equations of metal coils are not needed to analyze in detail in high school, some conservation laws or the average quantity or average thought in the process of motion changes are also utilized to think about and solve problems[1]. Nonetheless, whether these average quantities have the same effect is worth pondering. In this paper, common metal coil models (rectangular, triangular, circular) are taken as examples, and the situation that metal coils frictionlessly moving in and out of an uniform magnetic field is studied by adopteing the method of differential equation. The kinematical rule of this kind of electromagnetic damped motion is summarized.

2. Common way to test the understanding of electromagnetic damped motion

As shown in Fig. 1, there is a rectangular metal coil moving rightwards with an initial speed of v_0 on the smooth horizontal ground. There is a rectangular area with uniform magnetic field in front of it, which is perpendicular to the paper and facing inward (the area with magnetic field is large enough). It is known that the speed of the metal coil reduces by half once it enters the magnetic field completely. And then, in the entire process from entering the magnetic field to leaving the magnetic field completely[2], how the speed of the metal coil changes with time. The correct answer may be ( )

In this example, it is indicatedthat the metal coil is in a deceleration motion with a decreasing acceleration. That is not enough to identify the correct option. Compared with determining the correct option, we prefer to know what kind of motion the metal coil is in, whether the coil frame will be "stuck" and unable to enter or leave, and what the kinematical equation of the metal coil is. Next, the motion process from the perspective of Newton's second law will be covered[3].

2.1 Kinematical equation of the rectangular metal coil as it enters magnetic field

The metal coil is subjected to an Ampere force as it enters the magnetic field. According to Newton's second law, the equation below can be obtained (where m is the mass of the coil, l is the length of the coil cutting the magnetic field, v is the speed of the coil, R is the resistance of the coil, B is the magnetic induction intensity[4],

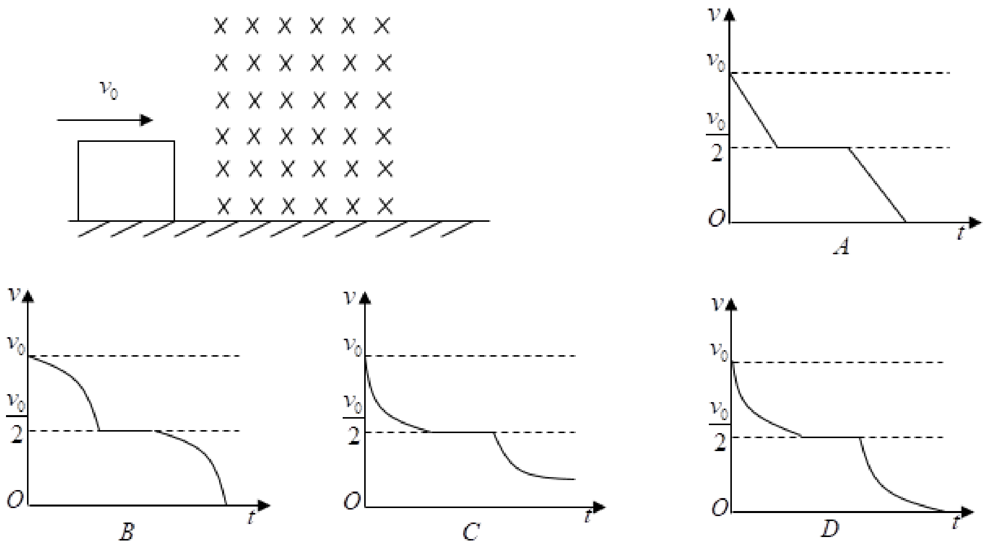

Figure 1: Schematic diagram

And the specified direction of motion is the positive direction)

$$-\frac{B^2 l^2 v}{R} = m\frac{dv}{dt} = ma \tag{1}$$

Multiply dt by the left to find

$$-\frac{B^2 l^2 v}{R} dt = mdv \tag{2}$$

Where $vdt = ds$ is the length of the coil that moves in the time of dt. Equation (2) is the differential form of the momentum theorem. If the length of metal coil that enters the magnetic field is s, the corresponding speed is v. The process is integrated by applying equation (2)

$\int_0^s -\frac{B^2l^2}{R}ds = \int_{v_0}^{v} mdv$, the equation below can be acquired after calculation

$$-\frac{B^2l^2}{R}s = mv - mv_0 \tag{3}$$

Equation (3) is the integral form of equation (2). When teaching this part of content, students can be told that it describes the relationship between the length of the coil that enters the magnetic field and the speed of the coil. To get the kinematical equation of the coil, it is necessary to separate the variables on both sides of equation (1), and then integrate the process of the metal coil entering the magnetic field $\int_0^t -\frac{B^2l^2}{R}dt = \int_{v_0}^{v} \frac{m}{v}dv$. By calculation, $-\frac{B^2l^2t}{R} = m\ln\frac{v}{v_0}$ is acquried. After simplification, the following can be obtained

$$v = v_0 e^{-\frac{B^2l^2}{mR}t} \tag{4}$$

Equation (4) is the kinematical equation in the process of the metal coil entering the magnetic field. It is indicated that the speed of the metal coil attenuates exponentially with a base of e during the process of entering the magnetic field. Let $\tau = \frac{B^2l^2}{mR}$ be the attenuation factor, it can be seen that the greater the mass and resistance of the metal coil, the slower the speed attenuates[13].

The greater the cutting length and magnetic induction intensity is, the faster the speed of metal coil attenuation is.

This is consistent with our cognition. For ease of understanding, it can be compared with the decay law of atomic nucleus[5]. By citing the practice of half-life of nuclear decay, the kinematical equation of the metal coil can also be written as

$$v = v_0(\frac{1}{2})^{\frac{t}{T}} \tag{5}$$

Where $T = \frac{mR}{B^2l^2}\ln 2$ is the time for the speed attenuating by half. It can be seen that the time is determined by the magnetic field and the properties of the coil.

2.2 The conditions that whether the rectangular metal coil can completely enter or completely leave the magnetic field

The kinetic energy of the coil is consumed as the metal coil enters the magnetic field. If the metal coil is long (the area with magnetic field is larger than the coil length), it may not be able to enter the magnetic field completely [12]. The length of the metal coil that can exactly enter the magnetic field completely (at this time, the speed of the metal coil will not be zero mathematically, but will be infinitely close to zero, and it can be treated as static physically) is defined as the critical length L_0, then

$$L_0 = \int_0^{+\infty} v_0 e^{-\tau t}dt = \frac{v_0}{\tau} = \frac{v_0 mR}{B^2l^2} \tag{6}$$

When the length of the metal coil $L > L_0$, that is, the wire frame will not be able to enter the magnetic field completely when $v_0 < \frac{B^2l^2L}{mR}$, at this point, the metal coil is "stuck" and cannot enter. So it can be said that $v_L = \frac{B^2l^2L}{mR}$ is the critical speed required for the metal coil to enter or pass through the magnetic field. It can be indicated that the critical speed is determined by the magnetic field and the properties of coil, and has nothing to do with the initial speed of the coil as it enters (or passes through) the magnetic field. At this time, the ratio of the part that enters the magnetic field to the length of the coil is as follows

$$\beta = \frac{L_0}{L} = \frac{v_0}{v_L} = \frac{v_0 mR}{B^2l^2L} \tag{7}$$

As $v_0 \geq v_L = \frac{B^2l^2L}{mR}$, the metal coil can enter the magnetic field completely. Assume that the speed of the coil frame is v after it completely enters the magnetic field, then the speed and displacement equations can be combined as follows

$$\begin{cases} v = v_0 e^{-\tau t} \\ \int_0^{t_0} v_0 e^{-\tau t} dt = L \end{cases} \tag{8}$$

Where is t_0 the time for the metal coil to enter the magnetic field completely, and the speed of the metal coil after it enters the magnetic field completely can be obtained

$$v = v_0(1 - \frac{L}{L_0}) = v_0(1 - \frac{B^2l^2L}{v_0 mR}) = \alpha v_0 \tag{9}$$

Where $\alpha \in [0,1]$, it describes the degree of speed attenuation

$$\alpha = 1 - \frac{L}{L_0} = 1 - \frac{v_L}{v_0} \tag{10}$$

The law of the metal coil leaving the magnetic field is exactly the same as that entering the magnetic field, only needing to change the initial speed of leaving the magnetic field to αv_0. Similarly, from equation (4), the kinematical equation of the metal coil while leaving the magnetic field can be acquired

$$v = \alpha v_0 e^{-\frac{B^2l^2}{mR}t} \tag{11}$$

Similarly, when $\alpha v_0 < \frac{B^2l^2L}{mR}$, the metal coil cannot pass through the magnetic field completely. It gets "stuck" on the boundary of the magnetic field and cannot leave. By analogy with equation (7), the ratio of the part that passes through the magnetic field to the total length of the coil can be obtained

$$\beta_1 = \frac{\alpha L_0}{L} = \frac{\alpha v_0 mR}{B^2l^2L} = \frac{\alpha}{1-\alpha} \tag{12}$$

If $\alpha v_0 \geq \frac{B^2l^2L}{mR}$, the metal coil can pass through the magnetic field completely. Similarly, from equation (9), the speed of the coil after it passes through the magnetic field can be obtained, which is as follows

$$v_1 = (2\alpha - 1)v_0 \tag{13}$$

2.3 Summary of problems

If $v_0 < \frac{B^2 l^2 L}{mR}$, the metal coil cannot completely enter the magnetic field from the left. The ratio of the length that enters the magnetic field to the total length of the coil is $\beta = \frac{v_0 mR}{B^2 l^2 L}$

If $v_0 \geq \frac{B^2 l^2 L}{mR}$, the metal coil can enter the magnetic field completely. The speed of the coil after it completely enters the magnetic field is $v = v_0(1 - \frac{LB^2 l^2}{v_0 mR}) = \alpha v_0$

If $\alpha v_0 < \frac{B^2 l^2 L}{mR}$, the coil cannot pass through the magnetic field completely. The ratio of the length that passes through the magnetic field to the length of the coil is $\beta_1 = \frac{\alpha L_0}{L} = \frac{\alpha v_0 mR}{B^2 l^2 L} = \frac{\alpha}{1-\alpha}$

If $\alpha v_0 \geq \frac{B^2 l^2 L}{mR}$, the metal coil can pass through the magnetic field completely. The speed of the coil after it passes through the magnetic field is $v_1 = (2\alpha - 1)v_0$

In this example, let $\alpha = \frac{1}{2}$, so the metal coil can exactly pass through the magnetic field.

3. Equivalence proof about effective cutting length of the conductor

Sometimes, the conductor that enters the magnetic field is not even a regularly-shaped conductor, as shown in Fig. 2[11]. At this time, the projected length of the straight line connecting any two points ab in the direction of vertical motion can be used as the length of the cutting magnetic induction line to calculate the electromotive force between these two points. After connecting ab, the conductor becomes a closed loop[6]. But the entire loop will not generate current when moving in the magnetic field. This is because the magnetic flux of the loop does not change, and no electromotive force is generated. Even more, it can be considered that since the dashed line segment and the solid line segment of the loop are cut in the same direction. The two segments generate the same induced electromotive forces with the same direction, there is a lack of current in the loop. Then, the dashed line segment is separated into the parallel speed direction and the vertical speed direction. No cutting is generated in the parallel speed direction, thus the effective cutting length is the projection $l(x)$ of the dashed line segment ab in the vertical speed direction

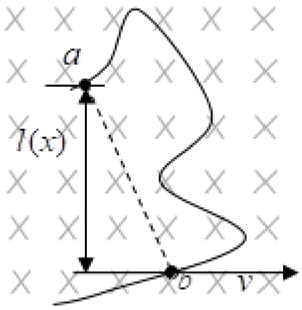

Figure 2: Schematic diagram

$$E_{ab} = Bl(x)v \tag{14}$$

$l(x)$ is called the "effective cutting length". It is also the "equivalent stress length" in the Ampere force. When the conductor is only subjected to Ampere force, according to Newton's second law, the equation below can be acquired

$$-\frac{B^2 l^2(x)v}{R} = m\frac{dv}{dt} \tag{15}$$

Multiply dt by the separation variable to the left to discover

$$-\frac{B^2 l^2(x)v}{R} dt = mdv \tag{16}$$

And because $vdt = dx$, by substituting into equation(16), the equation below can be acquired

$$-\frac{B^2 l^2(x)}{R} dx = mdv \tag{17}$$

The correspondence is established between the position change and speed of a conductor when it enters the boundary of magnetic field under the action of Ampere force only in Equation (17). To create the function $l(x)$ on displacement in the direction of motion, the key is to analyze the motion of the conductor with a different shape when it enters a magnetic field. For a rectangular metal coil, $l(x) = l$ is a constant [7].

4. Establishment of effective cutting length $l(x)$

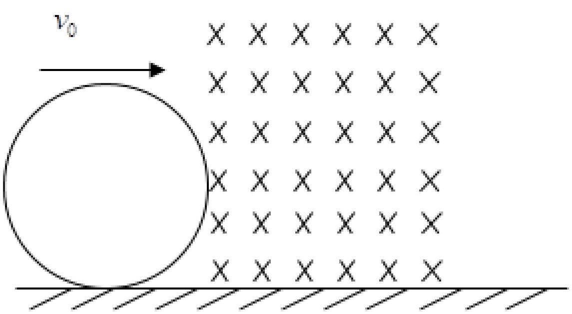

Figure 3: Schematic diagram

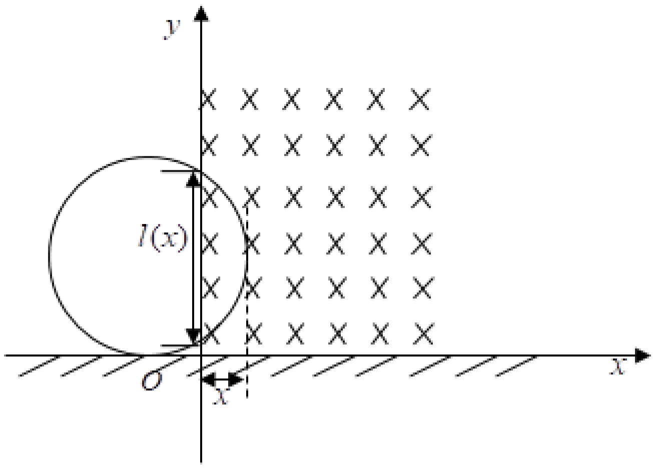

Figure 4: Schematic diagram

As shown in Fig. 3, to illustrate the problem, the rectangular metal coil in the first example is modified into a circular metal coil with a radius of r. Now, a rectangular coordinate system is established with the boundaries of magnetic field as the coordinate axes, as shown in Fig. 4[8]. Then $l(x)=2\sqrt{-x^2+2rx}, x\in[0,2r]$, by substituting into equation (17), the equation below can be obtained

$$-\frac{B^2(-4x^2+8rx)}{R}dx=mdv \tag{18}$$

By integrating the process that the circular metal coil enters the magnetic field on the basis of equation (18), the equation below can be acquired

$$-\int_0^s \frac{B^2(-4x^2+8rx)}{R}dx=\int_{v_0}^{v} mdv \tag{19}$$

Namely

$$\frac{(-\frac{4}{3}s^3+4rs^2)B^2}{R}=mv_0-mv \tag{20}$$

As $s\in[0,2r]$ is the length of the circular metal coil that enters the magnetic field. If $s=2r$, let $v=0$, the critical speed at which the circular metal coil enters the magnetic field completely can be obtained

$$v_L=\frac{16B^2r^3}{3mR} \tag{21}$$

When $v_0\geq\frac{16B^2r^3}{3mR}$, the circular metal coil can enter the magnetic field completely. The speed of the coil after it completely enters the magnetic field is $v=v_0-\frac{16B^2r^3}{3mR}$

When $v_0<\frac{16B^2r^3}{3mR}$, the circular metal coil cannot enter the magnetic field completely. The distance that enters the magnetic field is determined by equation (20), where $v=0$

To clarify the creation process of function $l(x)$, another example is given. As shown in Fig. 5, there is an isosceles triangular metal coil with a vertex angle of α, and its base line is parallel to y axis[9]. The metal coil enters the uniform magnetic field with an initial speed of v_0 along x direction (the area with magnetic field is large enough). In the Fig. 5, it can be seen that $l(x) = 2x \tan\frac{\alpha}{2}$. By substituting this into equation (17), the equation below can be obtained

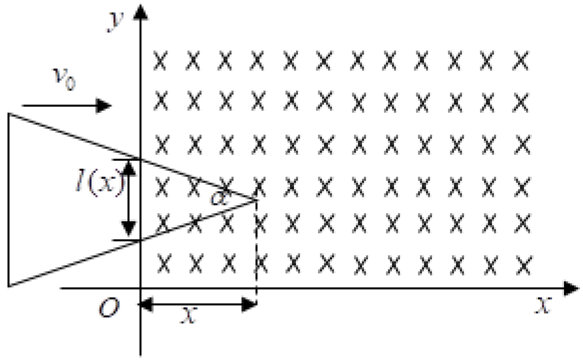

Figure 5: Schematic diagram

$$-\frac{B^2 4\tan^2\frac{\alpha}{2} x^2}{R} dx = mdv \tag{22}$$

Assume that when the distance of the triangular metal coil that enters the magnetic field is $s \in [0, L]$, the corresponding speed is v. By integrate the process that the triangular metal coil enters the magnetic field based on equation (22), the equation below can be acquired

$$\frac{4\tan^2\frac{\alpha}{2} B^2 s^3}{3R} = mv_0 - mv \tag{23}$$

When $s = L$, let $v = 0$, the critical speed at which the triangular metal coil can enter the magnetic field completely is

$$v_L = \frac{4\tan\frac{\alpha}{2} B^2 L^3}{3mR} \tag{24}$$

When $v_0 \geq \frac{4\tan\frac{\alpha}{2} B^2 L^3}{3mR}$, the triangular metal coil can enter the magnetic field completely. The speed of the coil after it enters the magnetic field is $v = v_0 - \frac{4\tan^2\frac{\alpha}{2} B^2 L^3}{3mR}$

When $v_0 < \frac{4\tan\frac{\alpha}{2} B^2 L^3}{3mR}$, the triangular metal coil cannot enter the magnetic field completely, and the length that enters the magnetic field is determined by equation (23), where $v = 0$

5. Conclusions

In this paper, the electromagnetic damped motion of a metal coil as it enters a large enough area with uniform magnetic filed (only subjected to Ampere force) is analyzed. For the rectangular metal coil, the critical conditions that the coil can enter the magnetic field completely as well as the kinematical equations when the coil enters and leaves the magnetic field were given[10]. While for non-rectangular metal coils, the correspondence between the length of the coil that enters the magnetic field and the coil speed as the metal coil enters the boundary of magnetic field was given $-\frac{B^2 l^2(x)}{R}dx = mdv$. By combining with this correspondence, the relationship between the speed and the length that enters the magnetic field when common non-rectangular metal coil models (circular metal coils, isosceles triangle metal coils) enter the magnetic field is specifically analyzed. It is revealed that the electromagnetic damped motion of this type of acceleration change, and helps high school students better learn and understand this part of knowledge points.

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