Journal of Precision Instrument and Machinery (2024)
Clausius Scientific Press, Canada

DOI: 10.23977/jpim.2024.040103
ISSN 2616-1990 Vol. 4 Num. 1

A Unified Method for Determining the Sign Convention of Bending Moments in Mechanics Courses

Zhonghu Zhao[1,a,*], Boqian Xue[1,b]

[1]College of Civil Engineering and Mechanics, Lanzhou University, Lanzhou, Gansu, 730000, China
[a]zhaozhh@lzu.edu.cn, [b]xuebq2023@lzu.edu.cn
[*]Corresponding author

Keywords: Elastic Mechanics, Teach, Bending Moment, Sign, Judgment Method

Abstract: Due to the varying conventions for determining the sign of bending moments across different mechanics courses, particularly in elasticity mechanics where there are no clear guidelines, beginners often find themselves confused during the learning process. This inconsistency not only increases the difficulty of learning but also leads to confusion when students study across multiple disciplines. This paper analyzes and summarizes the sign conventions of various mechanical quantities and their interrelationships in different mechanics courses. By comparing the sign conventions of bending moments in these courses, this paper proposes a unified method for sign determination applicable to all of them and further validates the method's reasonableness and applicability through specific examples. Although researchers in recent years have proposed different methods for determining bending moments within specific disciplines, no one has yet introduced a theory that unifies the sign determination of bending moments across theoretical mechanics, structural mechanics, material mechanics, and elasticity mechanics. The findings of this research provide valuable insights for the teaching of mechanics courses.

1. Introduction

For students in engineering disciplines such as civil engineering, hydraulic engineering, and mechanical engineering, it is generally required to systematically study courses such as theoretical mechanics, mechanics of materials, structural mechanics, and elasticity. From the perspective of knowledge structure, these courses are both interconnected and possess independent theoretical systems. This results in both similarities and differences in the notation used for mechanical quantities across these courses, which can easily lead to confusion and difficulties for beginners during problem-solving. In response to this situation, many educators have conducted research and analysis [1-3].

In these research findings, scholars have put forward various arguments regarding the issues related to the notation of mechanical quantities in each course. However, most of these discussions are limited to the specific applications within each course, without addressing the differences in the criteria for determining the sign of the bending moment across different courses, nor have they systematically examined the issue of unification. As a result, students inevitably encounter confusion and misunderstandings during their studies, which hinders a deeper understanding and application of mechanical concepts.

Furthermore, in elasticity, there is no explicit method provided for determining the sign of the bending moment, which leaves beginners even more perplexed.

In light of this, this paper systematically analyzes the conventions for determining the sign of the bending moment in courses such as theoretical mechanics, mechanics of materials, structural mechanics, and elasticity. It explores the distinct characteristics of each course and their interrelations. Based on this analysis, the paper proposes a new unified method for determining the sign of the bending moment, based on the principles of stress sign determination. This method not only standardizes the criteria for bending moment sign determination across different mechanical disciplines but also helps students reduce confusion during interdisciplinary study and application, thereby improving learning efficiency and depth of understanding. The unified method for bending moment sign determination aims to provide new insights and approaches for the teaching and learning of mechanics courses, contributing to the advancement of engineering education.

2. Notation Conventions for Mechanical Quantities in Mechanics Courses

2.1. Notation Conventions for Mechanical Quantities in Theoretical Mechanics

Theoretical mechanics, as a fundamental course in engineering mechanics, deals with the laws of motion and the analysis of forces acting on objects. Its primary focus is on particles, rigid bodies, and systems of particles, studying the equilibrium and motion of objects under the influence of force systems without considering internal forces and deformations. The mechanical quantities involved include force, bending moment, and couple, all of which are vector quantities. The number of mechanical quantities is relatively limited, and the notation conventions are relatively straightforward [4].

In theoretical mechanics, when determining the sign of a force, the first step in problem-solving is to establish a coordinate system. The direction of the force's projection along the positive axis of the coordinate system is considered the positive direction of the force. Stress and strain are key parameters in studying the deformation and strength of objects. Stress is typically denoted by σ, and unless otherwise specified, it is generally assumed that tensile stress is positive and compressive stress is negative. This convention is consistent with those in mechanics of materials and structural mechanics, aiding students in connecting and understanding concepts across different courses.

In planar problems, moments and couples are defined as scalar quantities. Typically, counterclockwise moments are considered positive, while clockwise moments are considered negative [5]. This convention simplifies the calculation of moments in planar problems and provides students with a consistent standard for solving moment equilibrium problems.

2.2. Notation Conventions for Mechanical Quantities in Structural Mechanics

Structural mechanics primarily focuses on the study of bar systems, such as trusses and frames. The goal is to analyze the forces and deformations within these bar systems to ensure the safety and reliability of structures. The key mechanical quantities involved in this analysis include axial force, shear force, and bending moment. The notation conventions for these mechanical quantities play a crucial role in structural mechanics, as they directly impact the accuracy of calculations and the rigor of the analytical process.

The notation conventions for axial force, shear force, and the bending moment in horizontal members in structural mechanics are consistent with those in mechanics of materials. Specifically, axial force is considered positive for tension and negative for compression. Shear force is defined as positive when it causes a clockwise rotation around the isolated section, and negative when it causes a counterclockwise rotation. For horizontal members, the bending moment is positive when the upper part of the member is in compression and the lower part is in tension, and negative in the opposite case.

In structural mechanics, the analysis of bending moments in frames is relatively complex. Typically, the analysis begins by assuming the sign of the bending moment, which is then verified through the calculation results. For instance, if a bending moment is initially assumed to be positive and the calculated result is also positive, this indicates that the assumption is correct and the bending moment is indeed positive. Conversely, if the calculated result is negative, it means that the bending moment is opposite to the initial assumption and thus negative. This method not only facilitates understanding and memorization but also enhances the accuracy of the analysis.

2.3. Notation Conventions for Mechanical Quantities in Mechanics of Materials

Mechanics of materials, as a crucial component of engineering mechanics, primarily investigates the relationships between forces and deformations in individual members. By analyzing the relationships between internal forces and deformations, it determines the sign conventions for relevant mechanical quantities. The notation conventions in mechanics of materials are essential for accurate analysis of internal forces, stresses, and deformations, which is critical for engineering design and analysis. The key mechanical quantities involved include normal stress, shear stress, axial force, shear force, and bending moment [6].

In mechanics of materials, internal forces are no longer treated as vectors and do not require vector notation; they are considered scalar quantities and can be either positive or negative.

Normal Stress and Axial Force: Normal stress is the perpendicular force acting on the cross-section of a material, typically denoted by σ. In mechanics of materials, normal stress is defined as positive if it acts away from the cross-section (tensile stress) and negative if it acts toward the cross-section (compressive stress). Similarly, axial force that causes longitudinal elongation of a member is positive, while axial force that causes shortening is negative. Therefore, the sign conventions for normal stress and axial force can be summarized as follows: tension is positive, and compression is negative. These conventions not only help standardize internal force analysis but also assist students in accurately determining and calculating the forces and deformations in practical problems.

Shear Stress and Shear Force: Shear stress acts parallel to the cross-section of a material and is typically denoted by τ. In mechanics of materials, the sign convention for shear stress is as follows: shear stress that causes a clockwise moment on a point inside the cross-section (near the section) is considered positive, and shear stress causing a counterclockwise moment is considered negative. This convention is the same as that for shear force, where a shear force that causes a clockwise rotation of the isolated section is positive, and a shear force that causes a counterclockwise rotation is negative. This approach helps simplify the analysis process and ensures consistency in the sign determination for shear force and shear stress. For example, when analyzing the shear force distribution in a cantilever beam, this convention allows for an intuitive understanding of the positive and negative shear force distribution.

Bending Moment: As illustrated in Figure 1, if the effect of the bending moment causes compression in the upper part of the beam segment and tension in the lower part (i.e., if the beam bends downward), the bending moment is considered positive; conversely, if it causes tension in the upper part and compression in the lower part, the bending moment is negative. This analysis reveals that there are differences in the sign conventions for moments in theoretical mechanics and bending moments in mechanics of materials. These differences arise primarily due to the distinct focus and analytical methods of the two courses: theoretical mechanics emphasizes the equilibrium and motion of particles and rigid bodies, while mechanics of materials focuses more on internal stresses and strains and the deformation behavior of components. To reduce confusion in interdisciplinary learning, it is important to highlight these differences in teaching and to seek a unified method for sign conventions.

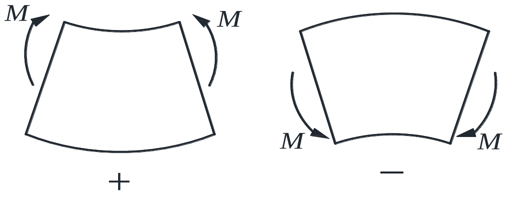


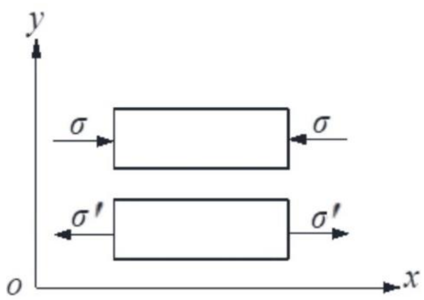
Figure 1: Symbol for moment M in material mechanics

2.4. Notation Conventions for Mechanical Quantities in Elasticity

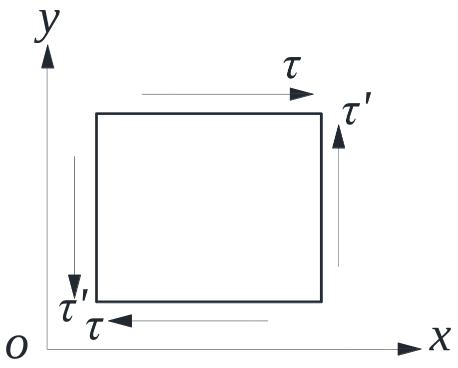
Stress: In elasticity, the definition of stress signs must consider the orientation of the surface on which the stress acts, which is a notable distinction from other mechanics courses. If the outward normal of a cross-section is aligned with the positive direction of the coordinate axis, the cross-section is termed a positive face. On a positive face, stress is considered positive if it acts in the direction of the coordinate axis and negative if it acts in the opposite direction. Conversely, if the outward normal of a cross-section is aligned with the negative direction of the coordinate axis, the cross-section is termed a negative face. On a negative face, stress is considered positive if it acts in the negative direction of the coordinate axis and negative if it acts in the positive direction. This convention can be simplified to: positive normal direction and negative face direction are positive, while negative normal direction and positive face direction are negative. In summary, stresses are positive if they act in the same direction as the normal vector and negative if they act in the opposite direction.

As shown in Figure 2(a), for the normal stress σ acting on a loaded member, in mechanics of materials, compressive normal stress is considered negative, whereas in elasticity, normal stress on the negative face is considered negative. For the normal stress σ' on the same member, tensile normal stress is considered positive in mechanics of materials, and in elasticity, normal stress on the positive face is considered positive. It has been found that the conventions for normal stress in elasticity are consistent with those in other mechanics courses, but the conventions for shear stress differ.

As shown in Figure 2(b), in mechanics of materials, shear stress that causes a clockwise rotation around an isolated section is considered positive, meaning that τ and τ' have opposite signs. In elasticity, the direction of the outward normal of the cross-section is taken as the positive direction of the first coordinate axis in the local coordinate system. The right-hand rule is used to determine the positive direction of the other two coordinate axes. Shear stress that is consistent with the positive direction of this local coordinate axis is defined as positive.



(a) Schematic diagram of axial force component



(b) Schematic diagram of shear stress

Figure 2: Schematic diagram of stress

Based on the above analysis, it is evident that the method for determining stress in elasticity is more general and that the conventions regarding positive and negative faces are applicable across other courses as well. Therefore, we can build on this more consistent stress determination method to explore and develop a unified approach for determining the sign of the bending moment in elasticity, which would also be applicable to other related courses. This method should be easy to understand and apply.

Bending Moment: As previously mentioned, the sign conventions for bending moments differ across various disciplines. For example, in theoretical mechanics, the sign of a bending moment is determined based on the direction of the force and its point of application. In mechanics of materials, the sign is based on the effect of the force on the deformation of the member. However, in elasticity, there is no explicit method provided for determining the sign of the bending moment. This lack of uniformity highlights the need for systematic research and the establishment of a unified method for determining the sign of the bending moment.

3. A Unified Method for Determining the Sign of Bending Moments in Mechanics Courses

3.1. Current Research Status

In recent years, researchers have proposed various methods for determining the sign of bending moments specific to individual disciplines. For example, Zhang Aijun et al. [7] suggested categorizing the determination of the sign of moments into two types: one caused by stress and the other by external force, and assessing them separately. Liu Xiaomei et al. [8] explained the determination of the sign of forces and moments in engineering mechanics by projecting these vectors onto axes, integrating principles from theoretical mechanics and mechanics of materials. Liao Shukuan [9] studied the determination of the sign of bending moments in beams undergoing planar bending deformation in architectural mechanics. Jiang Ke [10] proposed a method in engineering mechanics teaching that involves initially assuming the unknown bending moment to be positive, followed by verification through subsequent calculations.

In summary, although there have been numerous studies on methods for determining the sign of bending moments within specific disciplines, these methods are mostly confined to individual fields and have not achieved cross-disciplinary unification. The methods for determining the sign of bending moments in courses such as theoretical mechanics, structural mechanics, mechanics of materials, and elasticity differ from each other, leading to confusion and misunderstandings among students when they study across these courses. Currently, no researcher has proposed a theory that unifies the methods for determining the sign of bending moments across these mechanic courses.

This situation indicates the need for a systematic analysis of the sign conventions for bending moments across different mechanics courses, to explore their commonalities and differences, and ultimately to propose a unified method. Such a method could not only help students reduce confusion and enhance their learning efficiency and comprehension but also provide new insights and approaches for the teaching of mechanics courses.

3.2. A Unified Method for Determining the Sign of Bending Moments

Inspired by the consistent sign conventions for normal stress across the various disciplines, this paper proposes a method for determining the sign of bending moments: In a coordinate system that follows the right-hand rule, let the axis perpendicular to the paper plane be the z-axis. If the z-axis points into the paper, then a bending moment M on the front face that rotates counterclockwise around the z-axis, and on the back face that rotates clockwise around the z-axis, is considered positive. Conversely, if the bending moment M on the front face rotates clockwise around the z-axis, and on the back face rotates counterclockwise around the z-axis, it is considered negative. If the z-axis points out of the paper, the sign convention is reversed.

This method differs from the sign determination methods in theoretical mechanics and mechanics of materials by introducing a third coordinate axis, adding another dimension to the analysis. It proposes a unified method based on the consistent sign convention for normal stress on the front face and can also be applied to determine the sign of bending moments in elasticity. A simple example is provided below.

As shown in Figure 3, first establish a coordinate system that adheres to the right-hand rule, with the z-axis pointing out of the paper. If the bending moment M on the front face rotates counterclockwise around the z-axis, and on the back face rotates clockwise around the z-axis, then the bending moment M is negative in this case.

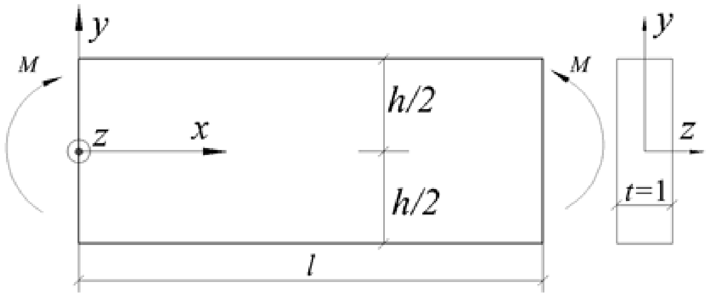


Figure 3: Unified judgment of positive and negative bending moment symbols

Example 1: Referring to Figure 2-9 on page 23 of the textbook *Concise Course on Elasticity* (Third Edition) by Xu Zhizhi [11], as shown in Figure 4 of this paper, by applying Saint-Venant's principle, the integral conditions obtained on the small boundary are:

$$\int_{-h/2}^{h/2} (\sigma_x)_{x=l}\, y dy = M \tag{1}$$

Next, let's proceed with determining the sign of M using the previously mentioned method: with the z-axis pointing into the paper, if the moment M on the positive face rotates counterclockwise around the z-axis, then the moment M is positive. This result aligns with the findings in the textbook.

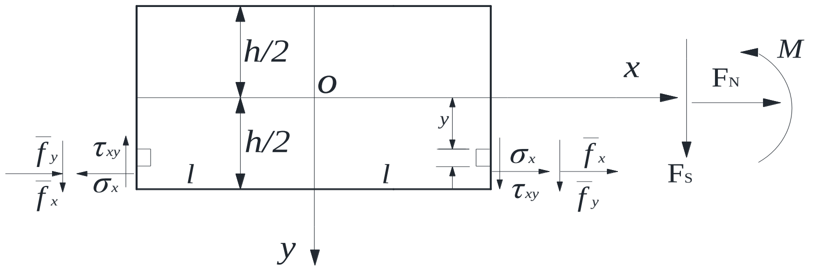


Figure 4: Force distribution on the left and right ends of the beam

Example 2: Referring to Figure 3-4 on page 51 of the textbook *Elasticity* by Guowei Wo and Yuanchun Wang (Shanghai Jiao Tong University Press) [12], as shown in Figure 5, the component bends under the action of moments applied at both ends. By applying Saint-Venant's principle, the resultant moment of the normal stress σ_x on the left and right boundary surfaces must equal the couple moment M of the surface forces, that is:

$$\int_{-h/2}^{h/2} (\sigma_x)_{x=0,l}\, y dy = M \tag{2}$$

Next, let's determine the sign of M using the method described in this paper: with the z-axis pointing into the paper, if the moment M on the positive face rotates counterclockwise around the z-axis, then the moment M is positive. The result is consistent with the textbook. These two examples demonstrate the correctness of the method proposed in this paper.

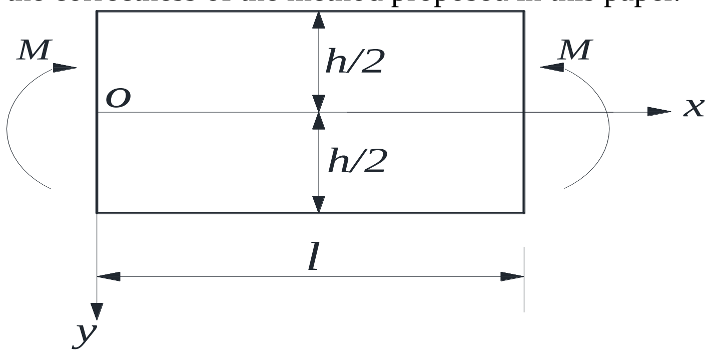


Figure 5: Moment of M beam on the unit width of the couple

Further analysis of Figure 5 reveals that if the coordinate system is established as shown, then according to the principles of mechanics of materials, the moment M causes the beam to bend with the top concave and the bottom convex, which is considered a positive moment. From the perspective of elasticity theory, this moment is also positive. Therefore, this method demonstrates a certain consistency with the approach used in mechanics of materials when applied within a specific coordinate system.

4. Conclusion

This paper systematically studies the methods for determining the signs of various mechanical quantities in theoretical mechanics, mechanics of materials, structural mechanics, and elasticity theory. It analyzes the similarities and differences in sign determination methods across these disciplines, comparing and contrasting them. The study finds that the method used in elasticity theory for determining stress signs is more comprehensive and consistent with the sign conventions of normal stress in other mechanics courses. Based on this, the paper proposes a method for determining the sign of moments in elasticity theory: In a coordinate system that follows the right-hand rule, with the z-axis perpendicular to the plane of the paper, if the z-axis points into the paper, a moment M on the positive face rotating counterclockwise around the z-axis is considered positive, while a moment M on the negative face rotating clockwise around the z-axis is also positive. Conversely, if the moment on the positive face rotates clockwise and on the negative face rotates counterclockwise, both are considered negative. If the z-axis points out of the paper, the signs are reversed.

This method has been shown to be applicable to theoretical mechanics, structural mechanics, and mechanics of materials as well.

References

[1] Liu Ronggang, Liu Wei, Bian Wenfeng. Mathematical Explanation of Material Mechanics Symbols. Mechanics and Practice, 2014, 36 (6): 791-793.

[2] Wang Renpeng, Shen Zuyan. Coincidence expression system of elastic mechanics equation. Journal of Tongji University: Natural Science Edition, 1999 (1): 123-126.

[3] Zhang Yufen. The integrated solution of the sign of force in the course of basic mechanics. Higher Education Journal, 2016 (06): 70.

[4] Hjelmstad K D. Fundamentals of structural mechanics. Springer Science & Business Media, 2007.

[5] Jin Shangnian, Ma Yongli. Theoretical Mechanics, 2nd edition, Higher Education Press, Beijing, 2002.

[6] Sun Xunfang, Fang Xiaoshu, Guan Laitai. Mechanics of Materials, 5th edition, Higher Education Press, Beijing, 2009.

[7] Zhang Aijun, Wu Luxiang. Discussion on several difficult problems in the teaching of Elastic Mechanics. Heilongjiang Education (Higher Education Research and Evaluation), 2019, (01): 13-15.

[8] Liu Xiaomei, Li Peichao, Pan Ying. Determination of Symbols of Engineering Mechanics Parameters. Education and Teaching Forum, 2016, (30): 210-211.

[9] Liao Shukuan. Positive and negative analysis of section bending moment of beam under plane bending deformation. Shandong Industrial Technology, 2016, (12): 276.

[10] Jiang Ke. Simple method for judging bending moment sign. Occupation, 2007, (29): 70.

[11] Xu Zhilun. Concise Course on Elastic Mechanics, 3rd edition, Higher Education Press, Beijing, 2002.

[12] Wo Guowei, Wang Yuanchun. Elastic Mechanics, Shanghai Jiao Tong University Press, Shanghai, 1998.