Environment, Resource and Ecology Journal (2024)
Clausius Scientific Press, Canada
DOI: 10.23977/erej.2024.080206
ISSN 2616-3756 Vol. 8 Num. 2

Optimal Water Level Study Based on Great Lakes Water Issues

Hanyu Yang[1,#,*], Zhixuan Du[2,#], Qi Zheng[3,#]

[1]*College of Transportation Engineering, Dalian Maritime University, Dalian, 116026, China*
[2]*Marine Electrical Engineering College, Dalian Maritime University, Dalian, 116026, China*
[3]*Information Science and Technology College, Dalian Maritime University, Dalian, 116026, China*
[*]*Corresponding author: xingfangdong2@163.com*
[#]*These authors contributed equally.*

Keywords: Violin Diagram, SQP, SA, AHP

Abstract: Lakes maintain ecological balance and natural beauty through their consistent and variable water levels. To explore the complexities of lake water level regulation, this article develops a series of mathematical models based on available data. First, the article identifies the optimal water level range for each time period by drawing a violin diagram and considering the problem's requirements. Then, an AHP evaluation model is constructed, which evaluates the optimal water levels in Ontario using the sequential least squares programming (SQP) algorithm. The article visualizes the optimal water level range obtained. Subsequently, a network model of river flow covering the Great Lakes, connecting Lake Superior to the Atlantic Ocean, is built. The article also develops two control algorithms based on the Simulated Annealing (SA) algorithm to regulate dam outflow. The relationship equation between river flow and the difference in river level is derived, which is then used to conduct a sensitivity analysis of the algorithm. This analysis aims to provide an optimized solution and verify the model's stability. Finally, the article analyzes the model's advantages and disadvantages and summarizes the findings. Finally, the article analyzes the advantages and disadvantages of the model and summarizes the model.

1. Introduction

Lakes play a critical role in our ecosystem and a primary source of freshwater that supports surrounding plant and animal life. The management of water in lakes is a complex problem that requires a thorough understanding of the various factors that determine the water level[1]. The natural factors responsible for the fluctuation of water levels in lakes are mainly temperature, wind, and precipitation, while human-induced factors such as dam construction and reservoir policies can also impact water levels[2].

To ensure the sustainability and longevity of lakes, it is essential to accurately analyze the dynamic network flow problem of lakes. By doing so, researchers can identify and extract key factors that significantly impact lake water levels and establish control algorithms that can help manage the inflow and outflow of dams[3]. These algorithms can be designed to optimize the water level based on the overall health of the lake and the needs of surrounding communities.

The key factors affecting water levels in lakes may vary depending on the specific context in which the lake is located. Some common factors include climatic conditions, water demand, and population growth.

In conclusion, managing water levels in lakes is a crucial issue that requires the collaboration of researchers, policymakers, and local communities to ensure that water levels are maintained at an optimal level for the health of the lake's ecosystem and the well-being of local communities. By using advanced techniques and control algorithms, researchers can analyze the dynamic network flow problem of lakes and identify the key factors that influence water levels, paving the way for the development of effective management solutions to this complex problem.(Data sources http://en.mcm.edu.cn/registry.moma)

2. Water level data analysis

2.1 Analysis and Visualization of optimal water levels in the four lakes

In addition to Lake Ontario, article show the remaining four lakes in fiddle charts. Here I'll start by showing LAKE SUPERIOR as an example as shown in fig.1:

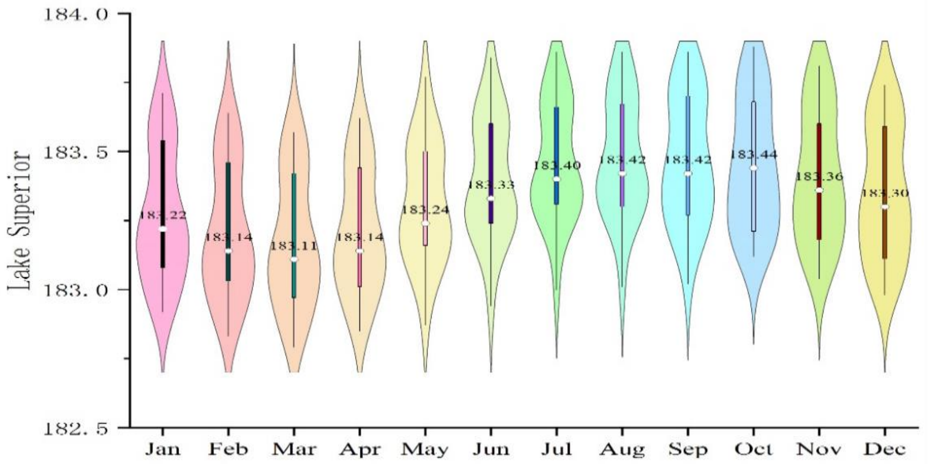

Figure 1: Distribution of water levels in Lake Superior by month

Since the month corresponding to the attached data varies very little from year to year and basically tends to a specific range, article chose a range of 0.15m. above and below the point of highest probable occurrence as the optimal water level corresponding to that month, and the optimal water levels for the remaining three lakes can be obtained in the same way, and the processing is visualized as shown in the following fig.2:

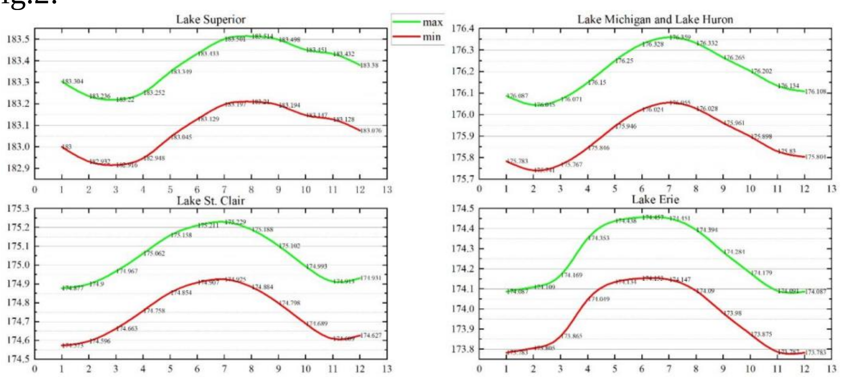

Figure 2: Optimal water level range for the four lakes

As shown in the fig.2 above, the black and red curves represent the maximum and minimum values of the optimal water level, respectively, and the range they enclose is the optimal water level for each time period.

2.2 Model I: AHP-based water level analysis of Lake Ontario

The AHP system for this problem is divided into two layers, the upper layer is the target layer A, and the lower layer is the indicator layer B[4]. The effective indicators are the shipping company (B1), article who manage shipping docks or live near Montreal harbor, (B2), environmentalists (B3), property owners on the shores of Lake Ontario (B4), recreational boaters and fishing boats on Lake Ontario (B5), and hydro-power generation companies (B6), which are six in total.

Here, article mainly consider safety followed by economy and use expert scoring method to create judgment matrix A-B as shown below Table 1:

Table 1: Judgment matrix A-B

	B1	B2	B3	B4	B5	B6
B1	1.00	0.20	4.00	0.33	3.00	0.50
B2	5.00	1.00	9.00	2.00	7.00	3.00
B3	0.25	0.11	1.00	0.14	0.33	0.20
B4	3.00	0.50	7.00	1.00	5.00	2.00
B5	0.33	0.14	3.00	0.20	1.00	0.25
B6	2.00	0.33	5.00	0.50	4.00	1.00

The results obtained through the judgment matrix obtained are as follows: $\lambda\text{max} = 6.182, \text{CI} = \frac{\lambda_{\text{max}} - n}{n-1}, \text{CI} = 0.036, \text{CR} = \frac{\text{CI}}{\text{CR}}$, CR=0.029<0.10.

This shows that the matrix passes the consistency test and finally article get the weights using the square and root method as: (9.969%, 40.835%, 2.942%, 25.225%, 5.097%, 15.932%).

2.3 SQP modeling and Solving the model

According to the weights obtained above, article can add up its various parameters to process the following model:

$$\text{minZ} = \sum_1^6 p_i X_i \tag{1}$$

where Z denotes the adjustment article make to the optimal water level after taking into account the interests of the stakeholders, and X_i is the quantitative result of B_i.

Our analysis shows that the shipping company represented by X_1 wants the water level to be high, from which article can determine that X_1 is a positive parameter, and article follow the same method to analyze that X_6 is a positive indicator, X_2 is a negative indicator, and $X_3 X_4 X_5$ is a neutral indicator , and since a deviation from the normal value of the water level by two to three feet may have a great impact on some of the relevant stakeholders, article require that $Z < 0.6$ m.

In summary article can the following constraints:

$$\text{s.t.} \begin{cases} X_1 > 1 \\ X_2 < -1 \\ X_3 > 0 \\ X_4 > 0 \\ X_5 > 0 \\ X_6 > 1 \end{cases} \tag{2}$$

(1) Choose the initial point, article make $X_1 = 1, X_2 = -1, X_3 = X_4 = X_5 = 0, X_6 = 1$

(2) Set the initial multiplier λ_0 and set the iteration subscript $k = 0$.

(3) Quadratic programming solution: a quadratic programming problem is obtained by performing a second order Taylor expansion of the objective function and constraints at (X_k, λ_k). Estimates of the search direction p_k and multipliers are obtained by solving this problem.

(4) Update iteration point: update iteration solution:

$$X_{(k+1)} = X_k + \alpha_k * p_k \tag{3}$$

where α_k is the step size.

(5) Update multiplier: update the multiplier λ according to some strategy.

(6) Check for convergence: if the termination criterion is satisfied (e.g., the amount of change in the solution is too small or the value of the objective function is no longer decreasing significantly), stop, otherwise, set $k = k + 1$ and return to the second step[5].

After several iterations, article can get the following Table 2 result:

Table 2: Calculation of parameters

parameters	solution value
Z(target value)	0.12
X_1	1
X_2	-0.333
X_3	0.005
X_4	0
X_5	0.002
X_6	1

2.4. Visualization of Optimal Water Levels in Lake Ontario

Based on the obtained results, the influence of stakeholders on the water level will require an optimal increase of 0.12m, as shown in section 5.1. The article continues by plotting a violin diagram to determine the point with the highest probability distribution, which is used as a reference point. From this reference point, an arbitrary range of 0.15m above and below is selected. After accounting for the increased influence of stakeholders by an additional 0.12m, the article then plots the distribution of the optimal water level in Lake Ontario, as shown in Figure 3:

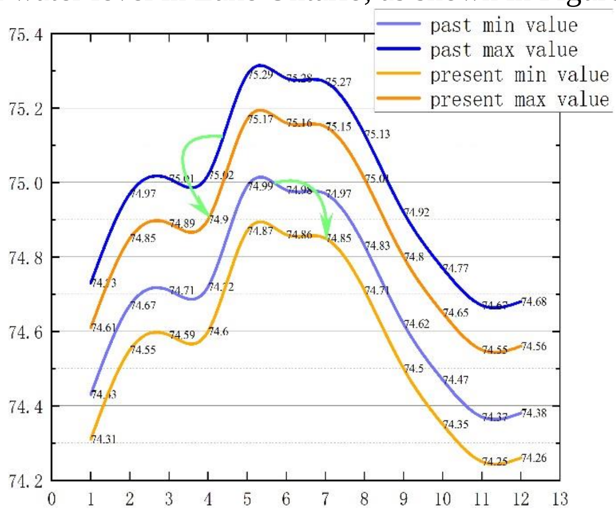

Figure 3: Before and After Comparison

3. Establishment of SA-based dam control system

3.1 Schematic diagram for constructing the Great Lakes system

In the second problem, the article simplifies the river planning to the river system shown in the figure below to facilitate analysis, as illustrated in Figure 4:

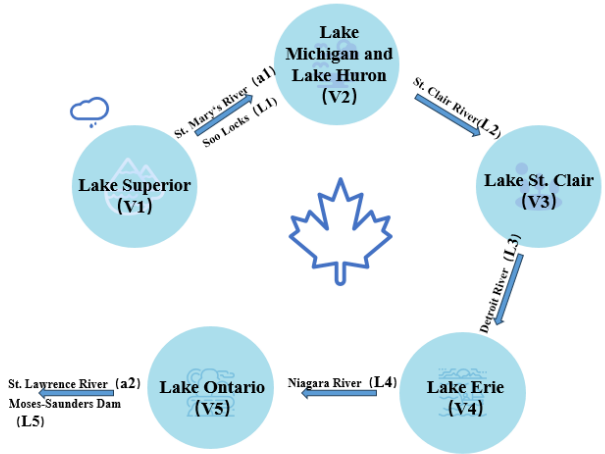

Figure 4: Simplified diagram of the Great Lakes system

After analyzing this schematic of the water system, article chose to use the month as the standard. In order to get the amount of water change in the lake, article can consider the amount of water change caused by natural factors and the amount of water change caused by the inflow and outflow of the river as two variables[6], where the dam can control the discharge of the river, so article introduce a 0-1 variable a_i to indicate whether the dam is open or not, And construct a water flow system model as shown:

$$\begin{cases} V_1' = b_1 t - a_1 L_1 t_1 + V_1 \\ V_2' = b_2 t + a_1 L_1 t - L_2 t + V_2 \\ V_3' = b_3 t + L_2 t - L_3 t + V_3 \\ V_4' = b_4 t + L_3 t - L_4 t + V_4 \\ V_5' = b_5 t + L_4 t - a_2 L_5 t_2 + V_5 \end{cases} \tag{4}$$

$$a_i = \begin{cases} 1 & Open\ the\ dam. \\ 0 & clese\ the\ dam. \end{cases} \tag{5}$$

where V_i denotes the volume of water in the ith lake in the previous month, V_i' denotes the volume of water in the ith lake in the current month, t denotes the time when the river is drained without dams, $t_j$denotes the time when the jth dam is opened, a_j denotes whether the jth dam is opened or not, and L_i denotes the volume of water discharged from the ith river per unit of time[7].

3.2 Model III Establishment of SA-based dam control system

3.2.1 Simulated annealing modeling

Simulated annealing is a generalized probabilistic search algorithm with strong search capabilities. This article uses it to build two control algorithms for managing the dam's outflow to find the optimal water level[8].

In the ontology, article model a kind of 2 dams, they determine the drainage time of the river $t_1 t_2$ are with Eqs.(1)and(2), the time t_i of the dam opening is used as the independent variable and

V_iV_i' are subtracted to obtain △V,leading to the following simulated annealing model:

$$\min \Delta V_i = b_i t_i + C_i L_{i-1} t_{i-1} - D_i L_i t_i \tag{6}$$

The article subtracts V_iV_i' to get △V, where C_i *and* D_i is a 0-1 variable defined by the article. If there is no dam upstream of the ith lake, C_i is always taken as 1. If there is a dam upstream, C_i is treated in the same way as a_i.

After that, for this simulated annealing model, article are given a prompt temperature T_0 and an initial solution $t(0)$, and from t(0), article generate the next $t_i' \in N(t_i(0))$, , and whether or not to accept t_i' as a new solution $t_i(1)$ depends on the following probability:

$$(t_i(0) \to t_i') = \begin{cases} 1 & f(t_i') < f(t_i(0)) \\ e^{-\frac{f(t_i') - f(t_i(0))}{T_0}} & \text{others} \end{cases} \tag{7}$$

If the resulting new solution t_i' has a smaller function value than the previous solution, then $t_i(1) = t_i'$ is accepted as a new solution, otherwise t_i' is accepted as a new solution with probability $e^{-\frac{f(t_i') - f(t_i(0))}{T_0}}$.This process iterates until t_i no longer changes.

3.2.2 Further analysis of the model

The river's flow is influenced not only by natural factors such as precipitation and dam operation but also by the water level difference between upstream and downstream. Therefore, the previously established model is optimized as follows:

$$\min \Delta V_i = b_i t_i + C_i L_{i-1} t_{i-1} - D_i L_i t_i \tag{8}$$

(1) A review of relevant literature shows that the relationship between water flow rate and upstream-downstream water level difference is:

$$v = 4.43\sqrt{h} \tag{9}$$

Where, v represents the flow rate of the river and h represents the water level difference between upstream and downstream.

Thus, the flow rate L of the river can be optimized as:

$$L_i = 4.43\sqrt{h} s_i \tag{10}$$

where s_i represents the cross-sectional area of the water flow, and fors_i article chose to replace it with the average stream cross-sectional area over the years;

(2) Since water level fluctuations of more than 3 feet can have significant impacts, article limit:

$$0 \le \Delta H \le 0.65 \tag{11}$$

Where $\Delta H_i = \Delta V_i / s_i$ represents the short-term lake water level change value.

(3) Therefore, the optimized model is:

$$\min \Delta V_i = b_i t_i + 4.43 C_i \sqrt{h_{i-1}} s_i t_{i-1} - 4.43 D_i \sqrt{h_i} s_i t_i \tag{12}$$

The optimized model has two independent variables, h_i, t_i, which are solved using new simulated annealing equations according to the principle of independent variable iteration.

4. Application of the model

4.1 Analysis and preparation of the problem

For the 2017 Great Lakes data, article find very little difference between the water level changes in the top four lakes and the years before and after, and article focus our discussion on the model's ability to release flood water through the dam for Ontario Lake.

Check the relevant information to know the surface area of the Great Lakes are, as shown below Table 3:

Table 3: Surface area of the Great Lakes

Great Lake	area (of a floor, piece of land etc)
Lake Superior	8.2wkm^2
Michigan and Lake Huron	11.6wkm^2
Lake St. Clair	0.1wkm^2
Lake Erie	2.5wkm^2
Lake Ontario	1.9wkm^2

It is difficult to quantify the natural influence factors, so article express the natural factor b_i by C_i (C_i represents the average change value of the water level of each lake affected by natural factors from 2010 to 2020 in millimeters), and the formula is as follows:

$$C_i S_i = b_i\, t \tag{13}$$

The monthly C_i for each of the specific five lakes is shown in the table 4 below:

Table 4: Volume of water in the Great Lakes due to natural variability

	Jan	Feb	Mar	Apr	May	Jun
C1	-91.89	-38.98	-44.40	82.63	86.87	125.09
C2	-19.43	-0.17	54.93	214.92	131.21	249.27
C3	37.16	23.90	60.56	44.24	25.90	-88.51
C4	56.56	27.87	60.54	111.55	104.82	31.91
C5	9.32	4.01	13.56	127.50	151.23	113.34

Continued

	Jul	Aug	Sep	Oct	Nov	Dec
C1	59.48	97.22	43.18	54.71	-100.87	-104.37
C2	135.29	49.52	-7.71	141.28	-71.80	-142.72
C3	-107.84	-19.70	-87.97	-4.71	70.01	-15.15
C4	-8.36	-65.65	-84.90	-89.59	-34.27	-52.66
C5	61.31	-2.61	-11.67	76.88	-12.83	-43.74

4.2 Compensatory engineering modeling of the Suez Canal locks

After article improve Eq. (5) according to Eq. (12) article can get the compensation engineering model for the locks of the Eas Canal as follows：

$$\begin{cases} V_1' - V_1 = \mathrm{S}_1 \Delta \mathrm{h}_1 = C_1 S_1 - a_1 L_1 t_1 \\ V_2' - V_2 = \mathrm{S}_2 \Delta \mathrm{h}_2 = C_2 S_2 + a_1 L_1 t_1 - L_2 t \end{cases} \tag{14}$$

Compensating Works of the Soo Locks at Sault Ste. Marie contains three hydroelectric power plants, and considering the benefits associated with the hydroelectric power plants, article regulate $t_1 = \frac{1}{n} t$,, so that the dam is in a cycle of periodic discharges. L_1 and t_1 have a certain relationship: $\sigma = L_1 t_1$ (σ is the total amount of water discharged).

The dataset shows that the river flow for January 2017 is L_2=5552.933m^3/s. According to the graph in 5.2, article make the model stabilize when the changes in Δh_1 and Δh_2 converge to the average monthly change. The Δh for January can be expressed as:

$$\begin{cases} \Delta h_1 \rightarrow -68mm \\ \Delta h_2 \rightarrow -42mm \end{cases} \tag{15}$$

Calculation gives $\sigma \rightarrow 3.2 \times 10^{10}$ m^3

It can be seen that the total water discharge tends to a fixed value, you can change t_1 and L_1 at the same time to maintain the lake level tends to a reasonable range of changes, when floods and other disasters, article can increase L_1 and t_1 at the same time to ensure that the water level of the lake will not fluctuate greatly in a short period of time.

4.3 Sensitivity Analysis of the Control System of the Soo Locks

Article are going to perform a sensitivity analysis by determining the values of L_1 and t_1. Article can determine through the simulated annealing algorithm that the first dam project gives a better operating solution when $L_1 = 37037m^3/s$, $t_1 = 10days$. The sensitivity analysis is carried out by comparing the value of the change in Δh_2 with half of the value of the permissible optimal water level change.

(1) Adjusting the value of L_1 upward by 1%, $L_1 = 37408m^3$, and keeping the value of t_1 unchanged, article get Δh_2^*= -39.2mm,After that, according to the formula:

$$\Delta\Delta h_2 = \left| \frac{\Delta h_2 - \Delta h_2^*}{\frac{1}{2}\Delta h_{max}} \right| \tag{16}$$

It can be calculated thatΔh_2 has changed by $1.84\% < 10\%$, which indicates that the model is more stable (less sensitive);

(2) Adjusting the value of t_1 upward by 1%, $t_1 = 10.1days$,, and L_1 takes the same value, article get Δh_2^*= -39.3mm, It can be calculated that Δh_2 has changed by $1.77\% < 10\%$,which indicates that the model is more stable.

4.4 Modeling of the compensation system at the Moses-Saunders Dam

Due to the influence factor of Niagara Falls and flooding, the outflow should not only take into account the maximum carrying capacity of the St. Lawrence River, but also work with the Iroquois Dam and Beauharnois Power Dam, at which time the dam will have a storage function, so article improved the model to:

$$V_5' - V_5 = C_5 S_5 + L_4 t - a_2 L_5 t_2 - L_6 t \tag{17}$$

The Moses-Saunders Dam at Cornwall has to control the outflow from the St. Lawrence River and the storage capacity of the Moses-Saunders Dam to control the water level in Lake Ontario, again taking into account the interests of stakeholders such as shipping companies, hydroelectric generators, and so on[9][10].

As in 7.1, article regulate $t_2 = \frac{1}{n'} t$, to keep the dam in a cycle of periodic discharges. L_5 and t_2 are related: $\sigma' = L_5 t_2$ (σ' is the total amount of water released).

The dataset shows that in mid-2017, Lake Ontario's water level was significantly higher than the average water level from May to September, and the river flow in May was L_4 =7320 m^3/s,L_6=8580m^3/s.According to the graph in 5.4, article made the model stabilize when the change in Δh_5 converged to the average change from month to month. The Δh for May can be expressed as:

$$\Delta h_5 \rightarrow -1mm \tag{18}$$

Calculation gives $\sigma' \rightarrow 2.5 \times 10^{10}$ m^3

When the total discharge tends to a fixed value, article can change t_2 and L_5 at the same time to maintain the lake level tends to a reasonable range of change, when natural disasters such as floods come, article can increase L_5 and t_2 at the same time to ensure that the water level of the lake will not fluctuate greatly in a short period of time.

4.5 Sensitivity analysis of control systems

Similar to in 7.1, the values of $\boldsymbol{L_5}$ and $\boldsymbol{t_2}$ are determined first. Based on continuous iterative debugging of the values of $\boldsymbol{L_5}$ and $\boldsymbol{t_2}$, article determined that the second dam project yields a better operating solution when $\boldsymbol{L_5 = 12056m^3/s}$, $\boldsymbol{t_2 = 24days}$. The sensitivity analysis is performed by comparing the value of the change in $\boldsymbol{\Delta h_5}$ to half of the value of the permissible optimal water level change.

(1) Adjusting the value of L_5 upward by 1%, $L_5 = 12177m^3$, and keeping the value of t_2 unchanged, article get Δh_5^*= -14.2mm, by Eq:

$$\Delta\Delta h_5 = \left|\frac{\Delta h_5 - \Delta h_5^*}{\frac{1}{2}\Delta h_{max}}\right| \tag{19}$$

It can be calculated that Δh_5 has changed by $8.6\% < 10\%$, which indicates that the model is more stable (less sensitive);

(2) Adjusting the value of t_2 upward by 1%,$t_1 = 24.2days$, and L_1 taking the same value, article get Δh_5^*= -11.9mm, It can be calculated that Δh_5 has changed by $7.2\% < 10\%$,which shows that the model is more stable.

In conclusion: since this model is more stable, it can make the water level changes tend to average out and can satisfy all stakeholders. Considering that the Moses-Saunders Dam is experiencing flooding phenomena in both 2017 and 2019, article can propose a more optimized solution based on the existing model: constructing a tributary near the headwaters of the St. Lawrence River to better cope with flooding phenomena.

5. Conclusion

In this study, the article analyzes the complexity of lake water level regulation and proposes an optimal water level algorithm that balances the influence of natural factors and the needs of stakeholders. Based on historical data and the balance of multiple factors, the article constructs a mathematical model that satisfies ecological needs—such as the hydrological cycle and biodiversity conservation—while considering human dependence on lake resources, including hydroelectric power generation, residential life, and tourism development.

Through sensitivity analysis, the article assesses the model's adaptability to parameter changes, verifying its stability and broad applicability. This analysis emphasizes that even in the face of extreme climatic conditions and changing demands from different stakeholders, the model can effectively adapt to environmental changes and provide scientific decision support for lake level management.

The findings highlight the importance of adopting a systematic approach and considering multiple factors in lake management. By building a reliable data-driven model, the article not only develops optimal water level regulation strategies for specific lakes but also provides a generalized quantitative decision-making tool for other natural resource management fields. This promotes the sustainable development of natural resource management and realizes the harmonious coexistence between humans and nature.

References

[1] Mahmood, K., and Fekete, B. M. (2020). A Review of Sedimentation Rates in Freshwater Reservoirs: Recent Trends and Future Implications[J]. Water Resources Research, 56(12).

[2] Zhang Wei, Impact of Reservoir Storage on Downstream River Flow[J]. Journal of Hydrology, 2019(4): 745-758.

[3] Liu Hong, Hydraulic Analysis of Spillway Gate Operations[J]. Advanced Water Resources, 2018(6): 1129-1142.

[4] Wang, H., Liu, Y., & Song, X. Assessment of Reservoir Sedimentation Using Satellite Remote Sensing and Digital Elevation Models[J]. Environmental Monitoring and Assessment, 2023,195, 622.

[5] Mishra, B. K., McBean, E., & Gomi, T. Present and Future Losses of Storage in Large Reservoirs Due to Sedimentation: A Country-Wise Global Assessment[J]. Sustainability, 2023, 15(1), 219.

[6] Sun Ming, Risk Assessment in Dam Discharge and Flood Management[J]. Risk Analysis, 2020(3): 376-390.

[7] Li Xiang, Modeling of Flood Release through Dam Gates[J]. Water Engineering Journal, 2019(7): 984-997.

[8] Cao, Z., Wang, D., Zhang, Y., & Li, Y. River Ice Regime Recognition Based on Deep Learning: Ice Concentration, Area, and Velocity[J]. Water, 2024, 16(1), 58.

[9] Ahn, J. H., Kim, S., & Park, K. Y. Community Flood Risk Management Using Hybrid Flood Models: A Case Study in South Korea[J]. Journal of Flood Risk Management, 2023, 16(2).

[10] Guo Lin, Long-term Effects of Reservoir Operations on Water Supply Security[J]. Journal of Water Resource Planning and Management, 2020(1): 88-102.