

# ***Research on undersea search and rescue program based on localization and equipment selection***

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**Abstract:** With the widespread application of submarine technology in military and scientific research fields, the importance of submarine search and rescue is becoming increasingly prominent. How to implement effective search and rescue in the shortest possible time and control search and rescue costs has become an urgent problem to be solved. This article explores the time and cost optimization strategies for submarine search and rescue, and constructs an optimization model for submarine search and rescue time and cost. Firstly, compare the position with the highest probability using Monte Carlo method and integration method, and calculate the coordinates. Secondly, a circular search algorithm is used for each position to obtain the shortest search and rescue path by comparing simulated annealing algorithm and genetic algorithm. Finally, the PCA method is used to obtain the solution with the lowest cost. The experimental results show that the optimization strategy proposed in this paper can significantly shorten search and rescue time and reduce search and rescue costs.

## **1. Introduction**

On the basis of existing positioning methods, this article studies submarine search and rescue methods based on existing technologies. The focus of this study is to approximate the location of the submersible and determine the most suitable portable device for use with the submersible [1-3].

The current literature mainly focuses on the position of submarines under ideal conditions, the current distribution of submarine positions, and how to fully report this. In the case of losing a submarine, the following model is proposed: under ideal conditions, assuming that the position distribution of the submarine is at its maximum directional angle, the position of the submarine at its maximum approximate point can be estimated [4] [5]. This involves calculating the maximum approximate point position. Subsequently, the same method can be used to identify the maximum approximate position in each iteration, thereby minimizing the search time for each position. Then apply the circle search algorithm to each point. To consider search and rescue costs, a cost-benefit model is used to analyze the strategy with the lowest equipment cost. The time cost-effectiveness model studied in this article aims to significantly reduce time and cost.

(Datasources:<https://www.comap.com/contests/mcm-icm>)

## 2. The establishment of the model

### 2.1 Preparation of the model under ideal conditions

This article involves the calculation of the initial position of a submersible and its position after a certain period of time. Using a formula, the probability distribution of the position at different times is derived as follows:

#### 2.1.1 Initial position distribution of the submersible

(1) Due to the fact that location information is usually obtained from detectors, there is considerable uncertainty involved. Therefore, this article makes two assumptions:

(2) The initial position coordinates of the submersible are  $x$  and  $y$ . At this point, there is a positioning error. However, the initial position of the submersible roughly follows a two-dimensional normal distribution, so the joint probability density distribution function of the initial position is expressed as formula (1):

$$p(x, y) = \frac{1}{2\pi\sigma_{0x}\sigma_{0y}} e^{-\left(\frac{x^2}{2\sigma_{0x}^2} + \frac{y^2}{2\sigma_{0y}^2}\right)} \quad (1)$$

(3) Assuming that  $x$  and  $y$  follow the same distribution and are independent, the standard deviation of direction  $\sigma_{0x} = \sigma_{0y} = \sigma_0$ . Choosing the  $r_0 = \sqrt{x^2 + y^2}$  in Cartesian coordinate system, the joint probability density function of the initial position of the submersible can be represented by formula (2):

$$p(r_0, \theta) = \frac{1}{2\pi\sigma_0^2} e^{-\left(\frac{r_0^2}{2\sigma_0^2}\right)} \quad (2)$$

Where  $D = \{(r_0, \theta) | r_0 > 0, \theta \in [0, 2\pi]\}$ , that is, the initial position of the submersible follows a circular normal distribution centered on the coordinate origin, in formula (2),  $\theta$  indicates the orientation of the target, and  $r_0$  represents the actual distance between the coordinate origin and the submarine target<sup>[6]</sup>.

#### 2.1.2 Position distribution of the submersible after sometime

After obtaining the initial position coordinate information of the submersible, this article focuses on collecting data from the surrounding area of the initial time dispersion position and initiating submarine search activities [7]. During this period, it is assumed that the submersible is traveling at a speed within the range of  $[v_1, v_2]$ , and the heading is evenly distributed within  $[\phi_1, \phi_2]$ . Furthermore, assuming that speed and heading are independent of each other, as shown in Figure 1:

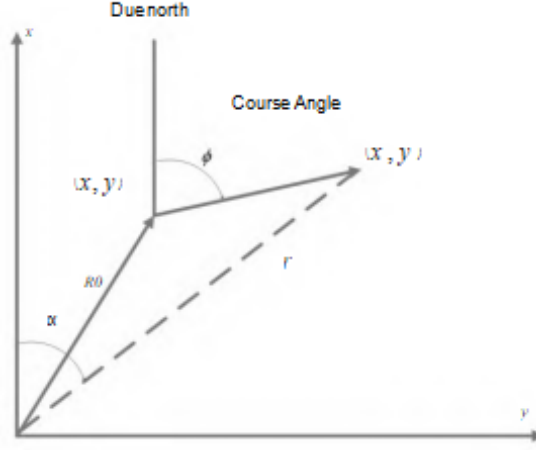


Figure 1: Schematic of submarine movement

This paper first use  $p_v(v)$  and  $p_\phi(\phi)$  to represent the probability distribution density function of submersible speed and heading, respectively, as shown in formula (3):

$$\begin{cases} p_v(v) = \frac{1}{v_2 - v_1} \\ p_\phi(\phi) = \frac{1}{\phi_2 - \phi_1} \end{cases} \quad (3)$$

The initial coordinates of the submersible are  $[x, y]$ , and after the period of travel, the coordinates changed to  $[x', y']$ , as shown in formula (4):

$$\begin{cases} \gamma_0 = \sqrt{x^2 + y^2} \\ \gamma = \sqrt{x'^2 + y'^2} \end{cases} \quad (4)$$

Furthermore, when the submarine travels at heading angle and with a speed  $v$  for a time  $t$ , the probability density function of its position is shown in formula (5):

$$p(\gamma, \alpha, t) = \int_{v_1}^{v_2} \int_{\phi_1}^{\phi_2} p(\gamma_0, \alpha_0, 0) p_\phi(\phi) p_v(v) d\phi dv \quad (5)$$

Secondly, using the cosine theorem, the results are shown in formula (6):

$$r_0^2 = r^2 + (vt)^2 + 2rvt \cos(\phi - \alpha) \quad (6)$$

Then, this paper takes the actual distance  $r_0$  between the coordinate origin and the submarine target, the standard deviation of the direction, and the submersible speed range  $v_1$ 、 $v_2$ ; heading range  $\phi_1$ 、 $\phi_2$ ; to obtain the probability distribution of the submersible's position and orientation over time, as follows:

$$\begin{aligned} p(r, \alpha, t) &= \int_{v_1}^{v_2} \int_{\phi_1}^{\phi_2} p(r_0, \alpha_0, 0) p_\phi(\phi) p_v(v) d\phi dv \\ &= \int_{v_1}^{v_2} \int_{\phi_1}^{\phi_2} \frac{1}{2\pi\sigma^2} e^{-\frac{r^2 + (vt)^2 - 2rvt \cos(\phi - \alpha)}{2\sigma^2}} \cdot \frac{1}{\phi_2 - \phi_1} \cdot \frac{1}{v_2 - v_1} d\phi dv \\ &= \int_{v_1}^{v_2} \int_{\phi_1}^{\phi_2} e^{-\frac{(vt)^2}{2\sigma^2}} \cdot e^{-\frac{rvt \cos(\phi - \alpha)}{\sigma^2}} d\phi dv \end{aligned}$$

$$= \frac{e^{-\frac{r^2}{2\sigma^2}}}{2\pi\sigma^2(\varphi_2-\varphi_1)(v_2-v_1)} \int_{v_1}^{v_2} e^{-\frac{(vt)^2}{2\sigma^2}} \left[ \int_{\varphi_1}^{\varphi_2} e^{\frac{rvt \cos(\varphi-\alpha)}{\sigma^2}} d\varphi \right] dv \quad (7)$$

Finally, it is solved using integration. This paper uses the control variable method to integrate the fixed  $\alpha$  and  $r$  pair formula (7) to obtain the submersible's distance and azimuth distribution density.

## 2.2 Establishment of the Submersible Position Prediction Model

After deriving the formulas for the probability distribution of different time periods, this paper used Monte Carlo and integration methods to simulate the probability distribution of the submersible to ensure the accuracy of the results.

The Monte Carlo model is a probabilistic statistical stochastic simulation method aimed at simulating and predicting physical phenomena through many random samples [8]. The basic principle is to link the solved problem with a specific probability model, and then use a computer for statistical simulation or sampling to obtain an approximate solution to the problem. Therefore, Monte Carlo models can be used to predict the position of submarines at different times [9].

This article predicts the position of a submersible at different times by calling the square point of Matlab's double integral function and Monte Carlo simulation.

## 2.3 Solution of the submersible position prediction model

### 2.3.1 Solving the probability distribution of the submersible distance

Determine the simulation conditions:  $v_1=11\text{km/h}$ ,  $v_2=16\text{km/h}$ ,  $\phi_1=\frac{\pi}{6}$ ,  $\phi_2=\frac{\pi}{2}$ , the standard deviation of the initial distribution of the submersible's distance is  $\sigma=1\text{km}$ .

Calculate the distance probability distribution of the submersible after driving 0.1h, 0.3h, and 0.5h using the integral method and the Monte Carlo method, respectively.

Get the results, as shown in the fig. 2 and fig. 3:

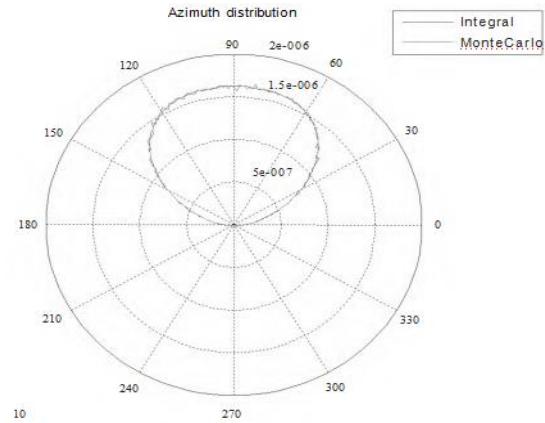
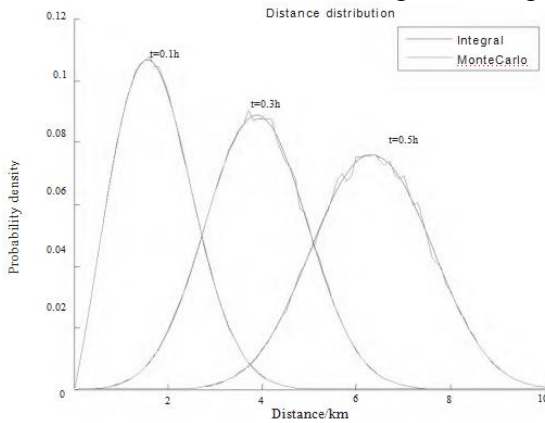


Figure 2: Distance probability distribution graph      Figure 3: Target orientation distribution at  $t=0.1h$  ( $r=6\text{km}$ )

From the shape of the curve, the probability density obtained through the two calculation methods follows a normal distribution. From the trend of the curve, it can be seen that the maximum point of the probability distribution of the distance between the submersible moves further with the passage of time; The maximum value gradually decreases, while the shape of the probability density curve gradually becomes "fat", indicating that the standard deviation is slowly increasing. Therefore, this article concludes that the distance probability peak of the submersible will gradually decrease. In

contrast, the maximum point will slowly move and its standard deviation will continue to rise.

### 2.3.2 Solving the orientation probability distribution of the submersible

Determine the simulation conditions:  $v_1=11\text{km/h}$ ,  $v_2=16\text{km/h}$ ,  $\phi_1=\frac{\pi}{6}$ ,  $\phi_2=\frac{\pi}{2}$ , the standard deviation of the initial distribution of the submersible distance is  $\sigma=1\text{km}$ .

Use the integral method and the Monte Carlo method to calculate the submersible's orientation distribution after driving 0.1h, 0.3 h, and 0.5 h, respectively.

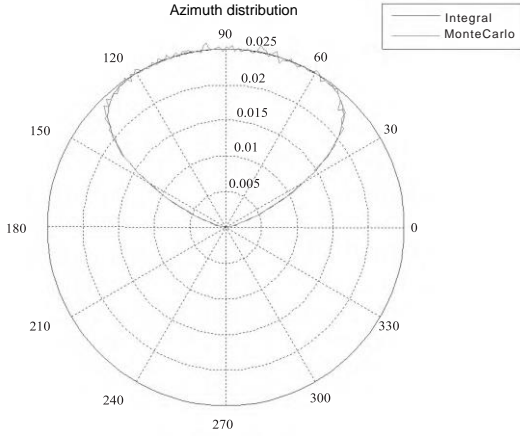


Figure 4: Target orientation distribution at  $t=0.3h$  ( $r=6\text{km}$ ) (Left)

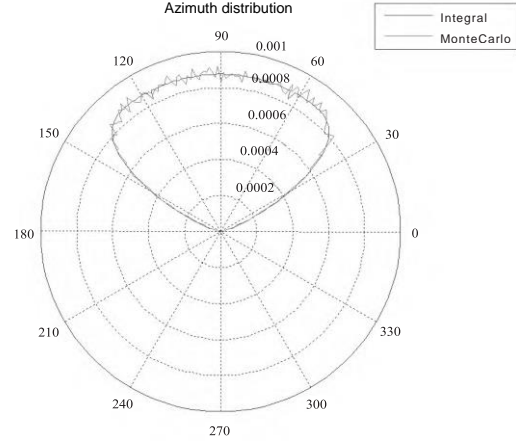


Figure 5: Target orientation distribution at  $t=0.5h$  ( $r=6\text{km}$ ) (Right)

As shown in Figures 4, 5, and 6, at different time points, the directional probability of the submarine is mainly concentrated within the preset heading range. Within this range, the probability of positions in the same direction shows a trend of first increasing and then decreasing.

## 2.4 Optimal paths and finding probabilities

### 2.4.1 Model building

Firstly, this article calculates the coordinates with the highest probability at each moment; Secondly, this article uses the circle search algorithm [10] to record the position of the total distance probability at each moment; Then, this article uses simulated annealing and genetic optimization algorithms to simulate and compare the results; Finally, based on the predicted area, time, and search results, this article identified the probability function for finding the submersible.

### 2.4.2 Solution of the maximum probability value

Firstly, this article identified the distances with the most significant probabilities at different time points; Secondly, this article calculates the azimuth angle at a certain distance and the distance with the most essential likelihood; Finally, calculate the coordinates with the highest probability at each time using equation (8).

$$\begin{cases} X_i = x + x \cdot \cos\theta \\ Y_i = y + y \cdot \sin\theta \end{cases} \quad (8)$$

This paper solves the coordinates as follows:

$$\left\{ \begin{array}{l} (x_1, y_1) = (1.8949, 1.9430) \\ (x_2, y_2) = (2.8061, 2.8703) \\ (x_3, y_3) = (3.3511, 4.2361) \\ (x_4, y_4) = (3.6781, 6.1447) \\ (x_5, y_5) = (6.6704, 4.7532) \\ (x_6, y_6) = (7.7669, 4.6741) \\ (x_7, y_7) = (7.7406, 6.9636) \\ (x_8, y_8) = (8.7478, 7.1629) \\ (x_9, y_9) = (6.9580, 8.9065) \\ (x_{10}, y_{10}) = (6.7207, 9.3237) \end{array} \right. \quad (9)$$

### 2.4.3 Establishment of the circle search algorithm

CSA (Circle Search Algorithm) is a global optimization algorithm whose basic idea is to find the optimal solution by searching within a circular area. This article mainly adopts the concept of circle search algorithm.

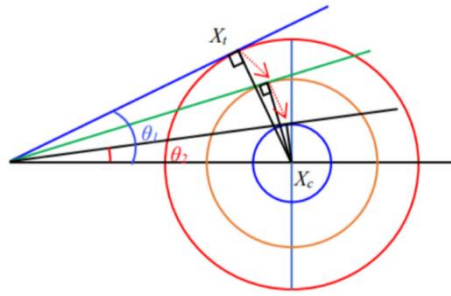


Figure 6: CSA Schematic

Firstly, this article considers the initial point as the center point, which serves as the starting point for the search, and sets the initial search radius. Adjust the radius of the search circle as the search time increases; Then, this article searches and evaluates all elements or nodes within the circle to find several coordinates that satisfy the significant probability of the submersible in that area. If a satisfactory solution cannot be found, the number of coordinates considered can be increased to increase the likelihood of finding the submersible.

### 2.4.4 Simulations this prepare performed for the optimization algorithm.

Based on the above steps, this article first uses simulated annealing algorithm and genetic algorithm to simulate the process.

The simulated annealing algorithm is based on probability, and its basic principle is the solid-state annealing principle. It is a heuristic search algorithm with better global search capability, which can quickly find solutions close to the optimal solution.

Genetic algorithm is a mathematical method that uses computer simulation operations to search for the optimal solution. When solving complex combinatorial optimization problems, better optimization results can be quickly obtained. Therefore, both algorithms are suitable for finding the optimal solution. Secondly, in order to ensure the accuracy of the results, this article compared the results of two algorithms and selected the optimal result.

By inputting the coordinates of the maximum probability value at each moment into the model, the results obtained are as follows, as shown in Figures 7 and 8:

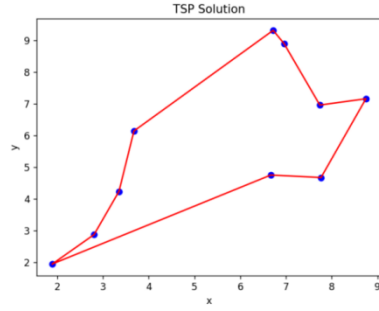


Figure 7: Simulated annealing

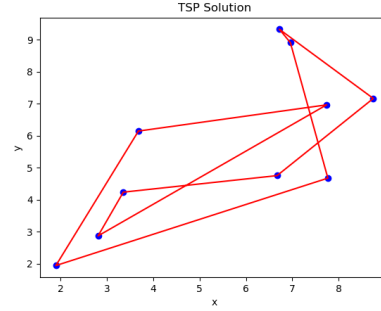


Figure 8: Genetic Algorithm

According to the genetic algorithm, the shortest search path is 35.603440524450676. In contrast, the simulated annealing algorithm generates a shorter search path of 22.024147159703634, and its search time is superior.

## 2.5 Search for best equipments

### 2.5.1 Model building

Firstly, this article determines the weight of each factor based on the variance contribution rate of the principal components and the coefficients of the corresponding indicators in each principal component. Next, this article calculated the cost data and usage frequency separately, and established a cost-benefit analysis model based on the weight, cost, and usage efficiency of the equipment. Finally, this article provides a conclusion. The specific modeling process is as follows.

### 2.5.2 Calculate the paperight of each factor:

Firstly, this article eliminates the square of component values in the component matrix to explain the initial eigenvalues of each indicator in terms of total variance. Next, this article will divide the product of principal component scores by the cumulative variance percentage. Finally, normalize the model score coefficients of each indicator based on the total score to obtain the weight of each indicator. The results are shown in Table 1:

Table 1: This paperight results of each indicator

Equipment	Paperight
Sonar	34%
Communication equipment	27%
Submersible	7%
Armarium	32%

### 2.5.3 Make statistics on the cost data and the use efficiency

The data obtained by reviewing the literature is shown in the following table 2:

Table 2: Statistical results

Equipment	Costing	Service efficiency
Sonar	36884	0.88
Communication equipment	4621	0.73
Submersible	23415	0.92
Armarium	342	0.61



#### 2.5.4 Establish a cost-benefit analysis model.

This paper constructs a cost-benefit analysis model based on the cost and usage efficiency of the equipment:

$$CER = \frac{E}{C} \quad (10)$$

In equation (10), CER represents the cost-benefit ratio, E denotes the efficiency value, and C stands for the cost value.

$$E = W \cdot S \quad (11)$$

Among them, S represents the utilization rate of each device, W represents the paper weight of related devices, and E represents the cost-effectiveness rate of each device.

#### 2.5.5 Solution of model

This paper get the cost-benefit rate of each device as shown in the following Table 3:

Table 3: Results

Equipment	Cost-benefit ratio
Sonar	0.000811
Communication equipment	0.00426
Submersible	0.000275
Armarium	0.057

According to the above analysis, in Country A, there is a high proportion of low-cost and high-efficiency medical equipment. In addition, the cost-effectiveness ratio is the highest, making medical equipment the best choice for carrying additional search and rescue equipment on the main ship.

### 3. Conclusion

The experimental results of this study indicate that calculating the probability distribution of submarines through Monte Carlo method can more accurately predict their location, providing important reference for search and rescue operations; The application of circular search algorithm in search and rescue operations has shown high efficiency and accuracy, which can help search and rescue personnel locate targets faster and improve the possibility of successful search and rescue. The simulated annealing algorithm performs well in optimizing search paths, effectively improving search efficiency, shortening search time, and providing strong support for the implementation of search and rescue operations. The cost-benefit model effectively reduces search and rescue costs. In summary, the time cost optimization strategy proposed in this article, combined with Monte Carlo method, circular search algorithm, and simulated annealing algorithm, can effectively improve the efficiency and success rate of search and rescue operations, provide useful exploration for maritime search and rescue work, and improve the accuracy and reliability of search and rescue tasks.

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