

Endowment fund investment strategy construction and risk assessment: An empirical analysis based on the EWMA methodology

Zhao Hongyu

*The Business School, University of Exeter, Finance and Investment, Exeter, EX4 4PY, UK
hz411@exeter.ac.uk*

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Abstract: This paper offers comprehensive investment guidance for an Endowment Fund by thoroughly analyzing six asset classes: equity ETFs, bond ETFs, real estate ETFs, bitcoin trusts, and insurance products. Utilizing the Exponentially Weighted Moving Average (EWMA) methodology, the study assesses volatility and optimizes portfolio composition. Furthermore, it explores cost-of-debt calculations and yield-to-maturity forecasting techniques, emphasizing the significance of portfolio diversification and risk-adjustment strategies. Ultimately, the Nelson-Siegel model is recommended as the preferred method for yield-to-maturity calculations, thereby enabling more informed and strategic investment decisions.

1. Introduction

This article will make recommendations for building the Endowment Fund. The following six assets will be considered.

- iShares Core FTSE 100 ETF (ISF.L)
- iShares Core MSCI Total International Stock ETF (IXUS)
- Vanguard Total Bond Market Index Fund ETF Shares (BND)
- Vanguard S&P 500 ETF (VOO)
- iShares U.S. Real Estate ETF (IYR)
- Grayscale Bitcoin Trust (GBTC)

Additionally, this paper incorporates an unconventional insurance product (INSUR) to assist in managing investment risks. The analysis relies on data from January 2016 to December 2020. Historical average returns are used as a proxy for expected returns. It is assumed that AB cannot short-sell any risky assets while constructing the risky portfolio, and the risk-free rate is fixed at 0%. The time-varying monthly return variance of the S&P 500 is calculated using the Exponentially Weighted Moving Average (EWMA) method with an attenuation factor of 0.85.

The technical section includes summary statistics on monthly returns for INSUR, the development of an underlying asset portfolio, a comparative analysis of 40-year zero-coupon bond yields using various methodologies, and a description of the data employed. The academic

methodology used in this paper will be detailed in the following section, with findings presented in the third section. Ultimately, empirical data is utilized to draw conclusions and assist AB in making more informed investment decisions.

2. Method

To compare the performance of portfolios across different asset classes, I introduce the concept of volatility. Volatility will be quantified to calculate a specific risk-adjusted performance metric [1]. This article employs the EWMA method to estimate conditional variance, operating under the assumption that volatility changes over time.

$$\hat{\sigma}_t^2 = \lambda \sigma_{t-1}^2 + (1-\lambda)r_{t-1}^2 \quad (1)$$

λ is decay factor

σ_{t-1}^2 is yesterday's variance

r_{t-1} is yesterday's return

Portfolio theory is grounded in the principle of risk aversion [2]. Financial markets offer a wide array of assets, and adding more assets to a portfolio enhances diversification. Investors typically seek investments that minimize risk while maximizing return potential [3]. Therefore, when integrating INSUR into a risk-based portfolio, our objective should be to optimize the Sharpe ratio of the portfolio, thereby determining the optimal weighting for each asset.

When entities issue bonds to finance their operations, it is imperative to consider the associated cost of debt, which is closely linked to the yield to maturity. This paper employs two distinct yield curve models: the Polynomial Model and the Nelson-Siegel Model. These models are used to estimate yields and subsequently identify the optimal solution.

Polynomial model:

$$y_i = a + bM_i + cM_i^2 + dM_i^3 + \varepsilon_i \quad (2)$$

y_i is log yield

M_i is time to maturity

ε_i is error term

a, b, c, and d are parameters to be estimated

Nelson-Siegel model:

$$y_i = \alpha_1 + (\alpha_2 + \alpha_3) \left[\beta \frac{1 - e^{-\frac{M_i}{\beta}}}{\frac{M_i}{\beta}} \right] - \alpha_3 e^{-\frac{M_i}{\beta}} + \varepsilon_i \quad (3)$$

α_1 determines level

α_2 determines slop

α_3 determines curvature

β determines the location of hump

Finally, when constructing an optimal and comprehensive portfolio that includes risk-free assets, it is crucial to consider the portfolio's expected return.

$$r_c = \begin{cases} wr_p + (1 - w)r_f & \text{for } w \leq 1 \\ wr_p + (1 - w)r_b & \text{for } w > 1 \end{cases} \quad (4)$$

r_p denote the return on the optimal risk portfolio

r_f denotes the risk-free rate

r_b denotes the borrowing rate

These are the methodologies covered in this paper, and I will now proceed to delve deeper into the obtained results.

3. Result

To begin, I will provide a summary of the INSUR data spanning from January 1, 2016, to December 1, 2020. The key statistical metrics related to INSUR's monthly earnings are presented in Table 1.

Table 1: Statistic data of INSUR from 1, Jan. 2016 to 1, Dec. 2020

mean	-0.0270646
variance	3.39279026
Standard deviation	1.84195284

To align the INSUR data with the time frame of the six assets under consideration, I have adjusted the dataset by removing the data from January 1, 2016, and adding the data from January 1, 2021. Consequently, the summary statistics for the revised monthly returns of INSUR are presented in Table 2.

Table 2: Statistic data of INSUR from 1, Feb. 2016 to 1, Jan. 2021

mean	-0.0594281
variance	3.3853212
Standard deviation	1.83992424

According to the INSUR statistics, if investors were to invest exclusively in this asset over the next four years, they would face losses rather than gains. The asset's relatively low variance suggests it carries minimal risk. I analyzed the INSUR data by calculating the time-varying monthly return variance of the S&P 500 using the EWMA method with a decay factor (λ). However, I contend that a λ of 0.85 is less appropriate; a lower λ is generally preferred, with 0.94 being more commonly used. Reducing the λ from 0.94 to 0.85 increases the emphasis on recent data while decreasing the weight of older observations. Consequently, volatility computed with a λ of 0.85 reacts more swiftly to new information and shows greater variability compared to volatility calculated with a λ of 0.94. Determining the optimal λ involves comparing our estimates with true values, identifying differences, and minimizing errors, but this is difficult due to the lack of actual values [4].

Additionally, AB has decided to construct a portfolio utilizing the six assets mentioned earlier. Details of the individual assets and the portfolio are presented in Table 3.

Given the constraint against short selling, it is essential that each asset in the portfolio has a positive weight. In constructing a portfolio with six assets, portfolio theory suggests that we should aim to maximize the Sharpe ratio, which requires determining the optimal weight allocations for each asset. As shown in Table 3, BND has the largest allocation at 81.38%, while ISF.L, IXUS, and IYR receive zero weight. Subsequently, AB decided to incorporate INSUR into the portfolio. The revised portfolio, now designated as portfolio 2 (P2), along with its detailed asset weights, is

presented in Table 4.

Table 3: Details of portfolio 1 (P1)

ISF.L	0
IXUS	0
BND	81.38%
VOO	17.05%
IYR	0
GBTC	1.56%
ER	0.72%
variance	0.02%

Table 4: Details of portfolio 2 (P2)

ISF.L	0
IXUS	0
BND	74.07%
VOO	24.07%
IYR	0
GBTC	1.56%
INSUR	0.30%
ER	0.77%
variance	0.02%

In comparison, the asset weights of P1 and P2 are markedly different. The incorporation of new assets into the portfolio and the increase in the total number of assets will inevitably alter the weightings. To achieve an optimal portfolio, it is essential to utilize the assets efficiently. For instance, while BND holds the largest proportion in both P1 and P2, certain assets still have a proportion of 0. Efficient and rational allocation of assets is crucial for helping investors maximize returns and minimize risk.

Subsequently, the firm issued a 40-year zero-coupon bond to finance the project. To estimate the cost of debt, I utilized both the polynomial model and the Nelson-Siegel model to construct the yield curve and estimate the yield to maturity. The results are presented in Table 5, with diagrams illustrated in Figures 1 and 2.

Table 5: Details of yield to maturity

The polynomial model	$y_i = 0.009006783 + 0.003306176M_i - 0.00010742M_i^2 + 0.000001288M_i^3 + \varepsilon_i$
yield to maturity	5.32%
The Nelson-Siegel model	$y_i = 0.055206787 + (-0.04605 - 0.02364) \left[3.7141 \frac{1 - e^{-\frac{M_i}{3.7141}}}{M_i} \right] - (-0.02364)e^{-\frac{M_i}{3.7141}} + \varepsilon_i$
yield to maturity	4.99%

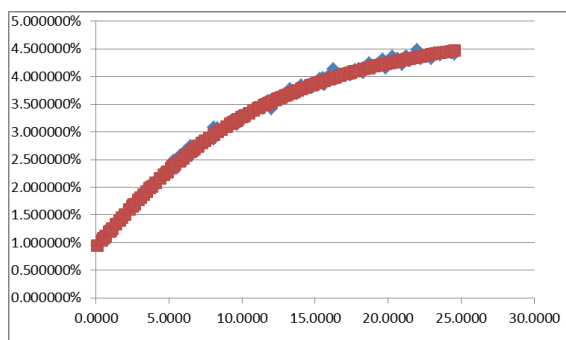


Figure 1: Polynomial model

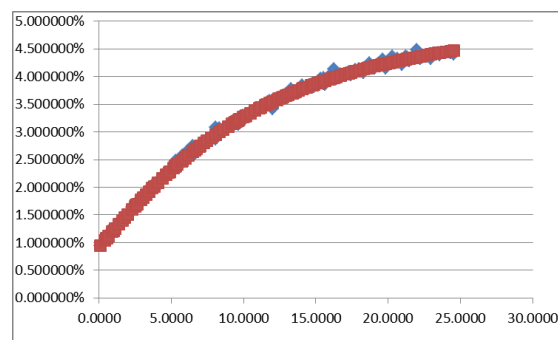


Figure 2: Nelson-Siegel model

According to Table 5, the two models yield different results for the yield to maturity. However, the graphs demonstrate that both methods fit the data remarkably well, with the lines overlapping significantly.

Based on the strengths of the model and the insights gained from this case study, I have chosen to use the Nelson-Siegel model. In this instance, the yield fitted using the Nelson-Siegel model proves to be more accurate than that calculated with the polynomial model, as it more closely approximates the log yield. Polynomial models, in an academic context, often struggle to provide reliable estimates of returns beyond the sample's maximum maturity. As maturity increases, polynomial functions can diverge, whereas actual returns tend to converge to a constant value [4]. Consequently, applying the polynomial model requires meticulous design and a solid understanding of the data to select the optimal exponent. Incorrectly chosen exponents can lead to overfitting.

In contrast, the Nelson-Siegel model provides a well-fitted yield curve and is widely used by central banks and monetary policymakers [5-6]. Fixed-income portfolio managers also employ this model to immunize their portfolios [5][7]. Its ease of linearization and parsimony make it highly effective, with parameter curves flexible enough to capture a broad range of term structure shapes. The model incorporates horizontal, slope, and curvature components, allowing it to be applied to a diverse and complex array of term structures [8].

To construct an optimal complete portfolio incorporating risk-free assets, we evaluate the role of INSUR in both scenarios where it is available and where it is not. At this stage, it is essential to convert the annual borrowing rate into a monthly rate. The conversion is as follows: $4.99\% / 12 = 0.41583\%$. Table 6 presents the complete portfolio constructed by AB.

Table 6: A complete portfolio constructed by AB

INSUR available		Weight
Issue bonds	Portfolio	1.085
	Bond issuing	-0.085
INSUR unavailable		
Issue bonds	Portfolio	1.263
	Bond issuing	-0.263

From Table 6, it is evident that AB should issue bonds to fund its investments, regardless of whether INSUR is available. When INSUR is available, the portfolio weight is 1.085, while the weight of the bond issue is -0.085. Conversely, when INSUR is unavailable, the portfolio weight increases to 1.263, and the weight of the bond issue becomes -0.263.

4. Recommendation

I advise AB to maintain a diversified mix of all major asset classes. This diversification strategy will allow them to achieve the highest long-term returns while minimizing risk. To meet their long-term goals and enhance returns, AB should regularly evaluate the portfolio to assess asset performance and make necessary adjustments.

I suggest that AB pre-determine an acceptable range of cost fluctuations from a cost and return perspective during the portfolio's initial phase. AB should then maintain the costs of individual components within this range throughout the portfolio's lifecycle. Furthermore, AB should consider the time value of money and establish a "return realization trajectory" in the introductory phase to ensure alignment with expected returns. Additionally, AB should take into account external factors, including competitiveness and prevailing market conditions.

In terms of asset classes, this case features a well-diversified portfolio. However, I recommend that AB choose gold over INSUR for portfolio construction. Gold has an annualized return of approximately 9% [9], whereas insurance products typically yield less than 5% [10]. When individual assets provide higher returns for a given weighting, the overall portfolio benefits from increased returns. Incorporating gold into the portfolio is expected to boost the overall return by 0.012% compared to the previous portfolio's expected return, while the associated risk of gold remains relatively low.

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Appendix

Appendix A: Details of the INSUR

INSUR is an innovative insurance product designed to help investors effectively manage their exposure to market risk. The product's mechanism requires the insured (i.e., the purchaser of INSUR) to pay a premium at the beginning of each month as a fee for the insurance service. INSUR's coverage extends for the entire month, concluding on the last day. The decision to provide a pay-out to the insured is based on the monthly performance of the S&P 500 Index.

INSUR activates its pay-out mechanism when two specific conditions are met: First, the monthly return of the S&P 500 Index is negative, and second, the standard deviation of the monthly return exceeds a threshold of 0.025. In such instances, INSUR will refund the full premium paid by the policyholder at the beginning of the month and provide additional compensation. This compensation is calculated based on two factors: the base amount, which is 150 times the premium, and the excess standard deviation above 0.025, which is used as the multiplier. The base amount is multiplied by this excess standard deviation to determine the final compensation amount.

Conversely, if the S&P 500 Index achieves a positive return for the month, or if the standard deviation of its monthly return remains at 0.025 or below, INSUR will not provide any payment to the policyholder at the end of the month. In these situations, the policyholder will bear the full loss of the premium without receiving any additional gain. Therefore, the monthly return from holding INSUR will be:

$$\begin{cases} -100\% & \text{if } r_t \geq 0 \text{ or } \sigma_t \leq 0.025 \\ (\sigma_t - 0.025) \times 15000\% & \text{if } r_t < 0 \text{ and } \sigma_t > 0.025 \end{cases} \quad (5)$$

r_t represents the return of the S&P 500 index at month t , while σ is the monthly (non-annualized) standard deviation of the index's return. It is worth noting that the method used to calculate σ^2 (i.e., the square of the standard deviation) is the EWMA, which specifically sets the attenuation factor to 0.8 to give higher weight to recent data. The key point is that when estimating σ for month t , we only consider historical monthly return data before month t , i.e. the return r_t for the current month t is not included in this estimation process. This setting ensures that the estimated standard deviation can be based on past information and is not affected by the immediate performance of the current month.