

Study on the relationship between multibeam bathymetric coverage width and overlap rate based on seafloor slope

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Abstract: Multibeam bathymetric technology, with its high-precision and full-coverage characteristics, has become an essential tool for seabed topographic mapping with the development of marine resources and the advancement of marine scientific research. Optimizing its coverage width and overlap rate is critical to enhancing the efficiency and accuracy of mapping. To examine the connection between the two, the mathematical model of multibeam bathymetry coverage width is created using the geometric and analytical methods, respectively, and the results obtained by the two methods are consistent. The investigation focuses on the relationship between multibeam bathymetry coverage width and overlap. Due to the uncertainty of the terrain, the coverage width is studied for the slope of multibeam bathymetry, and the results confirm that the slope has a certain influence on it. Therefore, accurate measurement of the coverage width of different slopes provides powerful technical support for ocean mapping.

1. Introduction

Efficient acquisition of marine topographic information is of great value in the fields of scientific research, resource exploration, and development, marine engineering construction, marine environmental protection, as well as military and national defense[1,2]. Multibeam survey technology has become the most popular technology in the field of marine topography. Multi-beam sounding technology has become an indispensable means of marine topographic mapping with its high-precision and full-coverage characteristics, which realize the acquisition of three-dimensional information on the seabed by transmitting and receiving multiple acoustic beams and processing them accurately[3]. In the practical application, the width of coverage and the accuracy of the sound beams are very high. In practical application, the coverage width and overlap degree are the key factors affecting the accuracy of detection, the coverage width determines the range of seabed area that can be covered in a single measurement, while the overlap rate affects the continuity and consistency of the mapping data. Therefore, how to optimize the survey line arrangement to achieve the best coverage effect under specific seafloor topography and survey conditions is one of the key issues in improving the performance of multibeam bathymetric systems.

When it comes to seafloor topography, multibeam bathymetric technology is more effective than single- and four-beam bathymetric systems[4]. However, as science and technology advance, the need for accurate maritime topographic data grows, leading academics to extensively study the error analysis of multibeam. Du et al. rejected the noise generated by beam propagation by filtering

algorithm[5]. Ma et al proposed a topographic data acquisition and modeling method based on a multibeam bathymetry system[6]. Wang et al. proposed to evaluate the accuracy of fringe beam data by using the overlapping area of neighboring survey lines[7]. The factors affecting the error of multibeam bathymetry are diversified, so the research on the relationship between coverage width and overlap rate in this paper is very innovative and feasible.

The data in this paper are obtained from <http://www.mcm.edu.cn/>, and the width of the survey line coverage and the overlap rate are first solved by establishing a right-angled coordinate system and a trigonometric function, respectively. In addition, the slope angle is obtained by geometrical and analytical methods, to establish a mathematical model, and subsequently derive the depth of seawater out of the center point of the sea area.

2. Coverage width and overlap analysis for multibeam bathymetry

To ensure the consistency of the results, this study here uses analytical and geometric methods to analyze the coverage width and overlap rate of multibeam bathymetry respectively.

2.1 The structure of the BP neural network

BP neural network is a multi-layer network with error reverse propagation, which is composed of input layer nodes, hidden layer nodes, and output layer nodes. This process has been reduced to an acceptable level of error to the network output, or to a predetermined number of learning times. The network structure is shown in Figure 1.

Multibeam bathymetric systems have no postural bias[8]. In the case of multibeam bathymetric systems, the width of the strips measured by each device at the same depth is considered to be constant. A coordinate system was constructed using the projection point from the center of the sea area onto the seabed as the origin and the x axis parallel to the sea level. The multi-beam sounding schematic was plotted 1:

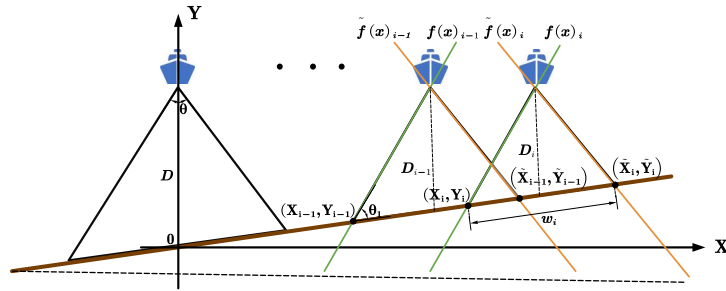


Figure 1: Schematic diagram of the bathymetric system in the coordinate system

From Figure 1, the formula for calculating the depth of seawater is as follows:

$$D_i = D - \tan\alpha * x_i (i = 0, \pm 1, \pm 2 \dots) \quad (1)$$

Where D_i is the depth of seawater at the i point of measurement. When i is positive, it indicates that the points are arranged to the right of the origin. When i is negative, it indicates that the points are arranged to the left of the origin; α is the seafloor slope; D is the depth of seawater at the centroid; x_i is the distance of the i line from the centroid; x is the distance of the sidetrack from the centroid; and θ the opening angle of the multibeam transducer.

From the figure it is easy to obtain $\theta_1 = \frac{\pi}{2} - \frac{\theta}{2}$, the expression describing the seafloor curve:

$$y = \tan\alpha * x \quad (2)$$

The expression describing the line where the leftmost beam is located in the beam emitted by the bathymetric system i is:

$$f(x)_i = \tan\theta_1 * (x - x_i) + D \quad (3)$$

The expression describing the line where the rightmost beam is located is:

$$\tilde{f}(x)_i = -\tan\theta_1 * (x - x_i) + D \quad (4)$$

Since the multibeam transducers have the same opening angle, the expression describing the straight line where the leftmost and rightmost survey beams are located corresponding to the bathymetric point located to the left of the i is:

$$f(x)_{i-1} = \tan\theta_1 * (x - x_{i-1}) + D \quad (5)$$

$$\tilde{f}(x)_i = -\tan\theta_1 * (x - x_i) + D \quad (6)$$

The expression on the right-hand side can be obtained in the same way. The above equation gives the coordinates of the leftmost part of the overlap corresponding to the i bathymetric point (X_i, Y_i) , and can obtain the coordinates of the rightmost part of the overlap $(\tilde{X}_i, \tilde{Y}_i)$, then the coverage width of the i bathymetric point W_i is:

$$W_i = \sqrt{(\tilde{Y}_i - Y_i)^2 + (\tilde{X}_i - X_i)^2} \quad (7)$$

Given that the overlap resembles the bathymetric section's shape, the overlap between the measured range of the i bathymetric point and the previous survey line is:

$$\eta_i = \frac{\tilde{X}_{i-1} - X_i}{\tilde{X}_i - X_i} \quad (8)$$

It is known that the opening angle of the multibeam transducer is 120° , the slope is 1.5° , and the depth of seawater at the center point of the sea is 70 m. The results are calculated by MATLAB and are compiled in the table 1 below:

Table 1: Statistics for each parameter under the localized line of measurement of the slope seabed

Distance of line from center /m	Sea depth D_i / m	Coverage width W_i / m	Overlap with the previous line η_i / %
-800	90.95	315.8	--
-600	85.71	297.6	35.7
-400	80.47	279.4	31.5
-200	75.24	261.3	26.7
0	70.00	243.1	21.3
200	64.76	224.9	14.9
400	59.53	206.7	7.4
600	54.29	188.5	-1.5
800	49.05	170.3	-12.7

2.2 Geometric methods to investigate the relationship between width and overlap rate

Owing to the shape's specificity, the overlap rate may be computed directly from the spacing and coverage breadth of two neighboring lines[9]. The geometric relationship can be used to obtain the lengths of the necessary edges when the seabed has a gradient because the bathymetric range of neighboring bathymetric points has a similar shape and the coverage width and spacing of two adjacent lines have a certain quantitative relationship, Figure 2:

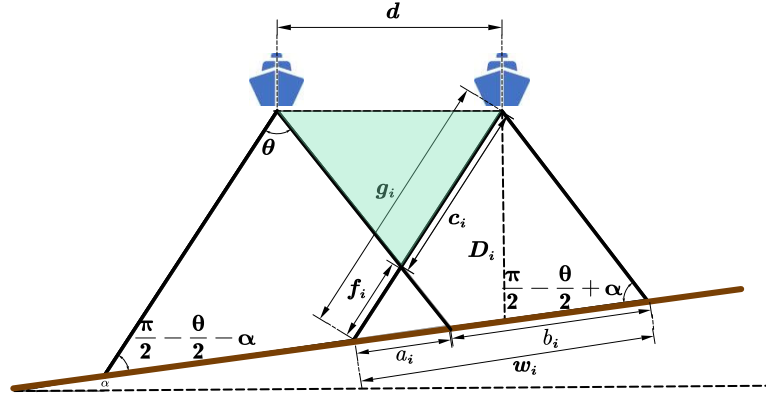


Figure 2: Main view of the bathymetric system

From the figure 2, the coverage width corresponding to the i measurement line is:

$$W_i = a_i + b_i \quad (9)$$

by the sine theorem:

$$\frac{a_i}{\sin \frac{\theta}{2}} = \frac{D_i}{\sin(\frac{\pi}{2} - \frac{\theta}{2} - \alpha)} \quad (10)$$

$$\frac{b_i}{\sin \frac{\theta}{2}} = \frac{D_i}{\sin(\frac{\pi}{2} - \frac{\theta}{2} + \alpha)} \quad (11)$$

Calculate a_i , b_i and then put them into the formula(9) The coverage width corresponding to the i line can be calculated:

$$W_i = \frac{D_i \sin \frac{\theta}{2}}{\sin(\frac{\pi}{2} - \frac{\theta}{2} - \alpha)} + \frac{D_i \sin \frac{\theta}{2}}{\sin(\frac{\pi}{2} - \frac{\theta}{2} + \alpha)} \quad (12)$$

Where, D_i is the depth of the sea at the first i measurement point and d is the distance between two neighboring vessels.

Based on the geometric features of isosceles triangles, the following equation can be derived for any triangle that resembles the shaded area in the diagram:

$$\frac{d}{2} = c_i * \cos(\frac{\pi}{2} - \frac{\theta}{2}) \quad (13)$$

$$\frac{g_i}{\sin(\frac{\pi}{2} - \frac{\theta}{2} + \alpha)} = \frac{W_i}{\sin\theta} \quad (14)$$

Since the overlap is similar in shape to the bathymetric range, the overlap rate is obtained using the quantitative relationship between edges and edges:

$$f_i = g_i + c_i \quad (15)$$

$$\eta_i = \frac{a_i}{W_i} = \frac{f_i}{g_i} \quad (16)$$

The equation between the overlap rate η_i and the coverage width W_i is obtained:

$$\eta_i = 1 - \frac{d \sin\theta}{2w_i \cos(\frac{\pi}{2} - \frac{\theta}{2}) \sin(\frac{\pi}{2} - \frac{\theta}{2} + \alpha)} \quad (17)$$

The coverage breadth and overlap rate can be computed using the equation above. The computation agrees with the constructive coordinate system analysis method results, demonstrating the high degree of reliability of the findings.

3. Slope impact analysis of multibeam bathymetry

The large data prediction model for the user's electricity consumption is implemented in the Clementine software. Create a rectangular image of the sea area to be measured, as shown in Figure 3, and the angle between the direction of the survey line and the normal direction of the seabed slope projected on the horizontal plane β .

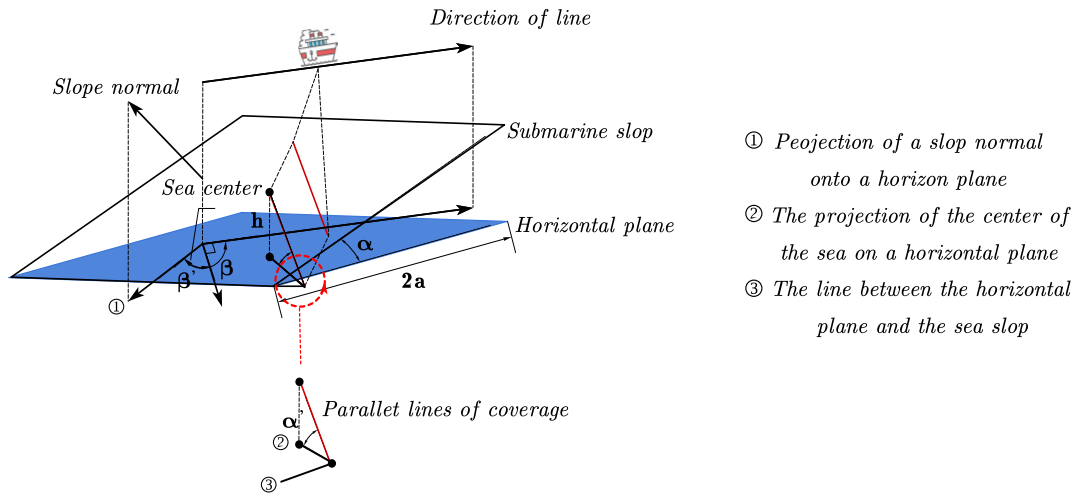


Figure 3: Direction of survey line and slope

Any survey line intersects the seabed's slope at an angle $\tilde{\alpha}$ with the horizontal at the junction of a plane perpendicular to the line's direction. h is the height of the projection point of the sea area's center on the seabed to the patch's lowest seabed level. The tangent of the seabed's slope is as follows:

$$\tan\alpha = \frac{h}{a} \quad (18)$$

The geometric relationship defines the angle between the survey line's direction and the seabed slope's normal projection on the horizontal plane. The following is the smaller angle $\tilde{\beta}$ that results from projecting the normal vector of the seabed slope on the horizontal plane onto the line that the survey line lies on:

$$\tilde{\beta} = \begin{cases} \beta, & (0 < \beta \leq \frac{\pi}{2}) \\ \pi - \beta, & (\frac{\pi}{2} < \beta \leq \pi) \\ \beta - \pi, & (\pi < \beta \leq \frac{3\pi}{2}) \\ 2\pi - \beta, & (\frac{3\pi}{2} < \beta \leq 2\pi) \end{cases} \quad (19)$$

The following relationship is derived from the trigonometric functions:

$$\frac{a}{\cos(\frac{\pi}{2} - \tilde{\beta})} = x \quad (20)$$

where x refers to the line segment formed by the projection of a plane perpendicular to the direction of the line of measurement onto the horizontal plane. The formula $\tilde{\alpha}$ is derived from $\tan\tilde{\alpha} = \frac{h}{x}$:

$$\tan\tilde{\alpha} = \tan\alpha * \cos(\frac{\pi}{2} - \tilde{\beta}) \quad (21)$$

The magnitude of the angle between the plane perpendicular to the direction of the line of measurement and the intersection line formed by the slope of the seabed and the horizontal plane can be found by calculating $\tan\tilde{\alpha}$.

$$W(x) = (D - x \tan\tilde{\alpha}) \cdot \sin\frac{\theta}{2} \cdot \left(\frac{1}{\sin(\frac{\pi}{2} - \frac{\theta}{2} - \tilde{\alpha})} + \frac{1}{\sin(\frac{\pi}{2} - \frac{\theta}{2} + \tilde{\alpha})} \right) \quad (22)$$

Where x refers to the distance of the survey vessel from the center of the sea, D refers to the depth of water at the center of the sea, and θ refers to the opening angle of the multibeam transducer.

The angle of different measurement line directions can be obtained from the size of the intersection line formed by different vertical planes and the seabed slope and the horizontal plane angle $\tilde{\alpha}$, known as the multibeam transducer opening angle of 120.0°, the slope of 1.5°, the depth of seawater at the center of the sea area of the depth of 120.0 m, you can derive the width of the coverage of the sounding[10]. The calculation results are shown in the table 2 below:

Table 2: Showing the findings for the measuring vessel's distance from the sea area's center

Coverage width/m		Distance/nautical mile from the center of the sea by the survey vessel							
		0	0.3	0.6	0.9	1.2	1.5	1.8	2.1
Measuring line direction angle/°	0	415.69	466.09	516.49	566.89	617.29	667.69	718.09	768.48
	45	416.19	451.87	487.55	523.23	558.91	594.59	630.27	665.95
	90	416.69	416.69	416.69	416.69	416.69	416.69	416.69	416.69
	135	416.19	380.51	344.83	309.15	273.47	237.79	202.11	166.43
	180	415.69	365.29	314.89	264.50	214.10	163.70	113.30	62.90
	225	416.19	380.51	344.83	309.15	273.47	237.79	202.11	166.43
	270	416.69	416.69	416.69	416.69	416.69	416.69	416.69	416.69
	315	416.19	451.87	487.55	523.23	558.91	594.59	630.27	665.95

4. Conclusions

The multi-wave velocity measuring system's excellent resolution, wide coverage, and accuracy make it an essential instrument for maritime topographic surveys. This research examines in depth the relationship between breadth and coverage using two alternative methodologies. First, a mathematical model of width and coverage is established by determining the functional expressions of the straight lines corresponding to the outermost beam of the survey line and the seabed slope by the geometrical relationship under the right-angled coordinate system (RCS); second, the coverage width of the survey line corresponding to each survey line is calculated using the trigonometric function, and it is then solved by the geometrical relationship. Finally, the coverage width of the line at the center of the sea area is 243.1m, and the overlap rate with the previous line is 21.3%. Consequently, the existence of a relationship between breadth and coverage is established. Since the seabed position's uncertainty is influenced by the slope topography within the spot range, this paper examines how slope affects multibeam bathymetry's coverage width, determines the new survey line's slope angle, creates a mathematical model of the coverage width of multibeam bathymetry, determines the coverage widths of the corresponding surveys under various slopes, and explores the impact of slope on it. This paper provides a research idea and framework for marine geomorphological exploration, marine engineering construction, offshore terrain-aided navigation systems, and other related fields, proving the feasibility of the method.

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