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Research on Real Estate Investment Project Valuation Based on the Binomial Tree Model

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Abstract: The valuation of real estate investment projects is a highly complex issue. Traditional net present value (NPV) evaluation methods largely depend on estimates of future variables, which are often characterized by certainty. This paper focuses on the characteristic of real estate investment projects that allows for delayed development, establishes project asset volatility, and then constructs a binomial tree option pricing model. Empirical analysis based on the binomial tree model shows that the value of real estate investment projects can be reasonably estimated by continuously adjusting the volatility of the project assets and thereby adjusting the expectations of future income changes.

1. Introduction

The valuation of real estate investment projects is an extremely complex issue. The net present value (NPV) method is a classical approach based on a discounted cash flow model to calculate the time value of money [1]. Traditional NPV has many limitations, such as overreliance on the determination of the benchmark rate of return and estimates of future cash flows, which makes it difficult to accurately evaluate the option value implied in investment projects [2]. It does not consider the uncertainty and volatility of the external environment and also ignores management flexibility [3]. According to China's "Regulations on the Management of Urban Real Estate Development", if a real estate enterprise does not start development within one year after obtaining the land use rights as stipulated in the contract, a land idleness fee of up to 20% of the land concession fee may be levied; if development has not started within two years, the land use rights may be revoked without compensation. Such regulations imply that real estate enterprises have an American-style call option with a maturity of two years. Incorporating this delay option into the valuation process of real estate projects can improve and enrich the assessment tools and methods for real estate investment projects, further enhancing the scientific accuracy of real estate project valuation and providing effective theoretical guidance for real estate valuation.

Options, as financial derivatives, have been widely studied in terms of pricing. In 1973, Black and Scholes established the options pricing formula (BS formula) [4], which propelled the in-depth development of the financial derivatives market. However, the assumptions of the BS formula do not match reality, and some scholars have made modifications [5]. Numerical methods commonly used in option pricing include finite difference methods, Monte Carlo simulation, and Fourier transform methods [6]. The binomial tree method, first proposed by Cox-Ross-Rubinstein in 1979 [7], is an intuitive, flexible, and easy-to-understand discrete model with a wide range of applications. Binomial tree theory is based on the hypothesis of efficient markets and is one of the simplest and most classic option theories [8]. One of the foundations for establishing the binomial tree option pricing model is the principle of risk neutrality, which assumes that the value of the option is independent of anyone's view of risk, and investors ignore risk without demanding compensation when encountering risk [9]. Furthermore, the introduction of volatility is premised on the assumption that the underlying asset's value is a random variable, not a fixed value. Through the impact of volatility on the parameters of the binomial tree model, the possible value of the underlying asset under uncertain conditions is fully realized [10].

2. Delayed Option Pricing Based on the Binomial Tree Model

Building on the Cox-Ross-Rubinstein (1979) binomial tree model, consider a non-dividend-paying asset option f. The current price of the underlying asset is S, and during the option's validity period T, the price of the asset rises to Su with a certain probability P or falls to Sd with another probability $1-P$. Under the no-arbitrage assumption, the risk-neutral valuation method is used to price the option. Assume a constructed portfolio consists of one unit of the underlying asset Δ and units of the option. When the price of the underlying asset rises, the value of the portfolio is $Su\Delta - fu$, and when the price falls, the value of the portfolio is $Sd - fd$. By the principle of risk neutrality, it follows that $Su\Delta - fu = Sd\Delta - fd$. Let $\Delta = \frac{fu - fd}{Su - Sd}$. represent the risk-free interest rate r, then the present value of the portfolio is equal to the cost of the constructed portfolio, $S\Delta - f = (Su\Delta - fu)e^{-rT}$, substituting Δ into the equation gives: $f = e^{-rT}\left[pfu + (1-p)fd\right]$

Where, $p = \frac{e^{rT} - d}{u - d}$.

From the above equations, option pricing can be determined. Cox, Ross, and Rubinstein extended the single-step binomial tree to smaller time intervals Δt, assuming that within each interval, the asset price moves to one of two possible values from S. This model can be illustrated by Figure 1, assuming $u = \frac{1}{d}$, the complete equations for the multi-step binomial tree model are obtained:

$$u = e^{\sigma\sqrt{\Delta t}} \tag{1}$$

$$d = e^{-\sigma\sqrt{\Delta t}} \tag{2}$$

$$p = \frac{e^{r\Delta t} - d}{u - d} \tag{3}$$

where σ is the volatility of the underlying asset's value.

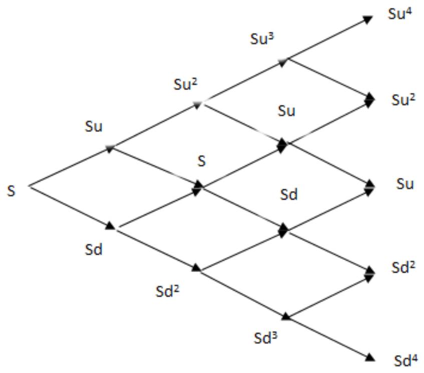

Figure 1: Illustrates a multi-step binomial tree for option valuation [11].

Let (i, j) represent a node in the binomial tree at time , j and step j, where nodes increase in value from bottom to top and from left to right, $F(i, j)$ representing the increasing values of the binomial options (i, j) . At time $i\Delta t$, there are $i+1$ possible prices for the underlying asset, expressed as $Su^j d^{i-j} (j = 0,1,2...i)$ The computation of option values starts from the final time T, moving backwards. Based on $S_{ij} = Su^j d^{i-j}$ the values obtained S_{ij}, X denotes the strike price at the final node, from which we derive:

$$F(i,j) = max[S(i,j) - X, 0] \tag{4}$$

At time $(i-1)\Delta t$, we have:

$$F(i, j) = \left[pF(i, j+1) + (1-p)F(i+1, j+1) \right] e^{-r\Delta t}$$

Considering that American options can be exercised early, the F_{ij} in the above equations must be compared with the intrinsic value of the call option, thus we have:

$$F(i,j) = max\{NPV(i,j), [pF(i,j+1) + (1-p)F(i+1,j+1)]e^{-r\Delta t}\} \tag{5}$$

Where $S(i, j) - X$, (i, j) represents the value corresponding to the node NPV, denoted as $NPV(i, j)$.

3. Empirical Analysis

In 2019, a developer acquired land use rights for a plot through an auction. The land is designated for residential use, with the project adopting a presale model. The project has received approval for initiation from Zhengzhou City and is currently in the preliminary planning and design phase. The loan interest rate from commercial banks is 10% (converted to a quarterly risk rate of 2.5%). The project is located in the southern suburbs of Zhengzhou, with a planned land area of 48,000 square meters, a building land area of 3,500 square meters, and a proposed building area of 84,200 square meters. The above-ground structure will have twenty-five floors, totaling 80,000 square meters, and an underground area of two floors spanning 4,000 square meters.

3.1. Traditional Net Present Value Method Evaluation

Given that the future development of Zhengzhou is mainly concentrated in the new district in the eastern suburbs of Zhengzhou, and this project is located in the southern suburbs, it is primarily targeted at mid-range residential development. Based on the development prospects of Zhengzhou and the current market situation, the residential sale price of this project is set at 5,800 yuan per square meter. The economic evaluation of the project primarily utilizes the total cost expense table and the complete investment cash flow statement See Table 1.

Table 1: Total Cost Expense Summary Table

Serial Number	Costs and Expenses	Total (in ten thousand yuan)
1	Development Costs	31890
1.1	Land Costs	6100
1.2	Preliminary Engineering Costs	1015
1.3	Construction and Installation Costs	20500
1.4	Infrastructure Engineering Costs	135
1.5	Public Facilities Costs	50
1.6	Taxes and Fees During Development	2700
1.7	Unforeseen Expenses	1390
2	Costs During Development	2504
2.1	Management Expenses	834
2.2	Financial Expenses	205
2.3	Sales Expenses	1465
3	Total	34394

Note: The unforeseen expenses are calculated as 5% of the sum of items 1.1 to 1.5; management expenses are calculated as 3% of the sum of items 1.1 to 1.5; sales expenses are calculated as 3% of sales revenue. 60% of the land cost is financed through loans, with an interest rate of 5.6%. See Table 2.

Table 2: Total Investment Cash Flow Table

Item Number	Item	Construction and Operation Period			
		0	1	2	3
1	Cash Inflow	0	0	29302	19534
1.1	Sales Revenue	0	0	29302	19534
1.2	Other Cash Inflows	0	0	0	0
2	Cash Outflow	6100	28089	2051	1367
2.1	Construction Investment	6100	28089		
2.2	Land Appreciation Tax	0	0	293	195
2.3	Sales Tax and Surcharges			1758	1172
3	Net Cash Flow	-6100	-28089	27251	18167
4	Net Present Value	-6100	-25080	21724	12931

Note: 1) The construction period for the project is 1 year, with sales progressing 60% in the first year after completion and 40% in the second year; land appreciation tax is calculated at 1% of sales revenue, and the baseline return rate is set at 12%. 2) Construction investment is the total cost minus land cost and financial expenses. Based on the total cost summary and the complete investment cash flow statement, if the project is developed immediately and calculated using the real estate industry's baseline yield of 12%, the net present value is 34.7527 million yuan. Compared to the investment, the profit level is low, with a profitability rate of 10.10%, just maintaining at the industry's benchmark return level. Using the net cash flow method to evaluate, this project is not considered ideal.

3.2. Real Options Method Evaluation

1) Determine the expiration time of the option Δt .According to the "Urban Real Estate Development and Operation Management Regulations", the two-year period after a real estate company acquires land use rights can be divided into 8 time segments, representing individual $T=8$ quarters and $\Delta t=0.25$ years.

2) Determine the project's asset volatility σ . Volatility σ is the only parameter not directly provided by the market, and the accuracy of its estimation significantly impacts many parameter values in the option pricing model, greatly affecting option pricing. The project's expected income can be adjusted by modifying the asset volatility. According to the volatility calculation method proposed by Timothy (1998), by using the product price fluctuation method based on investment return data from the same or similar industries, the asset volatility for this project is derived $\sigma=14\%$.

3) Determine u、d、p .Using the volatility $\sigma=14\%$,calculations can be derived $u=1.07,\ d=0.93,\ p=0.53$ from formulas 2.1, 2.2, and 2.3.

4) To calculate the delayed option value of the investment project, it is necessary to establish a price development table, NPV expansion table, and option expansion table, with the specific calculation process shown in Tables 3, 4, and 5.

Table 3: Residential Project Price Development Table

Development Time Point	0	1.1	1.2	1.3	1.4	2.1	2.2	2.3	2.4
	5800	6221	6672	7156	7675	8231	8828	9468	10154
		5408	5800	6221	6672	7156	7675	8231	8828
			5042	5408	5800	6221	6672	7156	7675
				4701	5042	5408	5800	6221	6672
					4383	4701	5042	5408	5800
						4087	4383	4701	5042
							3811	4087	4383
								3553	3811
									3313

Note: 1) The numbers in the table increase from left to right and decrease from top to bottom. 2) Example of calculation in the table: 6221 = 5800u; 5408 = 5800d. 3) In the table, 1.1 refers to the first quarter of the first year of operation; period 0 indicates immediate development.

Table 4: Residential Project NPV Expansion Table

Development Time Point	0	1.1	1.2	1.3	1.4	2.1	2.2	2.3	2.4
	3475	5990	8686	11578	14679	18001	21568	25033	29491
		1133	3475	5991	8686	11578	14679	18001	21568
			-1054	1133	3475	5991	8686	11578	14679
				-3091	-1054	1133	3475	5991	8686
					-4991	-3091	-1054	1133	3475
						-6760	-4991	-3091	-1054
							-8409	-6760	-4991
								-9551	-8409
									-11385

Note: The net cash flows in the table are calculated jointly from Tables 1, 2, and 3. Since the sales expenses are a small proportion, the calculation of NPV does not adjust the total cost and expenses, but only adjusts for value-added tax and sales tax surcharges.

Table 5: Residential Project Options Expansion Table

Development Time Point	0	1.1	1.2	1.3	2.1	2.2	2.3	2.4
	5403	7062	9899	12537	18580	21954	25585	29491
		3624	5142	7089	12156	15065	18197	21568
			1980	3040	6568	9073	11772	14679
				824	2349	3862	6183	8686
					364	694	1325	3475
					0	0	0	0
					0	0	0	0
					0	0	0	0
					0	0	0	0

Note: The final period is calculated according to the European option valuation formula (4), while the periods before the last are calculated according to the American option valuation formula (5).

Through the calculations of the options expansion table, the value of the delay option for this project can be determined $F = 5403$. Hence, the investment value of the project is the sum of the delay option value and the net present value (87.78 million yuan), achieving an investment profit rate of 25.81%. The project is feasible. Compared to traditional net present value assessment methods, which often underestimate real value by not considering the option value of delays, this approach emphasizes the value of flexibility and enhances the developer's investment decision-making opportunities.

4. Conclusion

While the traditional net present value (NPV) method has many advantages, such as simplicity and ease of implementation, it also has notable disadvantages, such as the certainty required in the input data and excessive dependence on accurate forecasts of future cash flows. Real estate investment projects are characterized by significant investments and high risks. An NPV analysis of a plot in Zhengzhou indicated that it was not an ideal project, while a real options approach suggested its feasibility. The real options approach, which considers the valuation results of project delay options, offers a more accurate reflection of the project's true value. Therefore, it is advisable to include the value of delay options brought by postponing project development into the investment value of real estate projects, using a binomial tree model for a more accurate assessment.

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