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Exploration of the Path of Computational Thinking Cultivation in University Mathematics Teaching

Jianwei Du, Shuxia Wang*, Xinhong Liu, Qinxian Shi

Zhiyuan School of Liberal Arts, Beijing Institute of Petrochemical Technology, Beijing, 102617, China

**Corresponding author*

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Abstract: Through a review of the concept of computational thinking and the current state of education, this article derives the essential connotations of computational thinking that should be cultivated in university mathematics teaching. It explores three main approaches to fostering computational thinking among college students: constructing classroom teaching scenarios, cultivating computational thinking through immersive learning modes, developing computational thinking in problem-solving, and fostering computational thinking through project-based learning. These approaches provide valuable references for reforming university mathematics teaching methods in the era of artificial intelligence.

1. Introduction

1.1 The origin of the concept of computational thinking

The concept of Computational Thinking originated in March 2006, when Professor Jeannette M. Wing, the head of the Computer Science Department at Carnegie Mellon University in the United States, introduced and defined it in the authoritative American computer journal "Communications of the ACM". Professor Wing believes that Computational Thinking is a series of thinking activities that encompass the breadth of computer science, such as problem-solving, system design, and understanding human behavior, using fundamental concepts from computer science. She argues that Computational Thinking is a basic skill for everyone, not just computer scientists, and that every child should learn Computational Thinking along with analytical skills[1].

1.2 The current situation of computational thinking education at home and abroad

In 2007, the European industrial and scientific communities convened a conference on "Thinking Science: Europe's Next Policy Challenge," emphasizing the importance of Computational Thinking. In 2011, the Computer Science Teachers Association in the United States included Computational Thinking in the "K-12 Computer Science Standards," marking its official entry into curriculum standards. In 2016, the European Commission's Joint Research Center released a "Report on the Development of Computational Thinking in Compulsory Education," and in 2017, the New Media

Consortium and the EDUCAUSE Learning Initiative's "NMC Horizon Report: K-12 Edition" identified Computational Thinking as a critical topic in 21st-century primary and secondary education, essential for students to master.

Domestically, in 2010, the C9 League of Universities emphasized in the "Joint Declaration on the Development Strategy of C9 Computer Basic Education" that "cultivating students' Computational Thinking ability is an important, long-term, and complex core task of computer education in universities." In 2012, the Ministry of Education proposed "focusing on cultivating Computational Thinking to promote the reform of university computer courses." In 2013, the Computer Course Teaching Steering Committee of the Ministry of Education issued the "Declaration on Computer Teaching Reform," aiming to improve students' computational thinking awareness and methods and enhance their computer application level. In 2017, the Ministry of Education launched the "Information Technology Curriculum Standards for Ordinary High Schools (2017 Edition)," which identified Computational Thinking as one of the core literacies of the information technology subject. In 2022, the Ministry of Education included Computational Thinking as a core literacy of the information technology subject in the "Information Science and Technology Curriculum Standards for Compulsory Education (2022 Edition)" [2].

Based on the current situation of Computational Thinking education both domestically and internationally, it can be seen that the cultivation of Computational Thinking has been included in the information technology curriculum standards for compulsory education in primary and secondary schools. At the university level, Computational Thinking serves as a core teaching task for computer science courses. As pointed out by Weintrop et al. [3], Computational Thinking, computer science, mathematics, and science should be mutually beneficial and symbiotic based on the practical application of their concepts and knowledge. This underscores the indispensable role of mathematics in cultivating Computational Thinking. However, research on cultivating Computational Thinking in university mathematics courses is still relatively lacking. A search on the CNKI database using the keywords "Computational Thinking" and "Mathematics" yielded 36 results. Most of these papers focus on cultivating Computational Thinking in computer-based courses such as discrete mathematics and combinatorics, totaling 29 articles. Only 10 papers are related to cultivating Computational Thinking abilities in university mathematics courses (including advanced mathematics, linear algebra, probability theory, and mathematical statistics).

1.3 The trend of technological advancement in mathematics

Mathematics is the science that studies the quantitative relations and spatial forms in the real world. It serves as the foundation of natural sciences, providing precise language and rigorous methods. In his article "10 Major Fundamental Issues of Artificial Intelligence" [4], Academician Zongben Xu argues that mathematics is also the basis for significant technological advancements and is playing an increasingly important role in social sciences.

According to Academician Zongben Xu [4], with the development of the new generation of information technology represented by the internet, big data, artificial intelligence, and so on, human society has entered a new era. In this new era, the role and value of mathematics have undergone fundamental changes, and the trend of mathematical technicalization has become increasingly apparent. This trend is perfectly reflected in various applications such as high-performance scientific and engineering computing, big data technology and industry, blockchain and digital economy, artificial intelligence and healthcare, all of which are fundamentally based on mathematics.

The connotation of mathematical research is rapidly expanding, with cross-integration, technicalization, and evolution towards data science. In the new era, the integration and mutual advancement of mathematics, information technology, and the digital economy is the direction, and mutual influence is inevitable [5].

In summary, not only the number of empirical studies on teaching computational thinking in China is small, but their rigor and scientificity are also relatively insufficient. Currently, most of the approaches to cultivating computational thinking are in computer-related courses. The trend of mathematical technicalization will inevitably make university mathematics courses a fertile ground for cultivating computational thinking. Therefore, it is inevitable to cultivate students' computational thinking ability in university mathematics courses, and it is worth exploring how to do so.

2. The cultivation path of computational thinking in university mathematics courses

Based on Professor Jeannette M. Wing definition of computational thinking, the core of computational thinking is to use the basic concepts of computer science for problem-solving. Some researchers have interpreted computational thinking from different perspectives. Reference [6] summarizes it into three perspectives: thinking skills, process elements, and practical operations. No matter which perspective, computational thinking focuses on how to solve problems. Computational thinking is an activity process that can be decomposed and studied, and ultimately becomes a solution to practical problems. Emphasizing the problem-solving characteristics of computational thinking is a general consensus in the academic community. Therefore, the computational thinking cultivated in university mathematics teaching should be the ability to digest and absorb engineering concepts and principles using mathematical ideas, concepts, and methods; the ability to describe practical problems using mathematical language; and the ability to use mathematical methods to deal with and solve practical problems with the help of computer software. In view of this, the cultivation path of computational thinking in university mathematics courses should start from the following aspects.

2.1 Construct classroom teaching situations and cultivate computational thinking through immersive learning modes.

The understanding and application of mathematical concepts are the foundation for engaging in mathematical thinking, formulating mathematical ideas, and applying mathematical methods. Therefore, in the teaching of mathematical concepts, students' computational thinking skills can be cultivated, and their awareness of the application of mathematics can be enhanced. However, mathematical concepts are often abstract and obscure. By constructing classroom teaching situations and starting from real-life backgrounds, students can personally calculate, observe, and think to gain an understanding of the concepts they are learning. This approach leaves a deep impression on students about the theoretical and practical value of the concepts they have learned, and enables them to solve related problems using these concepts, thus achieving the goal of cultivating computational thinking skills.

Example 1: The Formation and Application of the Concept of Differential

The introduction of the concept of differentials in most textbooks often uses the example of a square metal sheet undergoing thermal expansion, concluding that the differential is the linear principal part of the independent variable increment. This can be difficult for students to understand, affecting their comprehension and application of the concept of differentials. Therefore, we choose a relatively complex example, or after explaining the concept of differentials, we construct a teaching scenario where students can calculate by themselves (using a mobile phone calculator) to experience the difference between differentials and function increments.

Approximate value of $\sqrt[3]{999}$

$\sqrt[3]{999}=\sqrt[3]{1000-1}=\sqrt[3]{1000(1-0.001)}=10\sqrt[3]{1-0.001}$, set up $f(x)=\sqrt[3]{x}$, $x_0=1, \Delta x=-0.001$

$\sqrt[3]{999}=9.99666555$ (Using a calculator)

$\sqrt[3]{999}=10\sqrt[3]{1-0.001}\approx 10\times(1+\frac{1}{3}\times(-0.001))=9.99666666$

error $=|f(x)-dy|=\left|\sqrt[3]{1+\Delta x}-(1+\frac{1}{3}\Delta x)\right|=0.0000011$ ($(0.001)^2$)

The table below shows the nearby $x=1000$ cube root values, differential values, and a comparison of errors.

Students fill in the results in the table 1 by calculating with their mobile phone calculators, and draw conclusions in the last column through observation.

$$\Delta y=f'(x_0)\Delta x+o(\Delta x)$$

Students can more easily understand the meaning of the higher-order infinitesimal of Δx , and $o(\Delta x)<\Delta x^2$.

By observing Table 1, students can understand the conditions for the establishment of the following approximate formulas, namely when $|\Delta x|<<0.1$

$$f(x_0+\Delta x)\approx f(x_0)+f'(x_0)\Delta x$$

The smaller Δx is, the higher the accuracy of the approximate value calculated by the above formula will be. This is conducive to understanding the geometric meaning of differentials.

Table 1: The nearby $x=1000$ cube root values, differential values, and a comparison of errors.

Δx	$10\times\sqrt[3]{1+\Delta x}$ (Using a calculator)	$10\times(1+\frac{1}{3}\Delta x)$	error $\left\|\sqrt[3]{1+\Delta x}-(1+\frac{1}{3}\Delta x)\right\|$	Accurate to the decimal places
-0.001	$\sqrt[3]{999}=$ 9.99666555	9.99666666	0.0000011	five decimal places
-0.01	$\sqrt[3]{990}=$ 9.96655493	9.96666667	0.00011177	Three decimal places
-0.1	$\sqrt[3]{900}=$ 9.6548938	9.66666667	0.0117729	one decimal places
-0.3	$\sqrt[3]{700}=$ 8.8790400	7	1.8790400	large error
0.3	$\sqrt[3]{1300}=$ 10.9139288	11	0.0860712	large error
0.1	$\sqrt[3]{1100}=$ 10.3228011	10.3333333	0.0105322	one decimal places
0.01	$\sqrt[3]{1010}=$ 10.0332228	10.0333333	0.0001105	Three decimal places
0.001	$\sqrt[3]{1001}=$ 10.0033322	10.003333	0.0000011	five decimal places

Through the construction of the above immersive teaching background, students personally calculate and observe to draw conclusions about the concepts and application skills of differentials, which also lays the groundwork for explaining Taylor's formula later.

2.2 Cultivate computational thinking in problem-solving

Traditional mathematical problems generally refer to calculation problems, proof problems, and word problems. Solving mathematical problems means finding a result or conclusion. However, when focusing on mathematical techniques rather than the mathematical problems themselves, solving mathematical problems should pay more attention to the problem-solving strategies and methods that students form in the process of solving mathematical problems. Sometimes, it is necessary to use computers to obtain results and discuss them. The entire process covers various elements of computational thinking. Therefore, students' computational thinking can be cultivated through solving mathematical problems.

Example 2: Balancing chemical reaction equations

Balance the following chemical equation and make the coefficients of the equation the smallest possible integers.

$MnS + As_2Cr_{10}O_{35} + H_2SO_4 \rightarrow HMnO_4 + AsH_3 + CrS_3O_{12} + H_2O$

Let the balancing coefficients be a, b, c, d, e, f, and g, respectively.

Mn:1a+0b+0c-1d-0e-0f-0g=0

S: 1a+0b+1c-0d-0e-3f-0g=0

As: 0a+2b+0c-0d-1e-0f-0g=0

Cr: 0a+10b+0c-0d-0e-1f-0g=0

O: 0a+35b+4c-4d-0e-12f-1g=0

H: 0a+0b+2c-1d-3e-0f-2g=0

The above expressions can form a system of homogeneous linear equations, and a fundamental system of solutions can be found using MATLAB commands.

```
format rat
A=[1 0 0 -1 0 0 0;1 0 1 0 0 -3 0;0 2 0 0 -1 0 0;0 10 0 0 0 -1 0;0 35 4 -4 0 -12 -1;0 0 2 -1 -3 0 -2];
null(A,'r')
ans =
     16/327
     13/327
    374/327
     16/327
     26/327
    130/327
       1
```

It is found that there is only one vector in its fundamental system of solutions. Taking positive integer solutions, the balancing of the equation is as follows:

$$\mathbf{16}MnS + \mathbf{13}As_2Cr_{10}O_{35} + \mathbf{374}H_2SO_4 = \mathbf{16}\ HMnO_4 + \mathbf{26}AsH_3 + \mathbf{130}CrS_3O_{12} + \mathbf{327}\ H_2O$$

In this case study, students transformed a chemical problem into a mathematical one through the balancing of a chemical reaction equation. They then used software to obtain results and solve the practical problem.

2.3 Cultivating computational thinking through project-based learning

Project-based learning involves teachers assigning open-ended project tasks, and students working collaboratively in groups to solve problems. The method of completing the task is not predetermined or unique, but rather emphasizes students' ability to acquire knowledge in the process of problem-solving, design strategies to solve problems, use computer software to complete tasks, and summarize and report their findings. This approach requires team collaboration and effectively cultivates students' computational thinking skills, executive abilities, and teamwork capabilities.

Example 3: Project Name: Implementation of Color Image Filter - Matrix Representation of Color Images

Figure 1: Original image

The color image (Figure 1) is represented as a three-dimensional array in Matlab, and the structure of the three-dimensional array is shown in Figure 2.

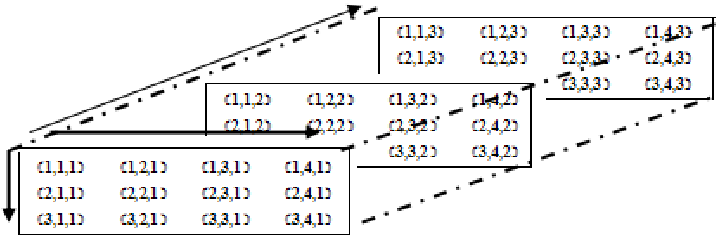

Figure 2: The structure of a three-dimensional array

The three dimensions include the size of rows, the size of columns, and the third dimension represents the size of layers. A three-dimensional array can be represented as A(:,:,3) to indicate the elements of the third layer. The first is to third layers of the color image correspond to the R, G, and B channel components respectively.

Multiplying different channel image matrices by corresponding weights can obtain images with different color enhancements (as shown in Figure 3), such as multiplying the R channel matrix by a factor greater than 1 to enhance red.

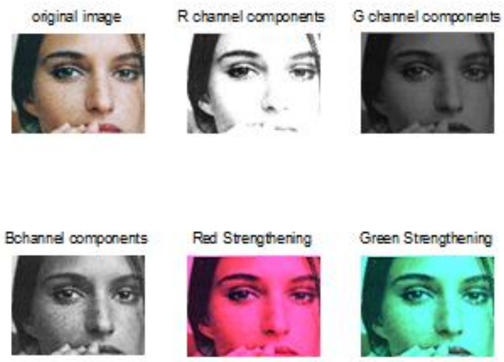

Figure 3: The influence of the change of channel weights on color images

The tasks for the linear algebra practical project are arranged as follows:

a. By consulting reference, students should achieve the image processing effects as shown in Figure 3, and answer which matrix operation is related to the implementation of this effect?

b. Use MATLAB to draw the corresponding image of a symmetric matrix.

c. Apply matrix addition and subtraction operations to process an image and observe how the image changes.

d. Based on the above practices and by consulting relevant image processing reference, create an animation of a color gradient image or an animation that gradually transitions from one image to another. Students should write the corresponding mathematical expressions.

e. Students should complete the above practical tasks and prepare a PPT presentation to showcase the results. The image used for processing should be different from the one used by the instructor. The reference consulted during the completion of the tasks needs to be listed as references.

In this practical project, students convert practical problems into mathematical problems by consulting reference, design strategies to solve them, and use software to fulfill tasks assigned by their teachers. They learn how to express practical problems using mathematical methods and solve them.

Example 4 Project name: Naive Bayes classifier

The Bayesian formula can be used to classify data. Here is an example of 1000 fruit data (Table 2). These fruits have three feature descriptions: shape (long), taste (sweet), and color (yellow). There are three types of fruits: apple, banana, and pear. These fruits have three features: long, sweet, and yellow. The Bayesian classifier can give the probability of each fruit for a new fruit based on known training data.

Table 2: Data List of 1000 Fruits

fruits	long	sweet	yellow	Total number
banana	400	350	450	500
apple	0	150	300	300
pear	100	150	50	200
Total number	500	650	800	1000

Naive Bayes classifier, where the word "naive" refers to the assumption that the information expressed in the data is mutually independent. In this example, the three characteristics of fruits, "long, sweet, and yellow," are mutually independent because they describe the shape, taste, and color of the fruit separately and are not related to each other. The word "Bayes" indicates that this classifier uses Bayes' formula to calculate the posterior probability, that is

$$\boldsymbol{P(A \mid B) = \frac{P(A)P(B \mid A)}{P(B)}},$$

where A represents a certain type of fruit and B represents “Long, yellow and sweet”.

Next, we will calculate the probability of this fruit being a banana, apple, or pear under the condition of "long, yellow, and sweet". For bananas:

$$\boldsymbol{P(banana \mid Long, yellow\ and\ sweet) \square\ P(banana \mid B) = \frac{P(banana)P(B \mid banana)}{P(B)}}$$

$$\boldsymbol{= \frac{P(banana)P(Long \mid banana)P(yellow \mid banana)P(sweet \mid banana)}{P(B)}}$$

$$\boldsymbol{= \frac{P(banana)P(Long \mid banana)P(yellow \mid banana)P(sweet \mid banana)}{P(B \mid banana)P(banana)+P(B \mid apple)P(apple)+P(B \mid pear)P(pear)}}$$

$$\boldsymbol{= \frac{0.5\times 0.8\times 0.7\times 0.9}{0.5\times 0.8\times 0.7\times 0.9+0.3\times 0\times 0.5\times 1+0.2\times 0.5\times 0.75\times 0.25} \approx 93\%}$$

Similarly, $P(apple \mid Long, yellow\ and\ sweet) = 0$, $P(pear \mid Long, yellow\ and\ sweet) \approx 7\%$ can be obtained.

The practical tasks of probability theory and mathematical statistics are to classify and distinguish different physical objects (as shown in Figure 4).

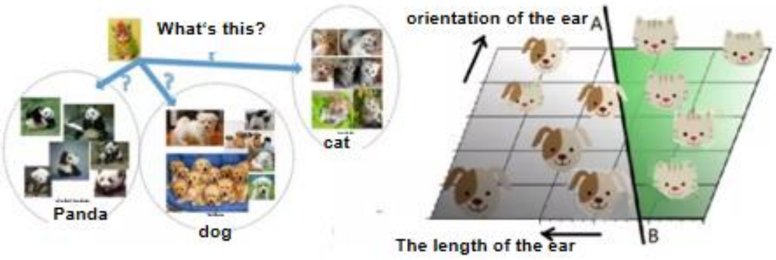

Figure 4: Classify and distinguish different physical objects

a. Review the reference and distinguish and identify a group of pandas, cats, and dogs to achieve the effect shown in Figure 4. Which probability knowledge point can be related to the realization of this effect?

b. Through the completion of the above practical tasks, create a PPT to complete the presentation task.

c. Requirements: The reference consulted during the completion of the task needs to be listed as references.

d. This practical project utilizes a naive Bayes classifier to classify physical objects, thereby understanding the application of Bayes' formula (posterior probability formula).

3. Conclusions

In today's era of artificial intelligence, computational thinking should become a basic skill for everyone, not only to be cultivated from an early age, but also to be an important thinking ability for students in college mathematics courses, which can also better connect seamlessly with middle school mathematics education. The path of cultivating computational thinking in college mathematics teaching should focus on building classroom teaching situations, immersive learning models to cultivate computational thinking; cultivating computational thinking in problem solving; cultivating computational thinking in project-based learning, etc.

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