

# *Analysis and reflection on the teaching of column volumes based on aesthetic education*

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**Abstract:** The connotation of the education curriculum reform calls for the enhancement of students' aesthetic interests and the promotion of their all-round development. This requires students not only to learn the necessary mathematical knowledge, but also to appreciate and apply the aesthetic values in mathematics. Therefore, the integration of aesthetic education into mathematics teaching has its important significance. This paper analyses the current situation of aesthetic education research around the world, explains how to integrate aesthetic education into mathematics, and proposes common approaches to aesthetic education in mathematics. Starting from the "beauty of images", we analyze the basic characteristics of "beauty of images" in mathematics teaching and the teaching strategies. Finally, the practical application of aesthetic education in the teaching of mathematics in senior high schools is analyzed by using the example of teaching the column volume in the hope that students will discover, understand and experience the beauty of mathematics.

## 1. Introduction

Mathematics is the crystallization of human civilization, and the structure, graphics, layout and form of mathematics embody mathematical beauty everywhere. In the teaching of mathematics, teachers should be good at mining the aesthetic elements in mathematics textbooks and combining them with appropriate teaching contents. Students also need to be guided to feel, appreciate and evaluate the sense of beauty within them, which can bring them delights and stimulate their interest in mathematics. Developing an aesthetic interest in mathematics not only provides students with vivid and distinctive images to inspire their scientific imagination, but also contributes to the development of the intellectual and creative abilities[1].

The vast majority of countries focus on the development of cultural literacy and emotional experiences in mathematics. The U.S. National Council of Teachers of Mathematics explicitly requires students to learn to appreciate the beauty of mathematics in the objectives of the senior high school mathematics curriculum [2]. Many foreign educators believe that in order for students to feel and appreciate the beauty of mathematics, teachers must make efforts in teaching methods

and carefully design mathematical activities so that students can participate in these activities to enjoy themselves and develop their thinking skills. There are numerous studies on mathematical aesthetics in education and psychology, however, most of them remain on the theoretical basis of the value, the curriculum and the overall understanding of aesthetic education in mathematics. In terms of application, despite the existence of several new editions of mathematics textbooks, aesthetic education in mathematics is not yet fully operational in teaching practice.

In recent years, the integration of aesthetic education into mathematics teaching has received great attention in China. When China's Ministry of Education introduced education curriculum reform in 2001, it explicitly stated that it was necessary to enhance students' aesthetic interests and promote their all-round development [3]. Furthermore, Yuan [4] pointed out that aesthetic education can be integrated into classroom teaching by using mathematical culture and realistic contextual materials. Guo [5] pointed out that integrating aesthetic education into classroom teaching is an important part of quality education and one of the core elements of the teachers' teaching process. Through a large number of literature researches, it can be found that frontline mathematics teachers are fully aware of the significance of integrating aesthetic education into mathematics teaching, and put forward the idea of using the history of mathematics and realistic materials to achieve this purpose.

Since the motivation to learn and research mathematics can only be generated by creating and using mathematical beauty in solving mathematical problems. Teachers have also consciously incorporated the beauty of symmetry and the simplicity of images into the teaching of mathematics. Therefore, this paper will use the teaching of column volume as an example to study how to integrate the beauty of images into mathematics teaching in senior high schools.

## **2. Approaches to aesthetic education in mathematics**

### **2.1. Revealing mathematical beauty in the mathematics class**

In the process of mathematics learning, students first come into contact with mathematical concepts, formulas, theorems, laws and so on. Although they contain elements of beauty, also embodied in the language of mathematics, there is a certain indirectness and ambiguity. Therefore, not all students can experience the existence of mathematical beauty, which requires teachers to consciously cultivate students' aesthetic perception of mathematics in teaching, and guide them to discover and appreciate beauty.

### **2.2. Embodying mathematical beauty in the organization of teaching**

The emotional factor in mathematics is embedded in the process of imparting knowledge and training skills in a particular environment. As the "Mathematics Curriculum Standards of Senior High School" [6] state: "Emotional development cannot be separated from the acquisition of knowledge and skills ". In order to improve the art of classroom teaching, Teachers, as the organizers, guides and collaborators of students' mathematical activities, should reprocess the teaching textbook according to the specific situation of students and creatively design the teaching process. These initiatives will make teaching an emotional as well as an intellectual experience.

Apparently, mathematical symbols are monotonous, mathematical formulas are boring. Teachers can use different methods in teaching according to the textbook content and the characteristics of the students. For example, mathematics teachers can make the content of mathematics come alive through explanation, analysis, demonstration, graphics, multimedia and other forms. Thus, the mathematical content is endowed with the life and connotation of beauty, so that students can raise their awareness of the implicit beauty of mathematics from the explicit beauty of mathematics and

from perceptual to rational understanding, which forms a sense of beauty in mathematics. In addition, teachers can intersperse some history of mathematics according to the textbook content, including the historical background of the emergence of mathematical theories, mathematical debates, the struggles of mathematicians, etc., so that abstract and profound mathematical knowledge can be visualized and made interesting. Furthermore, teachers can also create mathematical situations or group competitions in the classroom. Therefore, the diversity of teaching methods and means constitutes the formal beauty of mathematics teaching.

### 2.3. Pursuing mathematical beauty in applications

The spirit of mathematics is the root of learning mathematics, developing mathematics and applying mathematics. The process of cultivating this mathematical spirit is the process of creating mathematical beauty. Therefore, in the teaching of mathematics, the law of "practice - knowledge - practice again" should be always adopted to appreciate the beauty, to discover the beauty, and to form a regular understanding of the beauty of mathematics. And then it is used to conjecture, to explore, to discover, to analyze and to solve mathematical problems, so as the highest level of mathematical aesthetics is achieved through the application and creation of mathematical beauty.

## 3. Beauty of images in the mathematics class

### 3.1. Basic characteristics of the beauty of the image

**Symmetry:** Symmetry is an intuitive embodiment of beauty and it is manifold. The most prominent symmetries in senior high school mathematics are central symmetry and axial symmetry. They are not only beautiful in appearance, but also have an intrinsic mathematical essence that can be transferred from one knowledge point to another equivalent knowledge point.

**Uniformity:** Uniformity is about parts becoming a whole and differences coming together. The objects of study share commonalities and consistencies in a number of specific forms and contents, and in particular equations, functions and inequalities are often skillfully combined through the use of images.

**Compatibility:** Compatibility is the beauty of harmony and consistency represented by mathematical theories and laws. Although the external form and nature are different, they are intimately related.

### 3.2. Teaching strategies for the beauty of images

Based on the idea of combining numbers and shapes, the teaching strategies of the beauty of images are implemented from two entry points: space structure and quantitative relationship. The specific process is shown in Figure 1.

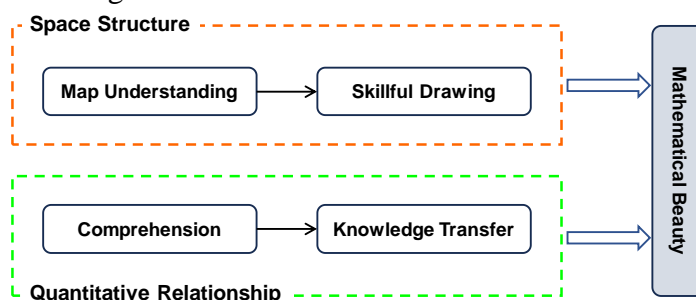


Figure 1: Teaching strategies for the beauty of images

## 4. Analysis for the teaching of column volumes

### 4.1. Difficulties in teaching

**Textbook Analysis:** "The volume of column" is the content of Chapter 11, Section 2 of the mathematics textbook for Book 3 of the Shanghai Edition. As the first lesson on volume of geometrical bodies, this lesson focuses on using the formula of special rectangular volume to derive the volume formula for general columns by Zu's principle (also known as Cavalieri's principle). On the basis that students have already learnt about spatial geometry in terms of both structural features and views of geometrical bodies, this lesson provides further insight into the structure of geometric bodies by understanding the spatial geometry in terms of metrics, as well as laying the foundations for the subsequent study of vertebral volumes.

**Focus of the lesson:** understanding of Zu's principle; derivation of the formula for column volumes.

**Difficulties in the lesson:** derivation and understanding of the formula for column volumes.

### 4.2. Teaching process

This lesson starts from practical mathematical problems, creates appropriate teaching situations, explains the Zu's principle with the help of the volume formula for rectangular bodies, and then introduces the formula for volume columns. Students will experience the process of "discovering the problem - exploring the problem - solving the problem" several times to deepen their understanding of Zu's principle and the formula for column volumes.

#### 4.2.1. Basic flow of the lesson

**Step 1:** Introducing a new lesson from the life situation of the volume problem of crushed stone in the roadbed of the Qingzang Railway, shown in Figure 2.

**Step 2:** Reviewing the formula for the rectangular body volume.

**Step 3:** Exploring the column volume using a simple model.

**Step 4:** Reviewing the history of mathematics and explaining Zu's principle.

**Step 5:** Exploring how to utilize the Zu's principle.

**Step 6:** Deriving the formula for column volumes via Zu's principle.

**Step 7:** Using of the formula for column volumes to solve the volume problem of crushed stone in railway roadbed.

**Step 8:** Explaining the example problem to consolidate the formula for column volumes.

**Step 9:** Summarizing to improve student understanding.



Figure 2: Volume problem of crushed stone in the roadbed of Qingzang Railway

### 4.2.2. Design Intentions

**Intention 1:** Using the volume problem of crushed stone in the roadbed of the Qingzang Railway, a mathematical problem is abstracted from the actual problem to stimulate the students' desire to explore. The new lesson is introduced by the life situation to stimulate the students' interest in learning.

**Intention 2:** Through the introduction of the mathematical history of Zu's principle, students can feel the brilliance of ancient Chinese mathematics, arouse their interest in learning mathematics and enhance their mathematical cultural literacy.

**Intention 3:** From the formula for the rectangular body volume to the formula for the general column volume, students are trained to think from the particular to the general, which promotes their mathematical thinking, and cultivates their observation skills and analytical abilities.

**Intention 4:** Through the example problem, students can master the column volume formula and experience the graphic beauty of mathematics.

## 5. Conclusions

Basic education focuses on the development of quality education for students, which is related to aesthetics. Learning aesthetics can not only cultivate one's sentiment, but also improve one's thinking quality. Since mathematics education has the fundamental task of establishing morality and developing quality education, the aesthetic commonality portrayed from the perspective of mathematics can enable students to move from an emotional to a rational sense of beauty. By encountering, feeling and understanding the beauty of mathematics, students can appreciate the powerful charm of mathematics and produce the emotional experience of loving mathematics. This can also arouse interest and desire to learn mathematics, stimulate the great potential of learning mathematics and improve learning efficiency. It is especially necessary to integrate aesthetic education into the whole teaching process of mathematics education in senior high schools, so that more students can know, discover and experience the beauty of mathematics. Therefore, aesthetic education in mathematics is a requirement of the times.

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