

# *A Comparative Study of Hedging Functions of Chinese and U.S. Bond Futures*

Cui Wei<sup>1,a</sup>, Qing Lei<sup>1,b</sup>

<sup>1</sup>*Faculty of Economics, Beijing Wuzi University, 321 Fuhe Street, Beijing, China*  
<sup>a</sup>18210516811@163.com, <sup>b</sup>1481235188@qq.com

**Keywords:** Chinese and U.S. bond futures; Hedging; Interest rate risk management

**Abstract:** With the gradual deepening of China's interest rate marketization reform, the management of interest rate risk is becoming more and more important. Treasury bond futures are the main standardized interest rate risk management tools in the world. This paper conducts a systematic research and comparison of the hedging function of the treasury bond futures markets in China and the United States. The VAR model and DCC-GARCH model are used to study the hedging function of the treasury bond futures market in China and the United States. It is found that the hedging efficiency of the U.S. treasury bond futures market is better than that of China's treasury bond futures market, and China's treasury bond futures market still needs further reform and development, and this paper puts forward some corresponding suggestions.

## 1. Introduction

Interest rate as the price of money is a very important basic indicator in the financial market. Interest rate marketization reform is also an important part of China's financial market reform. Treasury bond futures is a major standardized interest rate risk management tool in the world, and the healthy and efficient operation of the Treasury bond futures market is conducive to better interest rate risk management for investors. The U.S. Treasury bond spot market itself is an open and internationalized market, and U.S. Treasury bonds are important investment targets for investors from all over the world, and participants in the U.S. Treasury bond market are mainly international investors and institutional investors. Accordingly, the U.S. Treasury bond futures market also has a rich investor structure. The rich investor structure and open market make the U.S. Treasury bond futures market have good trading activity and liquidity, and such a financial market also tends to have high market efficiency. In this paper, we conduct an empirical study on the hedging function of the treasury bond futures market in China and the United States to find out the differences between the hedging efficiency of the treasury bond futures market in China and the United States.

## 2. Literature review

Academics have been studying hedging for a long time, in static hedging Leland L. Johnson (1960)<sup>[1]</sup> used OLS model to study hedging in commodity futures market. Later Asim Ghosh (1993)<sup>[2]</sup> established the ECM model on the basis of cointegration, which significantly improved the

effectiveness of the hedging ratio compared to the OLS model. Ederington Louis H. (1979)<sup>[3]</sup> proposed a minimum variance hedging strategy on the basis of the OLS model, which optimized the static hedging model. With the continuous evolution and development of hedging research, academics gradually began to study the dynamic hedging in futures market based on the GARCH model. Robert Engle (2002)<sup>[4]</sup> proposed the DCC-GARCH model to analyze and study the hedging in futures market. In recent years there have also been many innovations in the academic approach to the study of hedging Jonathan Dark (2015)<sup>[5]</sup> proposed the MS-VECM-FIAPARCH and MS-VECM-FIEGARCH models and empirically examined their performance when using futures to hedge the S&P 500 index, and the study found that the newly proposed models achieve a considerable variance reduction Zheng Chengli, Su Kuangxi, and Yao Yinhong (2021)<sup>[6]</sup> proposed a framework incorporating denoising and noise-assisted strategies to decompose the raw futures and spot returns using EMD techniques. The denoising and noise-assisted returns are obtained by gradually removing the decomposition terms or adding them in the opposite way. In the minimum CVaR framework, dynamic hedging portfolios based on raw and processed returns are constructed to test the effectiveness of hedging.

### 3. Empirical studies

#### 3.1 Selection and processing of data

Regarding the selection of data for the study, the daily closing price data of 10-year treasury bonds in the Chinese and U.S. bond futures and spot markets from January 2, 2020 to August 10, 2023 are selected as the sample for the study, and the series of the futures and spot data in the Chinese market are denoted as CPF, and each series of the CPS has a total of 875 data values; the series of the futures and spot data in the U.S. market are denoted as UPF, and each series of the UPS has a total of 908 data values. The U.S. market futures and spot data series are labeled as UPF, and each series of UPS has 908 data values. The above four time series data are logarithmized by first-order differencing to obtain four time series data, which are denoted as CRF, CRS, URF, URS.

Among them, 10-year China Treasury bond futures data, 10-year CSI bond index data, 10-year U.S. Treasury bond futures data are from the wind database, and 7-10 Year US Treasury Index ETF - iShares data are from the NASDAQ official website.

#### 3.2 VAR modeling

The dynamic effects of stochastic disturbances on a multivariate system can be studied by building a VAR model, and this paper explores the price discovery ability of the treasury bond futures market by building a VAR model on the price time series data of the treasury bond futures and spot markets.

The general VAR (p) model is as follows:

$$y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + B_1 x_t + \dots + B_r x_{t-r} + \varepsilon_t \quad (1)$$

where  $y_t$  is a vector of m-dimensional endogenous variables,  $x_t$  is a vector of d-dimensional exogenous variables,  $A_1, \dots, A_p$  and  $B_1, \dots, B_r$  are the parameter matrices to be estimated, with endogenous and exogenous variables having p- and r-order lags, respectively, and  $\varepsilon_t$  is a stochastic perturbation term, and the contemporaneous periods can be correlated with each other, but they cannot have an autocorrelation, and they cannot be correlated with variables on the right side of the model.

#### 3.3 Building the DCC-GARCH model

The establishment of the DCC-GARCH model first requires the establishment of GARCH models for the two time series separately, and then the residual series of the established GARCH model are

standardized, and the standardized residual series are fitted to the DCC-GARCH model.

The general GARCH model takes the following form:

$$x_t = f_1(t, x_{t-1}, x_{t-2}, \dots) + \varepsilon_t \quad (2)$$

$$\varepsilon_t = \sqrt{h_t} e_t \quad (3)$$

$$h_t = \lambda_0 + \sum_{j=1}^p \eta_j h_{t-j} + \sum_{i=1}^q \lambda_i \varepsilon_{t-i}^2 \quad (4)$$

Bollerslev proposed the CCC-GARCH model in 1990, which decomposes the conditional variance matrix into the product of the conditional standard deviation and the conditional correlation coefficient matrix, and assumes that the conditional correlation coefficient matrices of spot returns and futures returns are constant matrices. 2002 Engle relaxed the assumption of static conditional correlation coefficient matrices in the CCC-GARCH model, and proposed the DCC-GARCH model with time-varying conditional correlation characteristics. In 2002, Engle relaxed the assumption of static conditional correlation coefficient matrix in the CCC-GARCH model and proposed the DCC-GARCH model with time-varying conditional correlation characteristics, which has the following conditional variance equation.

$$H_t = D_t R_t D_t = \begin{pmatrix} \sqrt{h_{s,t}} & 0 \\ 0 & \sqrt{h_{f,t}} \end{pmatrix} \begin{pmatrix} 1 & \rho_t \\ \rho_t & 1 \end{pmatrix} \begin{pmatrix} \sqrt{h_{s,t}} & 0 \\ 0 & \sqrt{h_{f,t}} \end{pmatrix} \quad (5)$$

Define the matrix of conditional correlation coefficients  $R_t = \text{diag}(Q_t^{-\frac{1}{2}}) Q_t \text{diag}(Q_t^{-\frac{1}{2}})$ , with  $R_t$  being a positive definite matrix.

$Q_t$  is a 2 by 2 positive definite matrix:

$$Q_t = (1 - \alpha - \beta) \bar{Q} + \alpha \zeta_{t-1} \zeta'_{t-1} + \beta Q_{t-1} \quad (6)$$

The DCC-GARCH model uses maximum likelihood estimation, and the fitting result mainly focuses on the values of parameters  $\alpha$  and  $\beta$ , and  $\alpha$  and  $\beta$  need to satisfy the constraint that the sum of  $\alpha$  and  $\beta$  is less than 1. Where  $\zeta_t$  is the standardized residuals, and  $\bar{Q}$  is the matrix of unconditional correlation coefficients of  $\zeta_t$ . The DCC-GARCH model uses maximum likelihood estimation, and the fitting result mainly focuses on the values of parameters  $\alpha$  and  $\beta$ , and  $\alpha$  and  $\beta$  need to satisfy the constraint that the sum of  $\alpha$  and  $\beta$  is less than 1. maximum likelihood estimation, the fitting results are mainly concerned with the values of parameters  $\alpha$  and  $\beta$ , and  $\alpha$  and  $\beta$  need to satisfy the constraint that the sum of  $\alpha$  and  $\beta$  is less than 1.

### 3.4 Empirical study of hedging

#### 3.4.1 Unit root test and VAR model ordering

The ADF unit root test is performed on the logarithmized Chinese and U.S. bond futures spot time series data LCFP, LCSP, LUF, LUSP, and the logarithmized and first-order differenced time series data CRF, CRS, URF, URS, and the results of the test are shown in Table 1:

According to the results of the empirical test, the data after logarithmization and first-order differencing are all smooth time series, which can be modeled as time series.

The VAR modeling of the Chinese and U.S. bond futures and spot market data is carried out separately, and according to the AIC and SC principles, the lag order of the VAR model for the Chinese market is set as the 2nd order lag, and the lag order for the U.S. market is set as the 4th order lag.

Table 1: Unit Root Tests for U.S. Treasury Futures Spot Time Series in China.

market	China Treasury Bond Futures				U.S. Treasury Futures Market			
variable	LCPF	LCPS	CRF	CRS	LUPF	LUPS	URF	URS
Type(C,T,K)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)
t-statistic	0.674	2.343	-28.891	-24.705	0.061	-0.073	-30.736	-23.472
P-value	0.861	0.996	0.000	0.000	0.963	0.950	0.000	0.000
steady	no	no	yes	yes	no	no	yes	yes

### 3.4.2. Establishment of the DCC-GARCH model

The existence of ARCH effect in the residual series of the mean equation is a prerequisite for the establishment of GARCH model. After the establishment of the mean equation, it is necessary to test the ARCH effect on the residual series of the mean model of the China treasury bond futures spot time series and the residual series of the mean equation of the U.S. treasury bond futures spot time series, respectively.

The results of the ARCH effect test of the residual series of the mean equation of the Chinese and U.S. Treasury futures spot time series are shown in Table 2:

Table 2: ARCH effect test of residual series of Chinese government bond futures spot log yield data.

sequences	CRFresidual	CRSresidual	URFresidual	URSresidual
F-statistic	11.136	14.044	45.956	78.912
P-value	0.001	0.000	0.000	0.000
ARCH effect	Yes	Yes	Yes	Yes

The empirical test results show that there is an ARCH effect in the sequence CRF, CRS as well as in the residual series of the sequence URF, URS, so further DCC-GARCH modeling can be performed.

Firstly, a GARCH model is built for the sequences CRF, CRS, URF, URS, respectively, and the modeling process is not described in detail here due to space constraints. The residual series obtained from the GARCH model are normalized. A DCC-GARCH model is built for the standardized residual series of the sequences CRF, CRS, and a DCC-GARCH model is built for the standardized residual series of the sequences URF, URS, respectively. The fitting results of the models are shown in the Table 3:

Table 3: Parameter Estimates of the DCC-GARCH Model of China's Treasury Bond Futures and Spots.

market	China Treasury Bond Futures Market			U.S. Treasury Futures Market		
	$\alpha_c$	$\beta_c$	$\alpha_c + \beta_c$	$\alpha_u$	$\beta_u$	$\alpha_u + \beta_u$
coefficient	0.020	0.971	0.991	-0.011	0.950	0.939
P-value	1.05E-06	0.000000	-	5.44E-09	0.000	-

As can be seen from the model fitting results in Table 3, the coefficients  $\alpha_c$  and  $\beta_c$  of the DCC-GARCH model established in China's treasury bond futures and spot markets are significant at the 5% confidence level, and the sum of  $\alpha_c$  and  $\beta_c$  is less than 1, which indicates that the established model is stable, and the value of  $\beta_c$  is close to 1, which illustrates that China's treasury bond futures markets and the treasury bond spot markets have long-term dynamic correlation. correlation; Similarly, the coefficients of DCC-GARCH model established in terms of U.S. treasury bond futures and spot market,  $\alpha_u$  and  $\beta_u$ , are both significant at 5% confidence level, and the sum of  $\alpha_u$  and  $\beta_u$  is less

than 1, which indicates that the established model is stable, where the value of  $\beta_u$  is close to 1, which illustrates that U.S. treasury bond futures and spot market have long-term dynamic correlation with each other.

### 3.4.3 Hedging performance assessment

#### 1) Optimal hedging ratio $h^*$

For hedging transactions, it is critical to determine the appropriate ratio between futures market positions and spot market positions, i.e. the optimal hedging ratio  $h^*$ . In order to minimize the risk of the hedging transaction portfolio, the optimal hedging ratio is calculated as:

$$h^* = \frac{cov(\varepsilon_s, \varepsilon_f)}{Var(\varepsilon_f)} \quad (7)$$

In the above equation (1),  $\varepsilon_s$  represents the value of residuals in the spot market,  $\varepsilon_f$  represents the value of residuals in the futures market, the numerator position is the covariance between residuals in the futures market and residuals in the spot market, and the denominator position is the variance of residuals in the futures market.

Through the establishment of VAR model for static hedging research, the residual series value of futures and spot is obtained, which is brought into the above formula to calculate the optimal hedging ratio of the Chinese U.S. bond futures market, as shown in Table 4:

Table 4: Optimal Hedging Ratio of U.S. Bond Futures Market in the VAR Model.

futures market	China Treasury Bond Futures Market	U.S. Treasury Futures Market
$h^*$	0.108	0.483

The conditional variance series of  $\varepsilon_f$  and the conditional covariance series of  $\varepsilon_s$  and  $\varepsilon_f$  are obtained from the DCC-GARCH model above, and the optimal dynamic hedging ratio plot based on the DCC-GARCH model is calculated and derived as shown in Figure 1 and Figure 2 below:

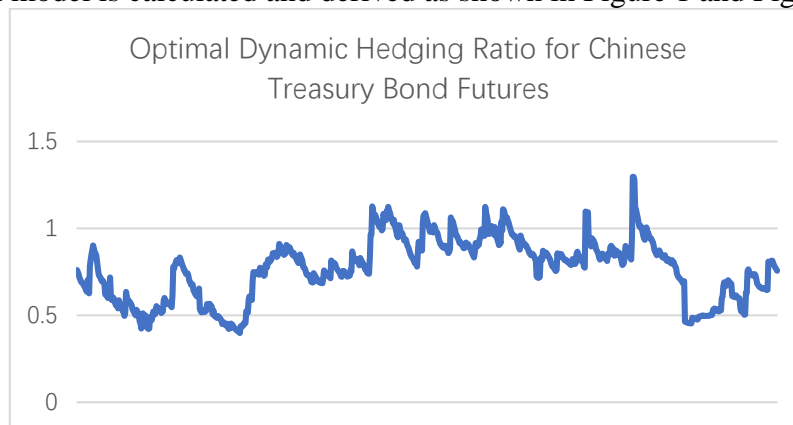


Figure 1: Optimal Dynamic Hedging Ratio of China's Treasury Bond Futures.

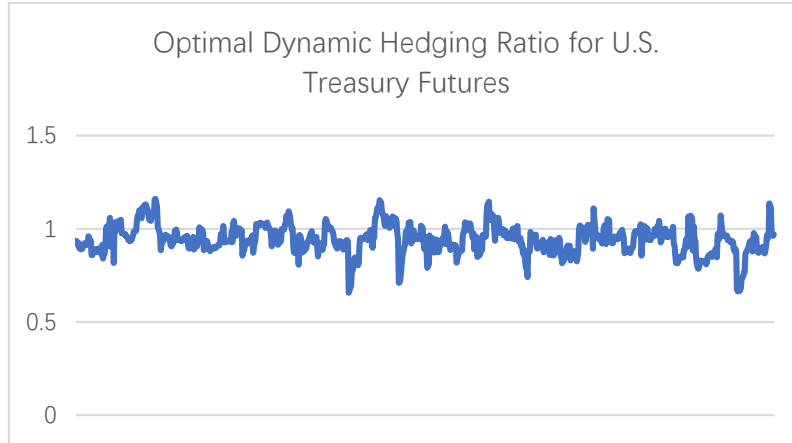


Figure 2: Optimal Dynamic Hedging Ratios for U.S. Treasury Futures.

The average value of the optimal dynamic hedging ratio for Chinese treasury bond futures is 0.768, i.e., for every treasury bond spot contract, 0.768 opposite positions can be established in the futures market for hedging to avoid risk. The average value of the optimal dynamic hedging ratio for U.S. Treasury futures is 0.943, i.e., for every Treasury spot contract, 0.943 opposite positions can be established in the futures market for hedging to avoid risk. From the optimal dynamic hedging ratio chart, the optimal hedging ratio of U.S. treasury bond futures changes relatively more smoothly, mainly around 1 in the range of 0.8 to 1.2 fluctuations up and down, while China's treasury bond futures of the optimal hedging ratio of the value of the value of the change of the amplitude of the maximum value of one time more than 1.3, the minimum value of close to 0.4, the law of change is not uniform, the beginning of the main in the 0.4 to 0.8 fluctuations, and then gradually evolved into the range in the 0.4 to 0.8 fluctuations. At the beginning, it mainly fluctuated in the interval of 0.4 to 0.8, and then gradually evolved into fluctuating in the interval of 0.8 to 1.1, and finally the gradual change was still concentrated in the fluctuation in the interval of 0.4 to 0.8.

## 2) Performance assessment of hedging

In order to minimize the risk of the asset portfolio, the effectiveness of hedging is usually measured by the reduction in the variance of the hedging portfolio's return compared to the variance of the return of the spot position without the hedging transaction, calculated as follows:

$$HE = \frac{Var(r_s) - Var(r_{st}^*)}{Var(r_s)} \quad (8)$$

In the above equation (2),  $r_s$  represents the yield of the unhedged spot market position,  $r_{st}^*$  represents the yield of the hedging portfolio, and the corresponding  $Var(r_s)$  represents the variance of the yield of the unhedged spot market position, and  $Var(r_{st}^*)$  represents the variance of the yield of the hedging portfolio.

Substituting the optimal hedging ratio data calculated by the VAR model and the DCC-GARCH model above with the data of the Chinese and U.S. bond futures and spot markets into the above equation yields the following results of the hedging performance assessment of the Chinese and U.S. bond futures markets in Table 5:

From the hedging data of China's treasury bond futures market in Table 5, it can be seen that the variance of the asset portfolio is reduced after hedging, whether it is static hedging through the VAR model or dynamic hedging through the DCC-GARCH model. In terms of hedging performance evaluation, hedging through the DCC-GARCH model can achieve about 19% risk aversion, while static hedging using the VAR model can achieve about 9% risk aversion, which can be seen that the dynamic hedging model is more efficient in hedging.



Table 5: Comparison of Hedging Performance of Chinese and U.S. Bond Futures.

models		Average rate of return		Yield variance		Hedging performance
		pre-hedging	post-hedging	pre-hedging	post-hedging	
China Treasury Bond Futures Market	VAR	0.015	0.014	0.155	0.141	0.095
	DCC-GARCH		0.011		0.126	0.191
U.S. Treasury Bond Futures Market	VAR	-0.017	-0.010	0.515	0.334	0.352
	DCC-GARCH		-0.002		0.194	0.622

From the hedging data of the U.S. Treasury bond futures market in Table 5, it can be seen that the variance of the asset portfolios are significantly reduced after the hedging transactions. From the performance evaluation of hedging, the static hedging through the VAR model can avoid about 35% of the risk, while the hedging through the DCC-GARCH model can avoid about 62% of the risk, and the hedging efficiency of the dynamic hedging model is significantly higher than that of the static hedging. Overall, the hedging efficiency in the U.S. Treasury futures market is better, and the efficiency of dynamic hedging is better than that of static hedging.

#### 4. Conclusions and recommendations

Through the establishment of VAR model and DCC-GARCH model on the Chinese U.S. bond futures market for static and dynamic hedging research found that the efficiency of dynamic hedging is better than static hedging. The hedging efficiency of the U.S. treasury bond futures market is higher than that of the Chinese treasury bond futures market. By conducting hedging transactions, the U.S. treasury bond futures market can avoid more risks for the asset portfolio, while the hedging efficiency of the Chinese treasury bond futures market is still relatively low. Synthesizing the results of empirical research and the current situation of China's treasury bond futures market, drawing on the experience of the U.S. treasury bond futures market, the following suggestions are made for the further reform and development of China's treasury bond futures market:

First, the optimization of investor structure should be a key direction of China's treasury bond futures market reform. The participation of joint-stock banks and some city commercial banks with good operating conditions in the treasury bond futures market can be further promoted to meet the interest rate risk management needs of these institutions on the one hand, and help to improve the liquidity of the market on the other.

Secondly, drawing on the development experience of the interest rate derivatives market in the United States and other developed countries, an open market tends to be more efficient. Combined with the actual situation of China's financial market, suitable international financial institutions can be gradually introduced to participate in China's treasury bond futures market, which is conducive to the development of the treasury bond futures market will also help promote the internationalization of the RMB to a certain extent.

#### References

- [1] Leland L. Johnson. *The Theory of Hedging and Speculation in Commodity Futures* [J]. *The Review of Economic Studies*, 1960, 27(3):139-151.  
 [2] Asim Ghosh. *Hedging with stock index futures: Estimation and forecasting with error correction model* [J]. *Journal*

*of Futures Markets*, 1993, 13(7): 743-752.

[3] Ederington Louis H. *The Hedging Performance of the New Futures Markets* [J]. *The Journal of Finance*, 1979, 34(1): 157-170.

[4] Robert Engle. *Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models* [J]. *Journal of Business & Economic Statistics*, 2002, 20(3):339-350.

[5] Jonathan Dark. *Futures hedging with Markov switching vector error correction FIEGARCH and FIAPARCH* [J]. *Journal of Banking and Finance*, 2015, 61: S269-S285.

[6] Zheng Chengli, Su Kuangxi, Yao Yinhong. *Hedging futures performance with denoising and noise-assisted strategies* [J]. *North American Journal of Economics and Finance*, 2021, 58:101466.