

An ocean exploration model based on geometric problems and least squares multibeam detection techniques

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Keywords: Multi-beam Detection, Geometric Calculation, Line Layout Design, Least Squares

Abstract: As the global exploration and use of ocean resources continues to progress, measuring ocean depth has evolved into a crucial aspect of ocean engineering and scientific studies. If the seabed is rugged and requires more beam coverage to obtain comprehensive and accurate bathymetric data, this will also increase detection time and costs. Therefore, it has become an important and complicated problem how to reasonably design the coverage width and overlap ratio of the transmission line in order to maximize the data quality and measurement efficiency. A mathematical model is suggested to address this issue, which is aimed at defining the span of a ship's coverage as it sails various routes from the sea's center and computing this coverage width under specific circumstances. Multi-beam sounding technology has been further developed and applied in the real life background. Multi-beam system can receive multi-beam return signal, and carry out signal processing and data processing, can get water depth data faster, speed up the survey.

1. Introduction

A multi-beam system can simultaneously transmit multiple sound beams, each of which provides information about the depth of the ocean. This makes it possible to achieve full coverage bathymetry in areas with large relief on the seabed, which is very important for marine surveys and ocean engineering. The intricacies involved in both data processing and energy consumption necessitate efficiency improvements in sounding. Thus, this paper suggests that the line design should encompass as much of the designated sea area as feasible. Moreover, it should maintain a maximum overlap rate of 20% between adjacent strips, and aim to limit the total line length.

In light of the complex and crucial problem described earlier, this research presents a novel mathematical structure for measuring the coverage width of multi-beam bathymetry. This measurement begins as the ship sets off from the center of the sea area, and continues as the ship travels in a direction diverging from the angle between lines. This methodology thus enables the estimation of coverage width in given situations [1-3]. At the same time, the paper discusses how to design the coverage width and overlap rate of the transmission line reasonably by using the sea depth data measured by a single beam in certain sea area several years ago, so as to improve the data quality and survey efficiency by using the least squares [4-7] data, to provide help for the measurement wiring of multi-beam survey ship.

2. Solution assumptions

- 1) The assumption is that the survey ship's path is steady and linear, unaffected by the speed of wind or waves.
- 2) It is assumed that the seabed topography is relatively stable, and the instantaneous changes of seabed morphology due to earthquake and sediment movement are not considered.
- 3) The model assumes that the transducer can emit the specified sound beam completely without considering the energy loss during the transmitting process.
- 4) The external factors, such as underwater obstacles, other sound sources and so on, are not considered.

3. Representation of scheme parameters

In order to effectively design sea area solutions, several key parameters will be introduced to provide assistance. The parameters are given in Table 1. By adding these effective parameters, we can better understand the location environment of the sea area so as to work out more reasonable and feasible solutions.

Table 1: Representation of concepts

parameters	Description	Unit
θ	Open angle of transducer	rad
α	Slope	rad
β	Angle between line direction and projection	Rad
D	Depth of sea water	m
D0	The depth of the sea at the center of the sea	m
W	Beam coverage width	M
d	The distance between two adjacent survey lines	m
η	The rate of overlap between adjacent bands	\
γ	Corner of the road	rad
λ	New slope angle	rad

4. Developing and implementing a solution

4.1 Measure strip coverage width

(1) Developing a solution

Figure 1 illustrates the ship's direction, symbolized by a yellow arrow. This direction is tangent to the ship's path, the slope, and the vertical axis of the ship, collectively forming a new slope angle, denoted as λ , with the existing slope.

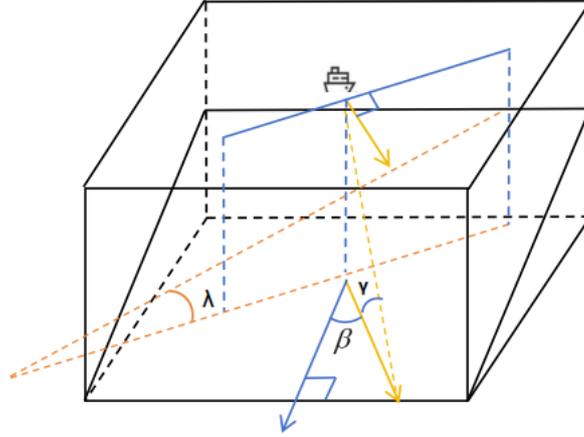


Figure 1: Three-dimensional diagram of the ship

Let the length of the rectangle be $2y$ and the width $2x$. The distance between the measuring vessel and the center of the sea within the rectangular area shall be as follows:

$$l_1 = \frac{Y}{\cos \beta} \quad (1)$$

The angle between the direction of the ship and the bottom of the sea is calculated as follows:

$$\tan \gamma = \frac{Y \tan \alpha}{\cos \beta} \quad (2)$$

To simplify:

$$\tan \gamma = \tan \alpha \cos \beta \quad (3)$$

Calculate the depth of deep water:

$$D = D_0 + t \tan \gamma \quad (4)$$

Calculate the distance perpendicular to the direction of the ship in a rectangular area:

$$l_2 = \frac{Y}{\cos(90 - \beta)} \quad (5)$$

The angle tangent between the vertical plane and the sea floor is obtained:

$$\tan \lambda = \frac{Y \tan \alpha}{l_2} \quad (6)$$

The angle between the vertical plane and the bottom plane is:

$$\lambda = \arctan(\tan \alpha \sin \beta) \quad (7)$$

Get the strip coverage width model:

$$W = (D_0 + t \tan \gamma) * \left(\frac{\sin \frac{\theta}{2}}{\sin(\frac{\pi}{2} - \frac{\theta}{2} - \lambda)} + \frac{\sin \frac{\theta}{2}}{\sin(\frac{\pi}{2} - \frac{\theta}{2} + \lambda)} \right) * \cos \lambda \quad (8)$$

Formula (11) inserted:

$$W = (D_0 + t \tan \alpha \cos \beta) * \left(\frac{\sin \frac{\theta}{2}}{\sin(\frac{\pi}{2} - \frac{\theta}{2} - \lambda)} + \frac{\sin \frac{\theta}{2}}{\sin(\frac{\pi}{2} - \frac{\theta}{2} + \lambda)} \right) * \cos \lambda \quad (9)$$

(2) Implementing a solution

We base our programming on Python using geometric problems and trigonometric relationships. Table 2 displays the results obtained from Python, showing the strip coverage width W achieved at varying angles of the survey line direction's horizontal projection and the seabed slope's normal direction. These results are also represented at various distances from the survey ship's central point to the marine area.

Table 2: Results of solution

Coverage width /m	The distance/nautical mile between a measuring vessel and the center of the sea								
	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	
Angle of line direction /	0	415.692	466.091	516.49	566.889	617.288	667.686	718.085	768.484
	45	416.157	451.834	487.511	523.189	558.866	594.543	630.22	665.898
	90	416.624	416.624	416.624	416.624	416.624	416.624	416.624	416.624
	135	416.157	380.48	344.803	309.125	273.448	237.771	202.094	166.416
	180	415.692	365.293	314.894	264.496	214.097	163.698	113.299	62.9002
	225	416.155	380.478	344.801	309.124	273.447	237.77	202.093	166.416
	270	416.617	416.617	416.617	416.617	416.617	416.617	416.617	416.617
	315	416.155	451.832	487.509	523.186	558.863	594.54	630.217	665.894

4.2 Curve fitting using least squares

(1) Developing a solution

The least squares technique, widely used in statistics for data fitting and parameter determination, is typically utilized to find a function or model that minimally reduces the sum of the squared variances between the model's projected value and the actual value of the observed data[8-10]. In this paper, we construct a plane from a point-to-plane least squares using a vector that satisfies the following formula:

$$Ax + By + z + D = 0 \quad (10)$$

$$z1 = N \sum xz - \sum x \sum z \quad (11)$$

$$a1 = \sum x^2 - \sum x \sum x \quad (12)$$

$$b1 = N \sum xy - \sum x \sum y \quad (13)$$

$$z2 = N \sum yz - \sum y \sum z \quad (14)$$

$$a_2 = \sum xy - \sum x \sum y \quad (15)$$

$$b_2 = N \sum y^2 - \sum y \sum y \quad (16)$$

Where N is the number of points in the selected region.

According to the above formula, we can find the expression of each parameter in the formula as follows:

$$A = -\frac{z_1 b_2 - z_2 b_1}{a_1 b_2 - a_2 b_1} \quad (17)$$

$$B = -\frac{a_1 z_2 - a_2 z_1}{a_1 b_2 - a_2 b_1} \quad (18)$$

$$D = -\frac{\sum z - A \sum x + B \sum y}{N} \quad (19)$$

From the sample data, we can get the following figure 2.

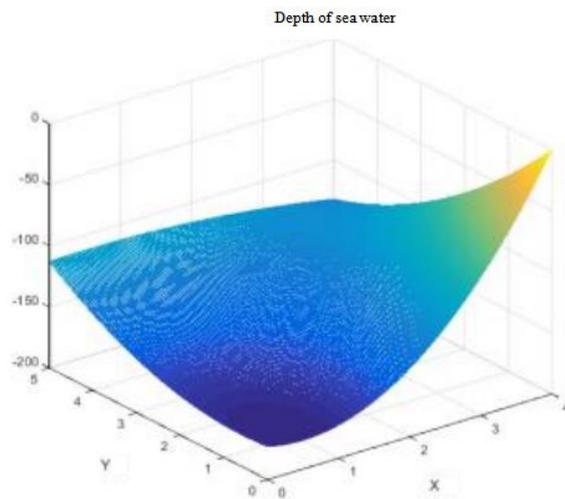


Figure 2: Three-dimensional chart of sea water depth

To sort out the terrain at the bottom, we can get the following figure 3.

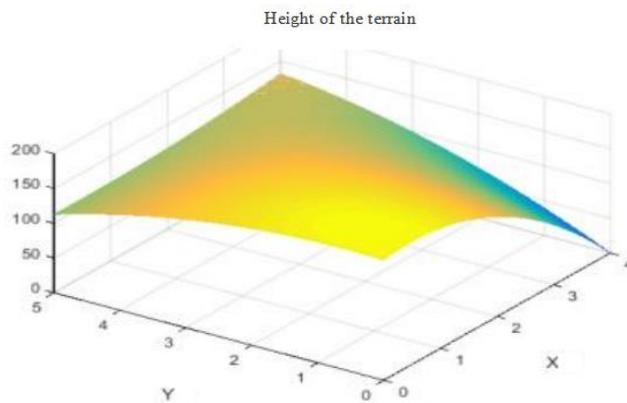


Figure 3: Three-dimensional map of terrain height

As shown, to calculate the terrain height separately, divide the terrain height into two slopes at different angles, with (0, 0) and (4, 5) as diagonals, the line divides the terrain height into two slopes.

(2) Implementing a solution

Derived from the above formula (17), Formula (18) and formula (19), we can get the following figure 4. The normal vector of the first plane: A is -1.7463, B is -14.3622, D is 195.2507. And we can get the following figure 5, the normal vector of the second plane: A is -40.9399, B is 15.2104, D is 208.318.

$$Ax + By + z + D = 0 \tag{20}$$

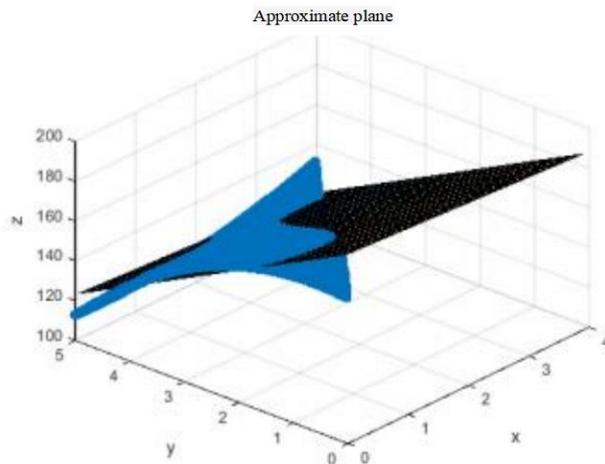


Figure 4: Approximate plane quantified by least squares

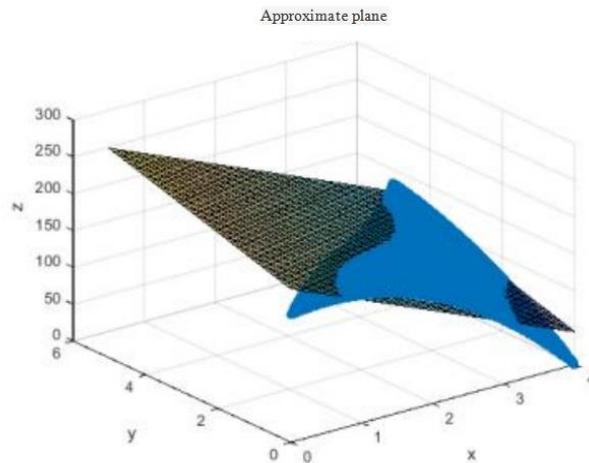


Figure 5: The approximate plane of the least squares

The two slopes formed by the diagonal line are respectively used in the last mathematical model to meet the requirements of the subject as far as possible to cover and survey line length as far as possible, as short as possible.

5. Assessment enhancement and adaptation of the approach

5.1 Assessment of the Approach

The model is based on geometric principles, and the theoretical foundation is solid.

A variety of choices are offered taking into account the arrangement of various conditions.

The practicability and accuracy of the model are enhanced by combining the actual bathymetric data.

5.2 Refinement of the Approach

Take into account the impact of outside environmental elements like water flow and waves, among others, on the measurement process.

Considering the effect of the speed of the survey ship on the survey layout, the survey plan is further optimized.

5.3 Application of the Approach

The model can be applied to the actual ocean exploration project to provide a scientific basis for survey wiring.

The model can be further extended to consider more complex environmental factors and improve the applicability of the model.

6. Conclusion

This document delves into and broadens the application of multi-beam sounding technology in Ocean Survey and marine engineering. The multi-beam system can receive and process multiple echo signals to improve the measuring speed and obtain faster bathymetric data. This paper suggests designing a comprehensive sea area survey line to enhance sounding efficiency, where the adjacent strips' overlap rate should be less than 20%, and the total line length should be the smallest possible. Simultaneously, it proposes a mathematical model to portray the multi-beam bathymetry's coverage width, which can elucidate the coverage width of ships moving from the sea area's center at varying angles. In addition, single-beam bathymetric data from several years ago are used to improve data quality and measurement efficiency through least squares, and to assist in the survey wiring of multi-beam survey ships.

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