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Multibeam bathymetry optimization problem based on geometric modeling and simulated annealing

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Abstract: Echo sounding is a technique commonly used in marine bathymetry to measure the depth and topography of water bodies. This paper combs the development process of echo sounding technology, briefly describes the principle of multibeam bathymetry, and its application in ocean bathymetry and water conservancy engineering. In order to get the optimized scheme of the survey line in the rectangular sea area, this paper firstly draws a spatial 3D scatter plot using the attached data to observe the general shape of the seabed surface. Then, polynomial fitting is utilized to fit the surface to all points to obtain the surface equation. From the scatter plot, it can be seen that the seafloor slope is relatively gentle, and if the formula for the coverage width when the seafloor slope is horizontal can be used for calculation, the model will be greatly simplified. The programming in this paper verifies the reasonableness of the conjecture, so the simplified formula can be used for subsequent calculations. In order to determine the number of survey lines, a simulated annealing algorithm was used, and finally, we designed 31 parallel survey lines in the north-south direction, with a total length of 155 nautical miles, and the omitted sea area accounted for 1.71% of the total area to be surveyed, and in the overlapping area, the overlap rate of the part of the overlap rate of more than 20% had a total length of zero.

1. Introduction

With the emergence of echo sounding technology, acoustic sounding has become the main technical means of marine exploration. Echo sounding equipment can be divided into two categories according to the number of beams.

One is single-beam bathymetry, single-beam bathymetry is the use of acoustic wave propagation characteristics in the water to measure the depth of the water body technology. By simple physical knowledge, sound waves in a uniform medium back to the uniform speed along the straight line propagation, and in the encounter with obstacles occur when the reflection. Then, you can design an acoustic transmitter to the seabed to transmit acoustic signals, record the acoustic wave propagation time from the launch to the reception, and then calculate the depth of the measurement point. 1913, the United States scientists Fessenden successfully developed the first echosounder, the single-beam bathymetric technology began to be applied.

Because the single-beam bathymetry process uses a single point of continuous measurement, the distribution of the measurement line data is characterized by dense measurement data along the trajectory and no measurement data between adjacent measurement lines.

On the research results of single-beam bathymetry, the U.S. Naval Research Service began to study multibeam bathymetry technology in the 1960s, and in 1962, the U.S. National Oceanic Survey conducted sea experiments with a narrow-beam echosounder. 1976, with the further development of computer systems and hardware, the first multibeam scanning bathymetry system came out, i.e., the SeaBeam, which has a measuring range of 0.8 times the depth of the water, and can simultaneously process the data from a single point, which can be used to measure the depth of the water, and can simultaneously process the data of the water. 0.8 times the water depth, and can process 16 beams simultaneously. After this, the multibeam bathymetric system has entered a stage of rapid development, the system from a single shipboard acquisition of land-based processing to the development of a set of data acquisition, synthesis, processing and display in one of the perfect system. In order to adapt to different water depths, the fan angle and sweep width of the multibeam bathymetry system have also been greatly increased, and the data processing speed has also been greatly improved, and for each acoustic pulse emitted by the transmitting transducer, dozens or even hundreds of bathymetric values can be obtained for the vertical sailing trajectory, which in turn has many advantages[1], as reflected in:

(1) High precision and high efficiency

Multibeam bathymetry technology uses a combination of amplitude and phase methods, using amplitude detection for the central beam and phase detection methods for the edge beams to make up for the errors and defects brought about by using a single detection method. Moreover, the number of beams in multibeam bathymetry is very large, reaching more than 1000 beams, which greatly improves the coverage of bathymetry. Overall, it is characterized by high precision and high efficiency.

(2) Integration and fusion of multiple technologies. Multibeam bathymetric system no longer relies solely on the acoustic wave transmitting and receiving devices, but the addition of a high-precision localizer, the measured seabed increased position information, so that high-precision position and bathymetry information can be obtained.

2. Subject matter

Multibeam bathymetry system overcomes the shortcomings of single-beam bathymetry, and can measure a full-coverage bathymetry strip with a certain width on the axis of the survey line of the survey vessel in the flat sea bottom, and its working principle is shown in the figure 1 and figure 2.

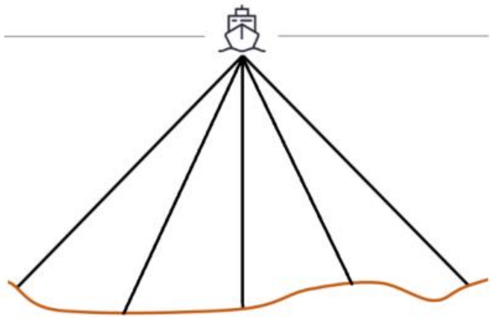

Figure 1: Working principle of multibeam bathymetry

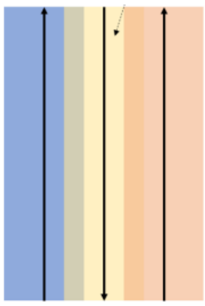

Figure 2: Strips, lines and overlapping areas

In the model, the coverage width W of the multibeam bathymetric strip varies with the transducer opening angle θ and the water depth D. In the model, the coverage width of a multibeam bathymetric strip varies with the transducer opening angle and water depth. If the survey lines are parallel to each other and the seafloor topography is flat, the overlap rate between neighboring strips is defined as $\eta = 1 - \frac{d}{W}$, where d is the spacing between neighboring survey lines and W is the coverage width of the strip. If $\eta < 0$, then it indicates a missed measurement. In order to ensure the convenience of measurement and data integrity, the overlap rate between adjacent strips should be 10%-20%. However, the real seabed topography varies greatly, if the average water depth of the sea area is used to design the line spacing, although the average overlap rate between the strips can meet the requirements, there will be omission of measurement in the shallow water depth, which affects the quality of the measurement; if the shallowest water depth of the sea area is used to design the line spacing, although the overlap rate of the shallowest water depth can meet the requirements, there will be excessive overlap in deeper water depths, and the redundancy of data will be large, which affects the efficiency of the measurement. This will affect the measurement efficiency, as shown in figure 3.

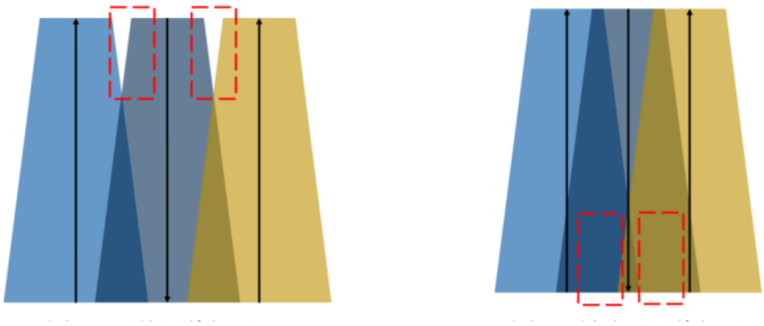

Figure 3: Schematic diagram of the overlap of adjacent survey lines

2.1 Solving for depth of coverage

It is known that the intersection line between a plane perpendicular to the direction of the survey line and the slope of the seabed constitutes an oblique line with an angle of α with the horizontal plane, and α is said to be the slope. Establish a mathematical model of the coverage width of multibeam bathymetry and the overlap rate between adjacent strips.

In this context, the coverage width refers to the width of the strip measured looking down on the sea, rather than the length of the line of intersection between the beam plane and the seafloor slope. With this understanding, an expression for the width of the overlap in relation to the slope and depth can be derived from the angular relationship within the triangle and the transformation of the trigonometric function. For the solution of the overlap rate, the ratio of the width of the overlapping portion to the coverage width of the previous survey line can be found through geometric relations, and then the recursive relationship equation can be written.

Let the angle between the outermost beam on the left and the slope of the seafloor be φ, the angle between the outermost beam on the right and the slope of the seafloor be γ, and the distance of the ship from the seafloor at a certain point in time be D, as shown in Figure 4:

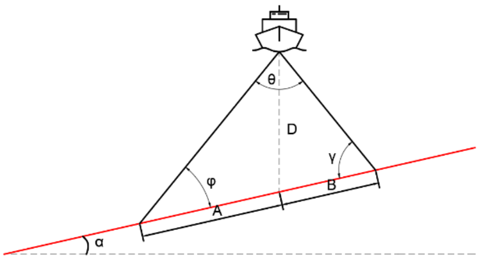

Figure 4: Schematic representation of the coverage width of a single survey line

It can be deduced from the angular relationship of the triangle:

$$\varphi = \frac{\pi}{2} - \frac{\theta}{2} - \alpha \tag{1}$$

$$\gamma = \frac{\pi}{2} - \frac{\theta}{2} + \alpha \tag{2}$$

In a triangle by the sine theorem:

$$\frac{A}{\sin\frac{\theta}{2}} = \frac{D}{\sin\varphi} \tag{3}$$

$$\frac{B}{\sin\frac{\theta}{2}} = \frac{D}{\sin\gamma} \tag{4}$$

Noting $A+B=L$, equation (3) is added to equation (4) to obtain:

$$\frac{L}{\sin\frac{\theta}{2}} = \frac{D}{\sin\varphi} + \frac{D}{\sin\gamma} \tag{5}$$

After simplification and organization the expression for L can be calculated as follows:

$$L = \frac{4D\cos\frac{\theta}{2}\cos\alpha\sin\frac{\alpha}{2}}{\cos\theta + \cos 2\alpha} = \frac{2D\sin\theta\cos\alpha}{\cos\theta + \cos 2\alpha} \tag{6}$$

The coverage width is the length of L the projection on the horizontal plane, so the calculation of the coverage width W is as follows:

$$W = L\cos\alpha = \frac{2D\sin\theta\cos^2\alpha}{\cos\theta + \cos 2\alpha} \tag{7}$$

2.2 Mathematical derivation of overlap rates

We still consider a simple geometrical relationship to make a schematic diagram of two neighboring survey lines emitting beams, as shown in Fig. 5:

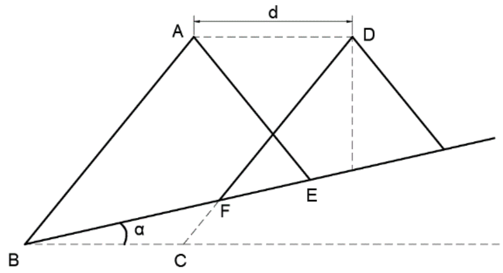

Figure 5: Schematic diagram of neighboring beam overlap rate calculation

Let the distance between two neighboring lines be AD, the length of the overlap on the slope be FE, the width of the overlap of neighboring strips be s, and the coverage width of the previous line be W. Remember $\angle BAE=\theta$, $\angle ABE=\varphi$, extend DF to intersect the horizontal line at C, obviously $\square ABCD$ is a parallelogram, from the geometric relationship:

$$\angle DAE=\frac{\pi-\theta}{2} \tag{8}$$

It follows from the equality of opposite angles within a parallelogram:

$$\angle BCF=\angle BAD=\theta+\frac{\pi-\theta}{2}=\frac{\pi}{2}+\frac{\theta}{2} \tag{9}$$

Can be obtained by applying the sine theorem in $\triangle BCF$:

$$\frac{BF}{\sin\left(\frac{\pi}{2}+\frac{\theta}{2}\right)}=\frac{BC}{\sin\varphi} \tag{10}$$

$$BF=BE-EF=\frac{W}{\cos\alpha}-\frac{s}{\cos\alpha} \tag{11}$$

Eqs. (12) and (13) can be solved to obtain the overlap rate η of two neighboring bands as:

$$\eta=\frac{s}{W}=1-\frac{d\cos\alpha\sin\left(\frac{\pi}{2}+\frac{\theta}{2}\right)}{W\sin\varphi} \tag{12}$$

Next, multiple neighboring survey lines are modeled, as shown in Figure 6:

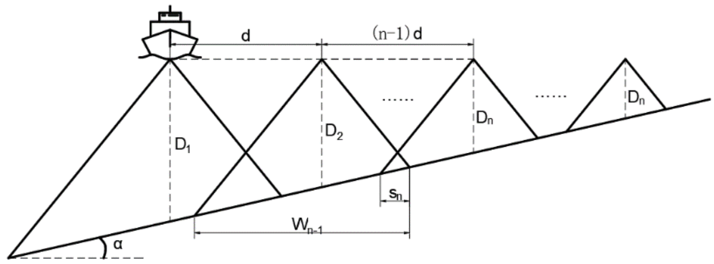

Figure 6: Schematic of the overlap rate of multiple isometric lines

Then the overlap rate η_n between multiple uniformly spaced survey lines is:

$$\eta_n = \frac{s_n}{W_{n-1}} = 1 - \frac{d\cos\alpha\sin\left(\frac{\pi}{2}+\frac{\theta}{2}\right)}{W_{n-1}\sin\varphi} \tag{13}$$

2.3 Building on data to help with calculations

Based on the above calculation results, we further consider the realistic situation. The survey line of the whole sea area to be measured is known, and the overlap rate between adjacent strips meets the requirement of 10%~20%.The seawater depth data for question B of the 2023 National Competition of Mathematical Modeling (Appendix.xlsx) is the bathymetry data of single-beam survey of a certain sea area (5 nautical miles long from north to south, and 4 nautical miles wide from east to west) a number of years ago, and we now hope to utilize the data for the help of the measurement routing for the multibeam measurement vessel. When designing the survey line, the following requirements were met: (1) the strips formed along the scanning line should cover the whole sea area to be surveyed as far as possible; (2) the overlap rate between neighboring strips should be controlled to be less than 20% as far as possible; and (3) the total length of the survey line should be as short as possible. After designing the specific survey line, please calculate the following indicators: (1) the total length of the survey line; (2) the percentage of the missed sea area to the total sea area to be surveyed; and (3) the total length of the part of the overlapping area where the overlap rate is more than 20%.

Using the attached data, we plotted a scatterplot of bathymetric data (Fig. 7), a scatterplot of seafloor depth (Fig. 8), and a simulation of seafloor slope (Fig. 9) using Matlab:

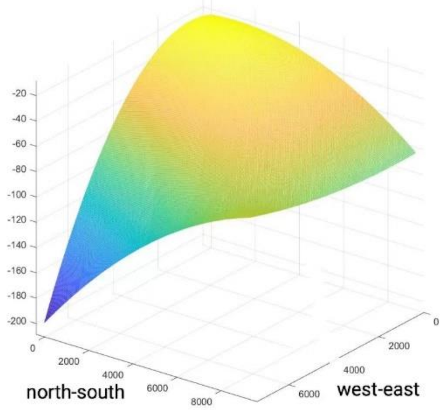

Figure 7: Scatter plot of bathymetric data

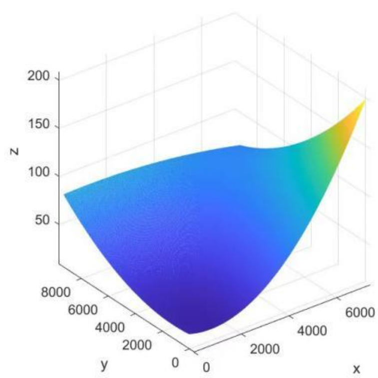

Figure 8: Scatter plot of seafloor depths

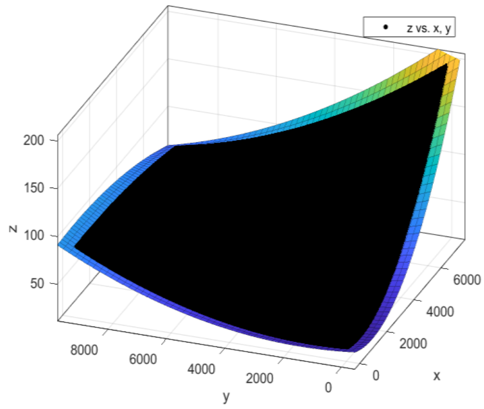

Figure 9: Seafloor Slope Simulation

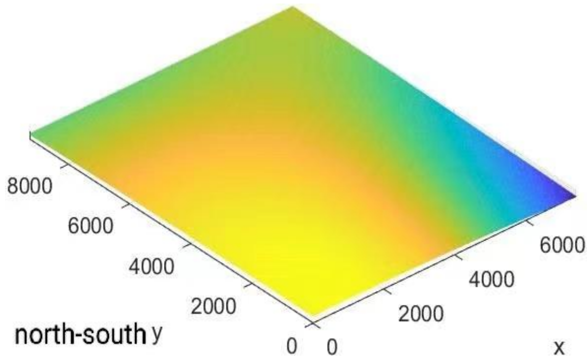

Figure 10: Seafloor Depth Fit

2.3.1 Curve Fitting

We tried to fit the surface equations using Matlab and got:

$$z = p00 + p10*x + p01*y + p20*x^2 + p11*x*y + p02*y^2 + p21*x^2*y + p12*x*y^2 + p03*y^3 \tag{14}$$

Calculate the goodness-of-fit R^2:

$$R^2 = 1 \tag{15}$$

It can be seen that the fit is good, so the surface equations shown in (14) can be directly utilized for the solution in the next analysis. In Fig. 10, the black part is the real seafloor depth map obtained by single-beam bathymetry, and the colored part is the seafloor depth map obtained by fitting, which can be seen that the fitting result matches well with the real value.

2.3.2 Model simplification

When the seabed slope is horizontal and flat, the width of cover is calculated as follows:

$$W_p = 2D\tan\frac{\theta}{2} \tag{16}$$

Observing the simulation of the seafloor slope, we can find that the seafloor slope is relatively gentle, and we guess that the relative error of using equation (16) to calculate the coverage width is very small. If the conjecture is valid, it will greatly simplify the calculation. To verify the conjecture, we use the program to find the maximum slope of the seafloor slope in the east-west direction α_{m1}、α_{m2}:

$$\alpha_{m1} = \arctan 0.0233 \tag{17}$$

$$\alpha_{m2} = \arctan 0.0141 \tag{18}$$

The error due to simplified calculation is 1.1736 m at the maximum slope α_1 along the east-west direction and 0.4269 m at the maximum slope α_2 along the north-south direction. The sea area is 9260 meters long and 7408 meters wide, so we think that the effect of the error caused by the simplified calculation can be ignored and the conjecture is valid.

2.3.3 Finding optimal survey lines using simulated annealing algorithm

From the above analysis, it can be seen that when measuring along the north-south direction, the error brought by the simplified model is smaller than that along the east-west direction, so the direction of the measurement line is made to be arranged in parallel along the north-south direction. We consider that the simulated annealing algorithm is suitable for solving the optimal measurement line in this problem. Simulated Annealing (SA) is a stochastic optimization algorithm used to solve large-scale combinatorial optimization problems, which starts from the similarity between the solution of the optimization problem and the annealing process of the physical system to solve the global optimization problem. [2]

It should be noted that when calculating the overlap rate here, the default overlap rate between two adjacent directions is equal everywhere.

After 10,000 iterations, the final optimal survey lines are 31 parallel lines along the north-south direction, of which the first strip covers the eastern boundary of the rectangular sea area.

The total length of the lines was calculated to be 155 nautical miles, the missed sea area accounted for 1.17% of the sea area to be surveyed, and the total length of the part of the overlapping area with an overlap rate of more than 20% was zero.

2.4 Practical applications of multibeam bathymetry

China's multibeam bathymetry started late, in 1997, Harbin Engineering University and the Navy jointly developed the first multibeam bathymetric instrument, in 2007, Harbin Engineering University developed China's first shallow water multibeam bathymetry system, the system has the advantages of easy installation, high resolution, with outstanding advanced, innovative and practical. However, at present, China's multibeam bathymetry system mainly relies on imports, the domestic multibeam commercial efficiency is not strong, the lack of mass production capacity, there are development difficulties.

The International Hydrographic Organization (IHO) in 1994 developed the international bathymetry standards clearly stipulated that: in high-level bathymetric surveys must be used in multibeam full-coverage bathymetry system. [3-4] The study of multibeam bathymetry is of great significance to the development of underwater measurement technology and the development and utilization of water resources. [5]

With the development of measurement equipment in recent years, multibeam bathymetric system has been greatly improved in terms of integration, lightweight, measurement accuracy, etc. Multibeam underwater topography measurement is gradually developed to shallow waters such as rivers and other waterways from the previous mainly applicable to marine surveying and mapping, and plays a role in the field of water conservancy. [6]

Traditional river monitoring uses single-beam measurement, with low bathymetric accuracy and efficiency. In order to better manage the river, the Yangtze River Channel Management Bureau of Anhui Province adopts multibeam bathymetry system to monitor the Yangtze River channel, which is able to quickly collect large-area and high-precision underwater point cloud data, make full-coverage and non-missing scanning measurements of the underwater topography of the river channel, which is convenient for making underwater topographic maps of various scales in CAD, as well as making use of the underwater data for three-dimensional modeling and fixed cross-section and scouring analysis in the GIS software[7]. It is convenient to produce various scales of underwater topographic maps in CAD and 3D modeling with underwater data in GIS software to produce fixed cross sections and siltation analysis maps for cross section comparison and siltation analysis. The monitoring provides a large amount of basic information for river management, and the analysis results can comprehensively reflect the change rules and trends of the river, and improve the technology and level of river management [8].

Engineering Survey and Inspection Department of Yangtze River Waterway Survey and Design (Wuhan) Co., Ltd. carried out prototype observation work in the middle reaches of the Yangtze River in the key sections of the river, for the first time used a more advanced multibeam bathymetry system for the large-scale measurement of underwater topography, high-efficiency collection of external data to provide technical support and protection, shorten the time of the prototype observation work, and strongly ensure that the construction and design work and even the key sections of the Yangtze River in the middle reaches of maintenance. This has strongly ensured the construction design work and even the implementation progress of the maintenance dredging project in the middle reaches of the Yangtze River. In the future, the multibeam bathymetric system will be further used in the field survey work to escort the high-quality development of the Yangtze River waterway. [9] It is reported that this prototype observation work, as the basic work of the maintenance dredging project for the middle reaches of the Yangtze River in the key sections of the Three Gorges follow-up work in the year 2022, will make a positive contribution to alleviating the negative impacts on the Yangtze River shipping caused by the storage and operation of the Three Gorges Project and fully guaranteeing the smoothness of the Yangtze River waterways.

In the underwater inspection, especially in the key process of immersed tube tunnel underwater topographic and geomorphological measurements have incomparable advantages, it can be clearly swept side to the suspected high point of the floating cloud navigation channel, you can have a more comprehensive understanding of the depth of the key waters, which is conducive to protect the immersed tube tunnel floating cloud, immersed and successful docking, it has a positive, irreplaceable role and significance in the immersed tube tunnel underwater topographic and geomorphological measurements.[10]

The water resources profession should actively use multibeam sounding technology to make the field of diving surveys more accurate and contribute to water resources projects.

3. Conclusions

Multibeam bathymetry is a fast and efficient method of collecting underwater topography. With the lightweight and integrated development of multibeam bathymetry system, its use has become more and more convenient. At the same time, with the advancement of localization, its technology is improving, the measurement accuracy of the equipment is improving, and the price of the equipment is decreasing, which have laid the foundation for the popularization of multibeam measurement system in the field of waterborne measurement. Using mathematical methods, this paper gives an example of applying fitting and programming methods to design a survey line within an irregular sea area, which can provide a reference for engineering. It is expected that the multibeam bathymetry system will be more mature and intelligent in the near future, which will provide strong support for the management and analysis of the river aspects such as ocean exploration, water conservancy engineering and digital twin.

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