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Application of a Modified Grey Model Based on Least Squares in Energy Prediction

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Abstract: The GM (1,1) model is a prediction method based on the grey system theory, which can be used to handle the prediction problem of time-series data. Compared with the traditional time series model, GM (1,1) model has the characteristics of a simple model, small calculation amount and small samples, so it has a wide application prospect in practical application. Grey GM (1,1) model is a commonly used prediction model in the energy industry, which can effectively deal with small amounts of data and incomplete data, as an accurate, reliable and efficient prediction model to predict energy consumption. In this paper, based on the classical grey GM (1,1) model, the constant free term is introduced, the modified grey GM (1,1) model is proposed, and the least squares method is used to construct an optimization problem related to the model parameters, and finally solve the general expression of the constant free term. Finally, the model is used to predict more accurately the per capita electricity consumption (kilowatt-hours). The results show that the improved GM (1,1) model is better than the traditional GM(1,1) model, which verifies the effectiveness and practicability of the improved model and is suitable for the forecast of per capita electricity consumption.

1. Introduction

Per capita electricity consumption refers to the ratio of the total amount of electricity consumed by a country or region in a certain period of time to the total population of a country or region. It is one of the important indicators to measure the energy consumption level of a country or region, and is closely related to the energy structure and production mode of a country or region. It reflects the level of economic development and living standards of a region. Accurate and reasonable prediction of per capita electricity consumption data can provide important reference information for the government, enterprises and the public to formulate energy policies and development strategies and promote sustainable development.

Since the 1980s, a large number of scholars began to make relevant research on the prediction of electricity power. Based on the statistical characteristics of China's social and economic data and the actual situation of power consumption, Guang Fengtao [1] builds a LEAP model containing three modules: key hypothesis modules, demand modules and conversion modules to predict China's future power demand. By combining Prophet, BPNN and other models, Cao Gangji [2] uses the hybrid model with multiple algorithms to improve the ability of power prediction. Yan Zhongyuan, Su Chenfei etc [3] based on smart meters (SM) data and combined with advanced metering infrastructure (AMI), the local level of short-term power demand forecast, and studied the power demand modeling and multiple node demand forecast aggregation effect, summarizes an AMI data into local short-term prediction method, in order to improve the distributed energy system, micropower grid and transactional energy prediction accuracy. Zhou Qinghua [4] uses the time series prediction method based on the group intelligence optimization algorithm combined with the fuzzy theory, and establishes the power prediction model. Shu Fuhua [5] uses the LSTM network with better performance to predict the electricity consumption data of the whole society in Henan province. Zhang Wujun, Cheng Yuanlin [6] et al. applied the genetic algorithm to optimize the initial weight and threshold of the BP neural network, took the BP neural prediction error as the fitness function of the genetic algorithm, and established the improved short-term power prediction method of BP neural network based on feature analysis. In addition, Hu Huanling [7] constructed a hybrid prediction model (VMD-DE-ESN) based on variational mode decomposition (Variational Mode Decomposition, VMD) and differential evolution algorithm (Differential Evolution, DE). Wu Fenghua [8] established the differential autoregressive moving average models of power generation (2,1,6) and power consumption (2,1,2) based on SPSS 22.0, respectively, to estimate the power supply and demand in Guizhou province from 2018-2027. Zhu Congcong [9] constructed a prediction model based on SSA-LSSVM for short-term (monthly) power demand forecast, and constructed a long-term power demand prediction model based on Kaya identity considering multiple factors. Gao Feng and Shao Xueyan [10-11] introduced the maximum relevance minimum redundancy (MRMR) algorithm to screen the key influencing factors of power consumption as predictors, and proposed the improved Jaya algorithm (iJay a) to optimize the hyperparameters of support vector regression (SVR), and then constructed the MRMR-iJayaSVR prediction model. Zhang Shu and Liao Xingwei [12] proposed a short-term power demand prediction method of long and short-term memory neural network based on characteristic analysis, and obtained the prediction results with high accuracy.

As one of the important indicators of energy consumption, the prediction accuracy of per capita electricity consumption is crucial for the development of the energy industry. The traditional grey GM (1,1) model has achieved some results in predicting the per capita electricity consumption, but due to the limitations of the model itself, its prediction accuracy needs to be improved. In this context, we propose a modified grey GM (1,1) model, introducing constant free terms and parametric solving using the least squares method. This model is more accurate and reliable than the traditional GM (1,1) model. In this paper, a constant free-term correction method can not only improve the prediction accuracy of GM (1,1) model, but also describe the characteristics and trends of the data more comprehensively.

2. The traditional grey GM (1,1) model

2.1 Generate additive data

k represents a moment, $X(k) = X_k$ represents observed value of a certain quantity at $t = k$, may set $X_k < X_{k+1}$, mark down the raw data column as a $X^{(0)}$

$$X^{(0)} = \left\{ X_1^{(0)}, X_2^{(0)}, \ldots, X_n^{(0)} \right\} \tag{1}$$

Let the generated cumulative data be listed as $X^{(1)}$

$$X^{(1)}(k)=\sum_{i=1}^{k}X^{(0)}(k)\ (k=1,2,\ldots,n) \tag{2}$$

2.2 Data expression after accumulation

The original data is monotonically increasing, so the accumulated data can be regarded as strongly monotonically increasing. Strongly monotonically increasing data can be approximated as exponential, which can be fitted with an exponential curve. The exponential curve must be a first-order linear constant coefficient differential equation. You can write the following expressions:

$$\frac{dX^{(1)}}{dt}+aX^{(1)}=u \tag{3}$$

The solution of the differential equation can be obtained by the following:

$$X^{(1)}=\left(X^{(0)}(1)-\frac{u}{a}\right)e^{-at}+\frac{u}{a} \tag{4}$$

In the equation, a,u are the only the unknown parameters to be determined. As long as the unknown parameters are solved, the expression of the cumulative sequence and time can be obtained, and the original sequence can be obtained naturally.

That is, the introduction time series k is:

$$X^{(1)}(k)=\left(X^{(0)}(1)-\frac{u}{a}\right)e^{-a(k-1)}+\frac{u}{a} \tag{5}$$

The original sequence is:

$$\hat{X}^{(0)}(k)=\hat{X}^{(1)}(k)-\hat{X}^{(1)}(k-1) \tag{6}$$

2.3 Unknown parameters are solved

$$\frac{dX^{(1)}}{dt}\approx X^{(1)}(k+1)-X^{(1)}(k) \tag{7}$$

Note that the difference of $X^{(1)}(t)$ between time $t=k+1$ and time $t=k$ is:

$$X^{(1)}(k+1)-X^{(1)}(k)=X^{(0)}(k+1) \tag{8}$$

And $\frac{dX^{(1)}}{dt}$ is the value of t at a certain point on $[k,k+1]$, since it is approximate, simply take the value at the point $k+1$ is the original differential equation:

$$\frac{dX^{(1)}}{dt}+aX^{(1)}=u \tag{9}$$

Can be approximated as:

$$X^{(0)}(k+1)+aX^{(1)}(t)\approx u\ (k\le t\le k+1) \tag{10}$$

Note that the function $X^{(1)}(t)$ takes values on the interval $[k,k+1]$, when approximated by the median:

$$X^{(1)}(t) \approx \frac{1}{2}\left(X^{(1)}(k)+X^{(1)}(k+1)\right) \tag{11}$$

The original differential equations can be converted into linear equations

$$X^{(0)}(k+1) = -\frac{1}{2}\left(X^{(1)}(k-1)+X^{(1)}(k)\right)a+u \tag{12}$$

$$B=\begin{pmatrix} -\frac{1}{2}\left(X^{(1)}(1)+X^{(1)}(2)\right) & 1 \\ -\frac{1}{2}\left(X^{(1)}(2)+X^{(1)}(3)\right) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}\left(X^{(1)}(n-1)+X^{(n)}(n)\right) & 1 \end{pmatrix} \tag{13}$$

$$Y=\begin{pmatrix} X^{(0)}(2) \\ X^{(0)}(3) \\ \vdots \\ X^{(0)}(n) \end{pmatrix} \tag{14}$$

Calculate coefficient by least squares method:

$$\begin{pmatrix} a \\ u \end{pmatrix} = \left(B^T B\right)^{-1} \cdot B^T \cdot Y \tag{15}$$

3. GM (1,1) model improved by least square method

First, introduce the constant free term, based on the classical grey GM (1,1) model, to obtain the modified grey GM (1,1) model:

$$X^{(1)}(k) = (X^{(0)}(1) - \frac{u}{a})e^{-a(k-1)} + \frac{u}{a} + \varphi \tag{16}$$

Where u, a, φ is the unknown parameter, and k is the time series, $\varphi \in \square$.

In fact, if the free term $\varphi = 0$, it degenerates to the classical grey GM (1,1) model, but due to the introduction of the free term, the original solution method is no longer applicable. To this end, we present a general method for solving such models. Let $\hat{Y}_t$ be the value calculated under the condition determined by parameter u, a, φ ,and Y_t be the given value, so the sum of squares of the two:

$$H(u,a,\varphi) = \sum_{t=1}^{n}(Y_t - \hat{Y}_t)^2 = \sum_{t=1}^{n}(Y_t - (X^{(0)}(1) - \frac{u}{a})e^{-a(k-1)} - \frac{u}{a} - \varphi)^2 \tag{17}$$

In the equation, n represents the number of raw data. The problem of solving u, a, φ is transformed into an optimization problem in operations research from (17):

$$\min_{u,a,\varphi} H(u,a,\varphi) = \min_{u,a,\varphi} \sum_{t=1}^{n} (Y_t - (X^{(0)}(1) - \frac{u}{a})e^{-a(k-1)} - \frac{u}{a} - \varphi)^2 \tag{18}$$

According to the extreme value theory of multivariate function, taking the derivative of H with respect to parameter u, a, φ:

$$\frac{\partial H}{\partial u} = \sum_{t=1}^{n} (2(-\frac{1}{a} + \frac{e^{-a(k-1)}}{a})(-\varphi - \frac{u}{a} - e^{-a(k-1)}(1 - \frac{u}{a}) + Y_t)) \tag{19}$$

$$\frac{\partial H}{\partial a} = \sum_{t=1}^{n} (2(\frac{u}{a^2} - \frac{e^{-a(k-1)}u}{a^2} + e^{-a(k-1)}(k-1)(1 - \frac{u}{a}))(-\varphi - \frac{u}{a} - e^{-a(k-1)}(1 - \frac{u}{a}) + Y_t)) \tag{20}$$

$$\frac{\partial H}{\partial \varphi} = \sum_{t=1}^{n} (-2(-\varphi - \frac{u}{a} - e^{-a(k-1)}(1 - \frac{u}{a}) + Y_t)) \tag{21}$$

Proof $\frac{\partial H}{\partial u} = F_1(u,a,\varphi)$, $\frac{\partial H}{\partial a} = F_2(u,a,\varphi)$, $\frac{\partial H}{\partial c} = F_3(u,a,\varphi)$ separately, and equations (19) - (21) are transformed into the following equations:

$$\begin{cases} F_1(u,a,\varphi) = 0 \\ F_2(u,a,\varphi) = 0 \\ F_3(u,a,\varphi) = 0 \end{cases} \tag{22}$$

The Levenberg-Marquardt algorithm is used to solve equation (22), and the following results are obtained:

$$\min_{x \in \mathbb{R}^3} F(x) = \frac{1}{2}\|F(x)\|^2 = \frac{1}{2}\sum_{i=1}^{3} F_i^2(x) \text{ , } x = (u,a,\varphi) \tag{23}$$

Its search direction:

$$d_k = -(J_k^T \cdot J_k + u_k I)^{-1} J_k^T F_k \tag{24}$$

Where $J_k = F'(x) = [F_1'(x), F_2'(x), F_3'(x)]$

And the gradient of equation (23):

$$G(x) \triangleq \nabla F(x) = \nabla(\frac{1}{2}\|F(x)\|)^2 = J^T(x)F(x) = \sum_{i=1}^{3} F_i(x)\nabla F_i(x) \tag{25}$$

4. Interpretation of result

4.1 Test of the model

In order to evaluate the prediction accuracy of the model, three key mean absolute percentage error indicators are used in this paper. These metrics provide a comprehensive and systematic way to measure the performance of the model under different scenarios. First, we examine the mean absolute percentage error ($MAPE_{fitting}$) of the fitted data, which measures the model's predictive accuracy over the range of training data. Second, we focus on the mean absolute percentage error ($MAPE_{projection}$) of the extrapolated predictions, which assesses the model's ability to generalize on previously unseen data. Finally, the average absolute percentage error ($MAPE_{overall}$) of the population is considered comprehensively, so as to understand the overall performance of the model more comprehensively. These error indexes not only provide the measurement of error quantification, but also help us to deeply analyze the calculation error level of the model, so as to provide a valuable reference for further optimization. Their calculation formulae are as follows:

$$MAPE_{fitting} = \frac{1}{N}\sum_{k=1}^{N}\left|\frac{X^{(0)}(k) - \hat{X}^{(0)}(k)}{X^{(0)}(k)}\right| \times 100\% \tag{26}$$

$$MAPE_{overall} = \frac{1}{n}\sum_{k=1}^{n}\left|\frac{X^{(0)}(k) - \hat{X}^{(0)}(k)}{X^{(0)}(k)}\right| \times 100\% \tag{27}$$

$$MAPE_{projection} = \frac{1}{n-N}\sum_{k=N+1}^{n}\left|\frac{X^{(0)}(k) - \hat{X}^{(0)}(k)}{X^{(0)}(k)}\right| \times 100\% \tag{28}$$

Where n is the number of total sample data, N representing the number of data used for the fitting modeling. Obviously, the smaller the sum $MAPE_{fitting}$, $MAPE_{projection}$, $MAPE_{overall}$, the better the model predicts.

4.2 Forecast and analysis of per capita electricity consumption

In order to verify the accuracy and reliability of the improved GM(1,1) model proposed in this paper, the per capita electricity consumption data of China released by the National Bureau of Statistics during the period from 2003 to 2020 were analyzed and modeled. First, the model was built and trained using the per capita electricity consumption data from 2003 to 2015. Then, the per capita electricity consumption data from 2016 to 2020 are forecasted. Among them, the per capita electricity consumption data of China from 2003 to 2020 are shown in the following Table 1:

Table 1: Data on Per capita Electricity Consumption for 2003-2020 (Unit: kWh)

Year	Per capita electricity consumption	Year	Per capita electricity consumption	Year	Per capita electricity consumption
2003	1477	2009	2782	2015	4231
2004	1695	2010	3135	2016	4439
2005	1913	2011	3497	2017	4754
2006	2181	2012	3684	2018	5134
2007	2482	2013	3993	2019	5356
2008	2608	2014	4239	2020	5501

According to the data in Table 1 above, the GM (1,1) model and the improved GM (1,1) model were established, and their prediction effect in per capita electricity consumption was compared. The data of per capita electricity consumption from 2003-2015 and the data from 2016-2020 were predicted.

The fitting results can be compared with the real data in Table 2 below. The prediction results and the real data are shown in Table 3, and the fitting prediction effect of different models is shown in Figure 1 below.

Table 2: Comparison of the fitting results of the two prediction models (unit: kWh)

Year	real value	GM (1,1)	Improved GM (1,1)
2003	1477	1477	1477
2004	1695	1892.8187	1757.0609
2005	1913	2049.9089	1918.8926
2006	2181	2220.0365	2122.4955
2007	2482	2404.2834	2435.0291
2008	2608	2603.8214	2512.9721
2009	2782	2819.9197	2723.3885
2010	3135	3053.9525	3045.7971
2011	3497	3307.4084	3358.6183
2012	3684	3581.8992	3588.2712
2013	3993	3879.1708	3905.4504
2014	4239	4201.1138	4169.2061
2015	4231	4549.7757	4339.3107
$MAPE_{fitting}$		3.6407	2.3150

Table 3: Comparison of the prediction results between the two prediction models (unit: KWH)

Year	real value	GM (1,1)	Improved GM (1,1)
2016	4439	4927.3741	4601.4242
2017	4754	5336.3103	4965.6721
2018	5134	5779.1853	5392.1608
2019	5356	6258.8157	5769.5469
2020	5501	6778.2519	6058.4679
$MAPE_{projection}$		15.1785	6.1990
$MAPE_{overall}$		6.8456	3.3939

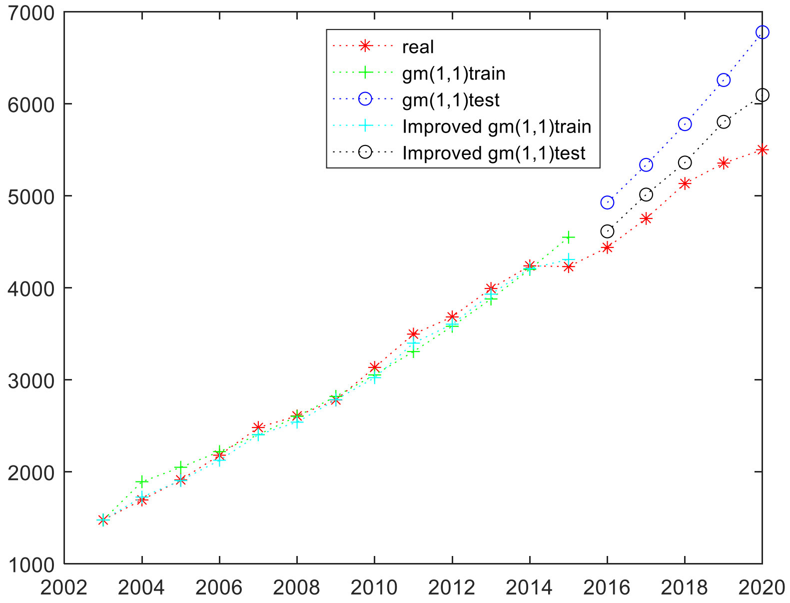

Figure 1: Fitting and prediction effects of the two models

By comparing the predicted data and fitting data in Table 2 and Table 3, it is obvious that the improved GM(1,1) model shows advantages in many aspects. First, from the point of view of mean absolute percentage error, in terms of fitting data, the mean absolute percentage error of the improved model is only 2.3150, compared with the error value of the ordinary GM(1,1) model is 3.6407, which shows that the improved model is more accurate in terms of the fitting ability of historical data. Secondly, in terms of extrapolated predicted values, the mean absolute percentage error of the improved model is 6.1990, while the error of the ordinary GM(1,1) model is as high as 15.1785, which further confirms the accuracy of the improved model in predicting the future trend. Most importantly, in terms of the total mean absolute percentage error, the improved model also far outperforms the ordinary model, with 3.3939 and 6.8456 respectively, indicating that our improved method has achieved a significant improvement in the overall prediction ability.

In addition, it can be clearly observed from Figure 1 that the predicted data of the improved GM(1,1) model is closer to the real data, and the forecast curve is closer to the actual trend. This further verifies the effectiveness and accuracy of the improved model, and also provides intuitive visual support for the research.

5. Conclusion

In this paper, based on the traditional grey GM(1,1) model, a modified grey GM(1,1) model is proposed by introducing the constant free term. By using the least square method, the optimization problem related to the model parameters is established, and the general expression of the constant free term is finally solved. Through comparative analysis and empirical research, we find that the improved GM(1,1) model has higher accuracy in data fitting and can more accurately capture the trend and change of historical data. At the same time, the performance of the model is more reliable in future trend prediction, and it can predict the changing trend of per capita electricity consumption more accurately. This not only has important significance in practical application, but also provides strong support for electric power planning and development, and also provides a new way of thinking and method for the research of similar problems.

In conclusion, the results of this study provide a new idea and method for power consumption forecast, and provide useful theoretical guidance for the planning and development of the energy field. We believe that in further research and practice, this improved model will play a greater potential and make positive contributions to solving practical problems and promoting progress in the field.

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