Research and Practice on the Teaching Reform of Probability Theory and Mathematical Statistics Based on Knowledge Visualization

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Abstract: The main research object of the course of probability theory and mathematical statistics is relatively abstract random phenomena, which requires students to have strong thinking level and learning ability. Due to the special nature of the course of probability theory and mathematical statistics, it is difficult for students to have an intuitive and profound understanding of the corresponding knowledge points in the course.Practice has proved that the visualization teaching method using Matlab, Office PowerPoint and Office Excel software as tools in the course of Probability Theory and Mathematical Statistics can convert "abstract" into "intuitive" to a large extent.The high requirement for students' "thinking level" is transformed into a low requirement for "observation ability", and the learning efficiency of the course "Probability Theory and Mathematical Statistics" is greatly improved.

1. Introduction

Probability Theory and Mathematical Statistics is an important mathematics course to study and reveal the statistical law of random phenomena [3], and it is also the only compulsory course for students majoring in science and engineering and management to study uncertain random phenomena in their undergraduate education.Due to the characteristics of mathematical tools of Probability Theory and Mathematical Statistics, the theories and methods of Probability Theory and Mathematical Statistics have been fully penetrated into the fields of science and technology such as artificial intelligence, big data science and technology, and blockchain, forming the characteristics of interdisciplinary development.However, Probability Theory and Mathematical Statistics is highly abstract and logical, with many content modules, abstract mathematical formulas, uncertainty of research objects and other factors that make students afraid of difficulties in the learning process.

Visualization means refers to the use of graphicable tools to recognize, perceive, imagine, and reason the attributes or change process of abstract objects, and realize the concretization and visualization of abstract objects through information processing technologies such as simulation and modeling, and finally display the abstract objects in static or dynamic graphics and images that are intuitive and easy to understand [4].Most of the objective laws, information and knowledge in various disciplines in reality are abstract and difficult to recognize. Visualization means can not only directly display the above objects, but also directly describe the dynamic process and change

law of the above objects, which is an excellent means to effectively solve the cognitive difficulties of abstract objects.

Based on the research and practice of knowledge visualization teaching method, this paper studies and solves the problem that the basic knowledge in the course of Probability Theory and Mathematical Statistics is abstract and obscure, which makes it difficult for students to have an intuitive and profound understanding. It is difficult to change from "teaching" to "learning" and to cultivate students' interest in learning.

2. Ideas of teaching reform

This paper researches and practices the visual teaching of Probability Theory and Mathematical Statistics, aiming at using static or dynamic graphs to describe relevant definitions, theorems and properties, providing students with an easy to accept sensory learning paradigm, and reducing the abstractness of relevant knowledge of Probability Theory and Mathematical Statistics.This can eliminate the problem that students have different ability to accept probability theory knowledge due to different level of basic knowledge reserve and different learning ability.The relevant knowledge points of probability theory are transformed from the original text description and mathematical description to the form of pictures and animations to unify the learning effectiveness of this course for students with different subject backgrounds and different learning abilities in this more vivid and interesting form, and the theoretical knowledge can be realized in practice through the visualization practice of theoretical knowledge, so as to enhance students' learning interest.The teacher must complete the transition from "teaching" to "learning".

In recent years, due to the distinctive characteristics and good teaching effect of visual teaching, domestic and foreign scholars have set off a research upsurge on visual teaching methods [5].The visualization teaching method has high application potential for the more abstract courses such as Calculus, Probability Theory and Mathematical Statistics and engineering mechanics.On the basis of studying the theories and methods of knowledge visualization teaching at home and abroad, this paper studies and practices the visualization teaching methods of Probability Theory and Mathematical Statistics, and further discusses the role and significance of introducing knowledge visualization teaching methods into the teaching process of Probability Theory and Mathematical Statistics.Based on the visual teaching practice of "Probability Theory and Mathematical Statistics" course, this paper studies and verifies whether Office PowerPoint, Office Excel, Matlab and other tool software are suitable for the visual teaching of "Probability Theory and Mathematical Statistics" course.This paper explores how to use visual teaching methods to encourage students to understand and master the course content through different sensory experiences through observation, perception, verification and application in the teaching process, and convert abstract mathematical symbolic calculation into intuitive results that can be obtained through visual senses to the maximum extent, so as to reduce the learning difficulty of the course and eliminate students' fear of difficulties in the course[6].

3. Visual tools and their application characteristics

3.1 Matlab

Matlab is one of the most widely used mainstream mathematical software, and its powerful modeling simulation, data operation analysis, data and graph conversion capabilities have been favored by the majority of users.

On the one hand, the contents of Probability Theory and Mathematical Statistics, such as mathematical analysis, random model and probability distribution of random variables, are abstract

and obscure, which are often difficult for students to learn.On the other hand, Matlab's excellent simulation, mathematical analysis and graphic output capabilities are very suitable for realizing the visualization of mathematical analysis content in the course of Probability Theory and Mathematical Statistics.In view of the compatibility between Matlab and the visual teaching of "Probability Theory and Mathematical Statistics" course, the visualization teaching of "Probability Theory and Mathematical Statistics" course is carried out with Matlab software, and the modeling and simulation are carried out with Matlab to further form the dynamic output of graphics or processes.The course knowledge of Probability Theory and Mathematical Statistics is analyzed by visual and concise graphical method[7]. The characters are changed into graphics, and the understanding is more intuitive, which can play a better role in auxiliary teaching[8].

3.2 Office PowerPoint

In the course of Probability Theory and Mathematical Statistics, random events and probability are generally taken as the first chapter of the course, which is the basic knowledge of probability theory and mathematical statistics.Among them, the probability calculation of random events can be transformed into the corresponding area calculation. If the relationship and operation of the events can be represented by a graph, the difficulty of understanding the corresponding knowledge points can be greatly reduced, so as to eliminate students' fear of difficulty in learning this course and enhance students' learning confidence[9].

Presentation software PPT can be used to solve this problem.PPT has powerful graphics production, animation production functions, the operation difficulty is low, easy to achieve, the effect is strong.In addition, the courses of probability theory and mathematical statistics are often taught by PPT, which makes the presentation of the definition of probability theory theorems by PPT graphics and animation more suitable for the teaching process and more operable.

3.3 Office Excel

Office Excel is one of the most widely used personal computer data processing software. It has the characteristics of powerful data computing ability, convenient chart tools and low practical operation difficulty.In the teaching process of Probability Theory and Mathematical Statistics, the random process and uncertain factors of variables make it difficult to learn the random variables and their distribution in the middle part of the course and the numerical characteristics of random variables.As this part of the content involves random process and a large number of numerical calculations, the output results are all numerical values, which is difficult to realize and show its regularity in traditional teaching methods, which is not conducive to students' learning and mastery.Spreadsheet Excel can quickly generate random numbers to simulate the random process, and can also quickly calculate a large number of numerical values. More importantly, spreadsheet Excel can convert the digital results into graphic output, and perfectly simulate and display the random variables and their distribution and digital characteristics that are difficult to describe clearly in the traditional teaching process.

4. Research and practice

4.1 Visual application of MATLAB in the teaching of Probability Theory and Mathematical Statistics

In the course of "Probability Theory and Mathematical Statistics", all the contents involving mathematical analysis can be realized by using MATLAB software to visualize related functions,

analyze mathematical knowledge in an intuitive and concise way, and show the process of the influence of variable changes on the results to students directly in a sensory manner.This can not only stimulate students' desire and interest in learning, but also enable students to understand the corresponding knowledge points more deeply and grasp more firmly.

Next, we will take the independent equally distributed central limit theorem as an example to compare the advantages and disadvantages of common teaching methods and visual teaching methods.

Teaching methods and contents:

Theorem 1: Set random variables

$$X_1, X_2,..., X_n,...$$

Independent of each other, subject to the same distribution, and

$$D(X_i) = \sigma^2 \neq 0, i = 1,2,... \ E(X_i) = \mu$$

Then for any one

$$x, -\infty < x < \infty$$

Always have:

$$\lim_{n \to \infty} P\left(\frac{\sum_{i=1}^{n} X_i - n\mu}{\sqrt{n}\sigma} \leq x \right) = \Phi(x)$$

Where $\Phi(x)$ is the distribution function of the standard normal distribution N(0,1).

The central limit theorem refers to a class of theorems in probability theory that discuss the sequence of random variables and their distributions asymptotically approaching normal distributions.This set of theorems is the theoretical basis of mathematical statistics and error analysis, pointing out the conditions for a large number of random variables to approximate normal distribution [1].

Visual teaching methods and contents:

The central limit theorem means that under proper conditions, the mean of a large number of independent random variables converges to a normal distribution after proper standardization.Each time n samples are randomly taken from these populations, a total of m samples are taken.The m samples are then averaged separately. The larger n is, the closer the distribution of these averages is to a normal distribution.

Below, we take binomial distribution as an example to observe the difference between the standardized simulation results (histogram) and the theoretical results (scatter plot) when the number of simulation experiments is m=50 and the sample space is n=10,20,80 and 150. See Figure 1-4.
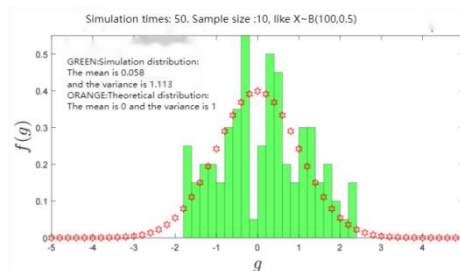

Figure 1: The number of experiments is m=50, and the sample space is n=10

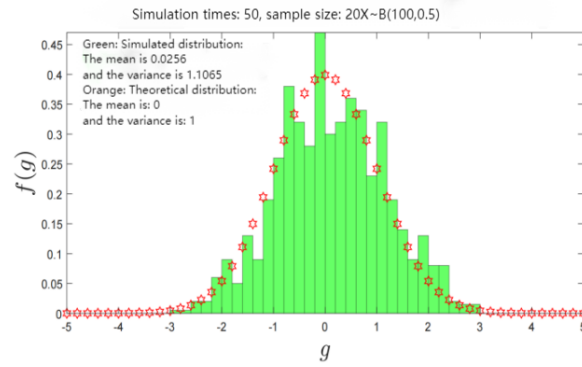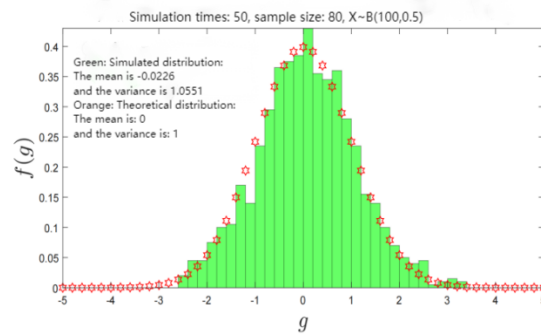Figure 2: The number of experiments is m=50, and the sample space is n=20

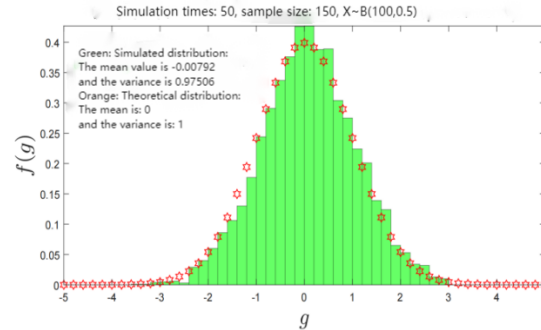Figure 3: The number of experiments is m=50, and the sample space is n=80

Figure 4: The number of experiments is m=50, and the sample space is n=150

Through the simulation results, we can intuitively observe that when the number of sample points n continues to increase, the system measure:

$$g = \frac{\sum\limits_{i=1}^{n} X_i - n\mu}{\sqrt{n}\sigma}$$

Gradually approximate the standard normal distribution, i.e.:

$$\lim_{n \to \infty} P\left(\frac{\sum\limits_{i=1}^{n} X_i - n\mu}{\sqrt{n}\sigma} \leq x \right) = \Phi(x)$$

Where $\Phi(x)$ is the distribution function of the standard normal distribution N(0,1).

Compared with the symbolic description of the book, the application of matlab to the visual teaching method can transform the complex mathematical analysis content into a dynamic process clear, intuitive and easy to accept the scene, so that students can better understand and master the corresponding knowledge points.

4.2 Visual application of Office PowerPoint in the teaching of Probability Theory and Mathematical Statistics

Because of its advantages in real-time presentation, information form conversion and information sharing, PPT is widely used in classroom teaching.Its advantages are low production difficulty, simple operation, intuitive and convenient.The most important thing is that PPT has the function of animation and dynamic process effect display, which is very suitable for application in the chapter of random events and probability in Probability Theory and Mathematical Statistics. Animation and dynamic process effect are used to show the relationship between events and their mathematical symbols such as operation, conditional probability and total probability formula.

Next, we will take the total probability formula as an example to compare the advantages and disadvantages of common teaching methods and visual teaching methods.

Teaching methods and contents:

Definition 1 If n events meet the following two conditions:

(1) B1,.....Bn is pairwise incompatible;

(2) $B_1 \cup ... \cup B_n = \Omega$。

So let's call these n events B1,.....Bn is a complete set of events.

Theorem 2 sets n events B1,.....Bn is a complete set of events for the sample space Ω, and(i = 1, 2,...n), $P(B_i) > 0$

then for any event: A

$$P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i)$$

This formula is called the total probability formula [2].
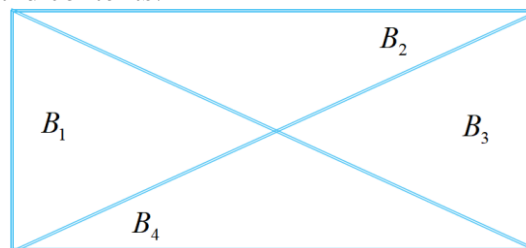
Visual teaching methods and contents:

Figure 5: Complete event group

If B1,.....B4, as shown in the Figure 5, does not overlap (incompatible), and B1+B2+B3+B4=Ω, then said B1,.....B4 forms a complete event group.
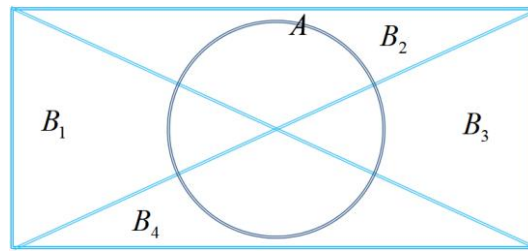
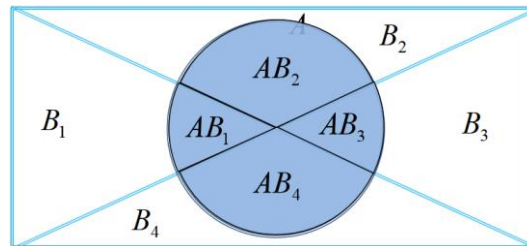Figure 6: An arbitrary event A in the sample space

Figure 7: Composition of any event A

As can be seen from Figures 6 and 7:

$$P(A) = \sum_{i=1}^{4} P(AB_i)$$

From the multiplication formula:

$$P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i)$$

Then you get

$$P(AB_i) = P(A|B_i)P(B_i), i = 1,...,4$$

In the conventional teaching method, it is difficult for students to understand and grasp the proof process of theorems such as total probability formula.If we can use the function of PPT animation and dynamic process effect display to analyze the composition of events in an intuitive graphical way, the total probability formula will become intuitive and easy to understand.

4.3 Visual application of Office Excel in the teaching of Probability Theory and Mathematical Statistics

Although the probability distribution of random variables can completely describe the statistical law of random variables, in many practical problems, it is difficult for learners to understand and master the knowledge points in the course of Probability Theory and Mathematical Statistics in a comprehensive way without examples.Office Excel is a simple and convenient operation, with powerful data processing and analysis and statistical capabilities, and has certain simulation functions.In the teaching of Probability Theory and Mathematical Statistics, visual teaching using Office Excel as a tool enables learners to observe random experiments and their dynamic process, which not only enables learners to comprehensively and accurately understand and master the corresponding knowledge points, but also makes the operation of Office Excel simple and convenient.Learners can complete most of the random experiments by themselves, which plays an important role in deepening learners' understanding of knowledge points and continuously

improving their learning interest.

Next, we will take the mathematical expectation of discrete random variables as an example to compare the advantages and disadvantages of common teaching methods and visual teaching methods.

Teaching methods and contents:

Definition 2 Let the distribution law of discrete random variable X be $P(X = a_i) = p_i, i = 1,2,...$,If the series $E(X)$,absolute convergence, then the value of this series is called a random variable X's

mathematical expectation, denoted as $E(X)$. That is to get

$$E(X) = \sum_{i=1}^{\infty} a_i p_i$$

.

By definition, the mathematical expectation of a random variable X is a description of the value center, which is an average property constant that characterizes X, similar to a weighted average.

Visual teaching methods and contents:

When studying mathematical expectations of random variables, it is more important to clearly recognize that the mathematical expectation is not necessarily equal to the sample mean, and in specific random experiments or samples, there may even be cases where the sample mean is not equal to the mathematical expectation.In fact, the mathematical expected value is the theoretical average of the output values of the variable, but the mathematical expected value is not necessarily contained in the set of output values of the variable.

Let's take coin flipping as an example, and let X be the average number of coin flipping heads 100 times in 10 rounds of experiments, which can be observed intuitively through Office Excel simulation relationship between $E(X)$ and $\bar{X}$.

Simulation simulation data acquisition area									
X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
1	1	1	1	1	0	0	1	0	1
1	1	1	0	0	1	1	1	1	0
0	0	1	0	0	1	0	1	0	0
0	1	1	0	0	0	1	0	1	0
0	0	1	0	0	0	0	1	0	0
0	0	0	1	1	1	1	0	0	0
1	1	0	1	1	1	1	0	1	1
1	1	0	0	1	0	0	0	1	1
1	0	1	1	1	1	1	0	0	0
0	0	1	1	1	0	1	1	0	0
1	0	1	0	0	1	1	1	1	1
0	1	1	0	1	0	1	1	1	1
1	1	0	1	1	0	1	1	1	0
0	0	0	0	0	0	1	0	0	0
0	1	0	1	0	0	0	0	1	0
1	0	0	0	1	0	0	1	0	1
0	0	0	1	1	1	0	1	0	1

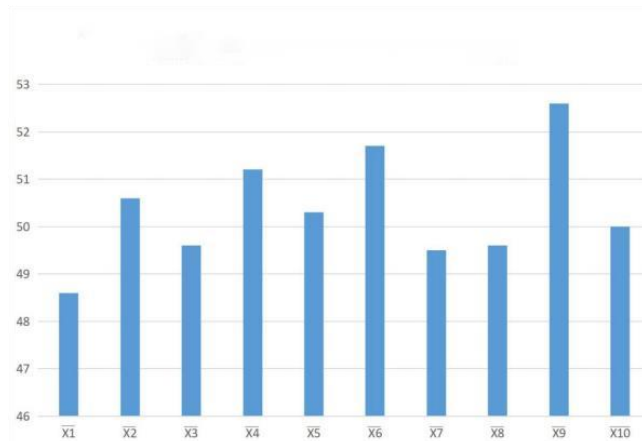Figure 8: Simulate 100 coin-toss experiments

Figure 9: Mean bars of X1-X 10 in 10 rounds of experiments

We can observe that the mean of each variable in Figure 8 and 9 closely fluctuates around the mathematical expectation, which is the essential relationship between the mathematical expectation and the sample mean.In addition, if the number of experiments increases, the tightness of the sample mean around the expected fluctuation of mathematics increases and the fluctuation decreases.

The advantage of applying Office Excel to the teaching of Probability Theory and Mathematical Statistics is that it can easily carry out simulation, data processing and graphical results display, and can intuitively show the difficult knowledge points to the learners by means of example repetition. This not only reduces the learning difficulty of Probability Theory and Mathematical Statistics, but also increases the learners' learning smoothness and enhances their learning confidence.

5. Conclusion

In practice, most learners in the course of Probability Theory and Mathematical Statistics have encountered the difficult problem of abstract concept and incomprehension.To solve this problem, the introduction of Matlab, Office PowerPoint and Office Excel in the teaching of Probability Theory and Mathematical Statistics can achieve the following two points: First, it can simulate some classic random phenomena and convert the results into graphs;Second, it can reproduce some important definitions and theorems by example, and show them in graphs or dynamic processes.The practice shows that in the course of "Probability Theory and Mathematical Statistics", the use of Matlab, Office PowerPoint, Office Excel software as the tools of visual teaching method, can indeed provide students with easy to perceive, intuitive understanding, but also well deepen students' understanding of knowledge.At the same time, due to the reduction of learning difficulty and the low difficulty of practical operation, students' learning confidence is constantly enhanced, and their learning interest in Probability Theory and Mathematical Statistics is also becoming stronger.

References

[1] Deng Guangming, He Baozhu, Liu Xiaoping, Jia Zhen.Probability Theory and Mathematical Statistics [M]. Hunan Normal University Press, 2013.

[2] Wu Xinjun, Guo Zhen, Pan Dong.Exploration and research on the teaching reform of Probability Theory and Mathematical Statistics [J]. University Education, 2021(10):125-127.

[3] Yang Li, Jiang Ruixia. Visual interactive teaching mode in the experimental teaching of mathematics application [J]. Journal of transformer, 2020, 57 (4): 98. DOI: 10.19487 / j.carol carroll nki. 1001-8425.2020.04.031.

[4] Chen Shan. Visual Comparative Analysis of Teaching design research at home and abroad [D]. Henan Normal University, 2013.

[5] Eppler, M. J. & Burkard, R.A. Knowledge Visualization. Towards a New Discipline and its Fields of Application, ICA Working Paper #2/2004[R]. University of Lugano, Lugano, 2004.

[6] R. A. Burkard. Learning from Architects: The Difference between Knowledge Visualization and Information Visualization; Proc. Eigth International Conference on Information Visualization (IV04), London, July, (2004b).

[7] Bera P, Burton-Jones A, Wand Y. Guidelines for Designing Visual Ontologies to Support Knowledge Identification [J]. Mis Quarterly, 2011, 35(4):883-908.

[8] Bruzelius, C. Teaching with visualization technologies how does information become knowledge? Material Religion, 2013, 9(2):246-253.

[9] Yuan Guomin, Zhou Ning. Comparative study on Information visualization and Knowledge visualization [J]. Science and Technology Information Development and Economy, 2006, 57 (3): 359-377.