

Optimized Application of Multibeam Bathymetry Technology in Seafloor Surveys

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Abstract: This study focuses on optimizing multibeam bathymetry technology, applying it to underwater terrain measurement, achieving a transition from point-to-line measurement, and minimizing the measurement path. Using the sine theorem and slope cosine theorem, a mathematical model for coverage width in the presence of slopes is established. The study calculates the seawater depth, coverage width, and overlap rate with the previous measurement line at various positions along the measurement line from the center point, with a distance of 800m resulting in a seawater depth of 49.05m and a coverage width of 170.27m. Subsequently, multiple β angles are selected to quantitatively analyze the impact of different angles on the measurement. Combining the established coverage width model, a mathematical model for coverage width at a certain distance from the center of the maritime area is developed for different β angles. The conclusion is drawn that at a β angle of 180 degrees, the farthest point from the center of the maritime area has the minimum coverage width of 63.03m, while at a β angle of 0 degrees, the farthest point has the maximum coverage width of 770.07m.

1. Introduction

Multi-beam sounding is a highly advanced acoustic sounding technique used to measure underwater terrain and depth. It is a side-scan sonar system capable of detecting a wide range of seabed topography and geological features underwater [1]. With the continuous development and improvement of science and technology, single-beam sounding which adopts a single-point continuous measurement method is abandoned, and the traditional single-beam sounding technology is changed to multi-beam sounding technology, which uses the reflection principle of the sound wave to work. The system transmits acoustic pulses underwater by transmitting multiple acoustic beams (usually tens or even hundreds). Each acoustic beam is a full-coverage water depth strip with a certain width. After these acoustic pulses are reflected on underwater terrain and objects, the information in the full-coverage water depth strip can be returned to the receiver [2].

Multi-beam sounding technology has a far-reaching impact on the seabed survey. It can provide high-precision seabed topography and geological data in the exploration of Marine geology, the analysis of seabed mineral resources, and the study and management of Marine ecosystems. In the construction and planning of ocean engineering, multi-beam sounding technology can provide submarine topographic data for cable, port construction, submarine pipeline, and other ocean engineering projects. By analyzing high-resolution and high-precision submarine topographic data, engineers can help determine the best construction plan, thereby reducing engineering risks and costs. Multi-beam sounding can be used to monitor submarine geological disasters such as earthquakes and volcanic eruption[3,4].

Multi-beam sounding technology is widely used in the ocean. This study firstly establishes a mathematical model about the overlap rate between adjacent strips and the coverage width of multi-beam sounding.

2. Establish the coverage width model of seabed multi-beam sounding

Multi-beam sounding uses the principle of acoustic reflection to measure the depth change caused by ground fluctuation, and obtains the depth value of multiple measuring points in the strip covering area, which realizes the crossing from "point-line" measurement to "line-plane" measurement[1,5]. The schematic diagram of multi-beam sounding is shown in Figure 1, to measure the coverage width of multiple independent beams, the vertical emission of the transducer and the opening Angle of the transducer θ are used, by using the open Angle bisection, the coverage width W of the center point is set as x_1 and x_2 respectively, assuming that the slope is α , the distance between the two adjacent measurement lines is d , the seawater depth at the center point is D , the distance between each measurement point and the center point is s_n , and the Angle between the beam and the slope is respectively k_1 and k_2 , then the equation of the coverage width W of the center point is obtained:

$$W = (x_1 + x_2) \cdot \cos \alpha \quad (1)$$

According to the sine theorem, the relation function of the coverage width of the center point can be obtained as follows:

$$W = \left(D \cdot \frac{\sin \frac{\theta}{2}}{\sin k_2} + \frac{D \cdot \sin \frac{\theta}{2}}{\sin k_1} \right) \cdot \cos \alpha \quad (2)$$

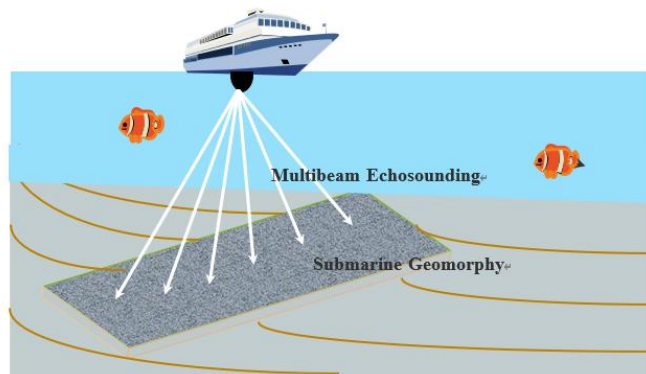


Figure 1: Schematic diagram of multi-beam sounding

With the progress of the survey line of the survey ship, the sea depth of the survey point is constantly changing. According to the relationship in Figure 2, the relationship between the sea depth D_1 of the next survey line point and the sea depth of the central point can be obtained:

$$\frac{D_1}{D} = \frac{\frac{D_1}{\tan \alpha}}{\frac{D_1}{\tan \alpha} + s_1} \quad (3)$$

According to the recurrence, the relationship between the depth of seawater at each measurement line point and the depth of seawater at the central point is obtained as follows:

$$D_n = D - s_n \cdot \tan \alpha (n = 1, 2, 3 \dots) \quad (4)$$

Then the relation function of the coverage width of each measurement line point is:

$$w_n = \cos \alpha \cdot \left[(D - s_n \tan \alpha) \cdot \frac{\sin \frac{\theta}{2}}{\sin k_2} + (D - s_n \tan \alpha) \frac{\sin \frac{\theta}{2}}{\sin k_1} \right] (n = 1, 2, 3 \dots) \quad (5)$$

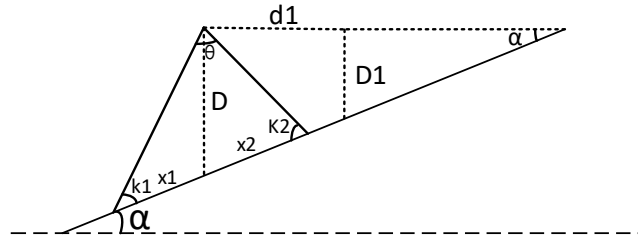


Figure 2: Depth plane analysis chart

Then this paper defines the formula according to the overlap rate between adjacent strips when the survey lines are parallel to each other and the seabed terrain is flat:

$$\eta = 1 - \frac{d}{w} \quad (6)$$

Calculate the overlap rate when there is a slope on the seabed, subtract d from the sum of x_2 and x_3 to obtain the length of the overlap part z , then the overlap rate formula between adjacent strips of each survey line point is as follows:

$$\eta_n = \frac{z - d}{w_{n-1}} \quad (7)$$

According to the diagram in Figure 3, the expression of the overlapping part z_1 between the central sea area survey line point and the next survey line point is calculated using the sine formula:

$$z_1 = \cos \alpha \cdot \sin \frac{\theta}{2} \left(\frac{D}{\sin k_2} + \frac{D_1}{\sin k_1} \right) \quad (8)$$

According to recursion, it can be inferred that the relationship between the overlapping part z of each measuring point and the previous measuring point is as follows:

$$z_n = \cos \alpha \cdot \sin \frac{\theta}{2} \left(\frac{D_{n-1}}{\sin k_2} + \frac{D_n}{\sin k_1} \right) (n = 1, 2, 3 \dots) \quad (9)$$

Therefore, it is calculated that the overlap rate between adjacent strips when the seabed has a slope is:

$$\eta_n = \frac{\cos \alpha \cdot \sin \frac{\theta}{2} \left(\frac{D_{n-1}}{\sin k_2} + \frac{D_n}{\sin k_1} \right) - d}{W_{n-1}} (n = 1, 2, 3 \dots) \quad (10)$$

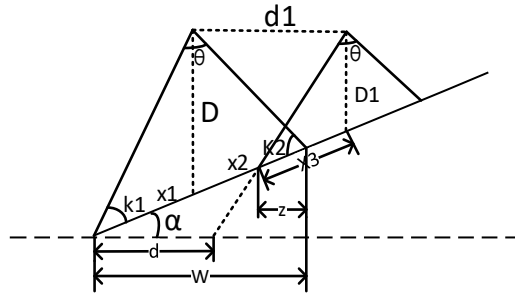


Figure 3: Plane analysis chart of overlap rate

When the opening Angle of most wave transducers is 120 degrees, the slope is 1.5 degrees, and the seawater depth at the center point of the sea area is 70m, the calculation results using MATLAB combined with the above model are shown in Table 1:

Table 1: Calculation results

Measure the distance of the line from the center point /m	-800	-600	-400	-200	0	200	400	600	800
Sea depth /m	90.95	85.71	80.47	75.25	70.00	64.76	59.53	54.29	49.05
Covering width /m	315.71	297.53	279.35	261.17	242.99	224.81	206.63	188.45	170.27
The overlap rate with the previous line /%	—	33.64	29.59	25.00	19.78	13.78	6.81	-1.39	-11.17

3. Analysis of the influence of Angle on seabed topography under multi-beam sounding technology

To study the influence of Angle on seabed topographic survey, the following mathematical model is established to calculate the coverage width of multi-beam sounding at each position.

First, this article defines the parameters: θ : The number of open angles of the multi-beam transducer, which here is 120 degrees; α : Seafloor slope, 1.5 degrees; D : The depth of the water at the center of the ocean, here is 120 meters; β : The Angle between the direction of the survey line and the normal direction of the submarine slope projected on the horizontal plane.

By analyzing the problem, we can randomly select the Angle β between the direction of a survey line and the normal direction of the submarine slope projected on the horizontal plane, and then make a plane perpendicular to the horizontal plane through the direction of the survey line, and the plane is a section, as shown in Figure 4.

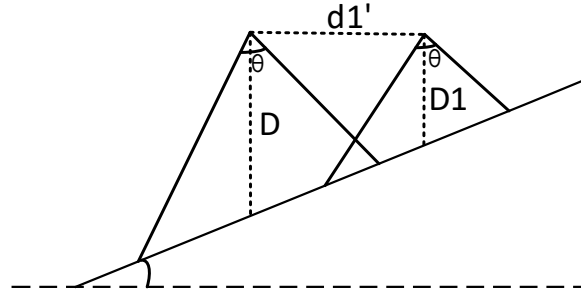


Figure 4: Profile map

From this, it can be analyzed that the slope of the section will change under different Angles β . As shown in Figure 5, through a simple analysis, we can obtain:

$$\cos \beta = \frac{X_1}{X_2} \quad (11)$$

$$\tan \alpha_j = \frac{h}{X_j} \quad (j=1, 2, 3 \dots) \quad (12)$$

Where α_j is the slope under different included angles β ,

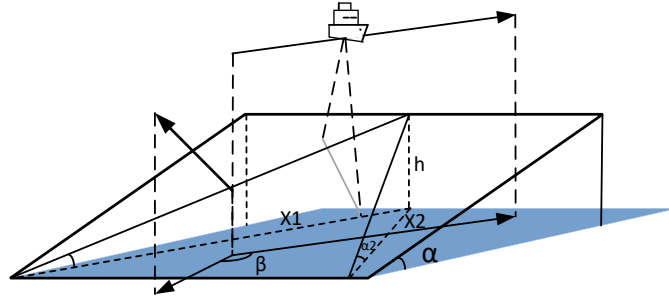


Figure 5: Stereogram of the Angle

Therefore, the tangent value of the changed slope can be found:

$$\tan \alpha_j = \tan \alpha \cos \beta \quad (j=1, 2, 3 \dots) \quad (13)$$

By combining the tangent value and the condition given by the problem stem, the seawater depth D_{nj} measured at different values of β can be obtained:

$$D_{nj} = D' - s' \tan \alpha_j \quad (n=1, 2, 3 \dots, j=1, 2, 3 \dots) \quad (14)$$

Where, D_{nj} is the seawater depth measured each time with different values of β , D' is the depth of the central sea area, and s' is the distance of the ship from the central point when β is different values.

A mathematical model covering the width w_k can be established:

$$\phi_1' = \frac{\pi - \theta}{2} \quad (15)$$

$$\phi_2' = \frac{\pi - \theta}{2} \quad (16)$$

Where ϕ'_1 , ϕ'_2 is the Angle between the strip and the slope when different values are β .

$$w_k = \left(\frac{1}{\sin \phi'_1} + \frac{1}{\sin \phi'_2} \right) \times D_{ij} \times \sin \frac{\theta}{2} \times \cos \alpha_j (k, n, j = 1, 2, 3 \dots) \quad (17)$$

It can be seen that the opening Angle θ of the multi-beam transducer is 120 degrees, the slope α is 1.5 degrees, and the depth of the seawater at the center of the sea area is 120m. The mathematical model established above uses Matlab software to calculate the distance of the measuring ship from the center point of the sea area with different directions of the measuring line, and the corresponding coverage width of the multi-beam sonar. The results are shown in Table 2:

Table 2: Calculation results

The distance from the center of the ocean / Sea mile	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	
Measurement line direction Angle / °	0	416.55	467.05	517.55	568.06	618.56	669.06	719.57	770.07
	45	416.12	451.79	487.47	523.14	558.82	594.49	630.16	665.83
	90	415.69	415.69	415.69	415.69	415.69	415.69	415.69	415.69
	135	416.12	380.45	344.77	309.10	273.42	237.75	202.07	166.40
	180	416.55	366.05	315.54	265.04	214.54	164.04	113.53	63.03
	225	416.12	380.45	344.77	309.10	273.42	237.75	202.08	166.40
	270	415.69	415.69	415.69	415.69	415.69	415.69	415.69	415.69
	315	416.12	451.80	487.47	523.14	558.82	594.49	630.16	665.84

In order to more intuitively show the relationship between the depth of the sea water, the direction Angle of the surveying line and the distance between the surveying ship and the central point of the sea area, the distance between the surveying ship and the direction Angle of the surveying line are taken as the X axis and the Y axis respectively, and the three-dimensional curve is drawn by MATLAB, as shown in Figure 6.

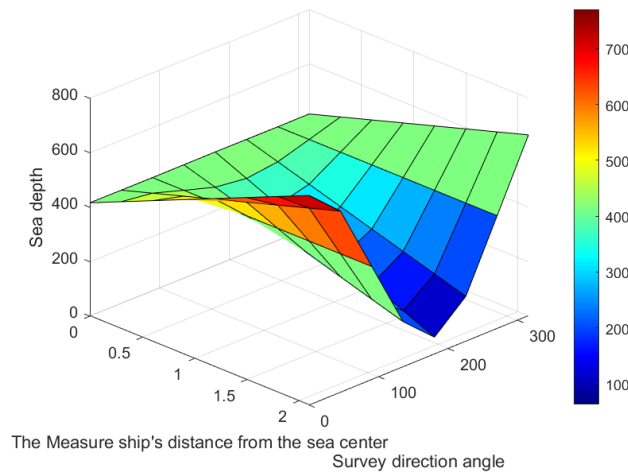


Figure 6: Visual 3D diagram

From the Table 2 and Figure 6, it can be seen that when the angle between the survey line direction and the projection of the slope normal in the horizontal plane is 180 degrees, the coverage width decreases fastest as the distance between the survey ship and the center of the sea area increases. At the farthest distance from the center of the sea area, the minimum coverage width is 63.03m. When the angle between the survey line direction and the projection of the slope normal in the horizontal

plane is 0 degrees, the coverage width increases fastest as the distance between the survey ship and the center of the sea area increases. At the farthest distance from the center of the sea area, the maximum coverage width is 770.07m.

4. Conclusions

This paper studies the application of multi-beam sounding technology in seafloor sounding. First of all, the sine theorem and slope cosine value are used to Establish the coverage width model of seafloor multi-beam sounding, that is the mathematical model of the coverage width of a single surveying line. When β Angle is 180 degrees, the measured distance between the surveying line and the center point is 800m. The calculated seawater depth is 49.05m, and the coverage width is 170.27m. Then, based on the above research, considering the influence of different β angles on the seabed topographic survey and the mechanism relationship between related variables, a vertical profile with the horizontal plane is drawn by selecting multiple β angles. The mathematical model of the coverage width at a certain distance from the center point of the sea area is established. When the β Angle is 180 degrees, the minimum coverage width at the farthest point is 63.03m. When the β Angle is 0 degrees, the maximum coverage width is 770.07m.

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