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The Stochastic Model of Stock Prices

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Abstract: The purpose of this research is to investigate the utilization of the Brownian model in the analysis of financial data. In the field of finance, the analysis of financial data is crucial for making investment decisions and managing risks. The classical stochastic process model, known as the Brownian model, has been extensively employed in this domain. This paper provides an overview of the Brownian model, discusses its advantages and limitations in analyzing financial data, and presents empirical research findings. These findings serve as valuable references for future studies. The primary focus of the Brownian model is to study the random fluctuations of a variable while adhering to a specific statistical pattern. In this study, we derive the Brownian equation and explore the meanings of the damping term and the random variable term within the equation. When the random variable exhibits complete randomness at different time points, it is referred to as white noise. However, when considering certain time correlation lengths, the solution to the Brownian equation becomes more intricate. We meticulously examine how different noise terms and damping terms influence the solutions of the equation. Furthermore, we establish connections between these variables and various financial variables, particularly stock prices, to gain a practical understanding of the Brownian equation. The evolution of stock prices under different parameters is graphically illustrated and analyzed in detail.

1. Introduction

Financial data analysis entails the organization, interpretation, and utilization of a vast amount of economic and market data within the finance field. In today's intricate and globally interconnected financial market environment, financial data analysis plays a crucial role in investment decision-making, risk management, and market forecasting. Precise interpretation and application of financial data enable investors to seize opportunities, mitigate risks, and make informed decisions for businesses and governments, thereby fostering stability and growth in financial markets. Firstly, financial data analysis significantly impacts investment decision-making. Through the analysis of market trends, company profits, and financial conditions, investors can better evaluate market dynamics and the investment potential of individual stocks. Data-driven investment decisions aid investors in formulating sound strategies for buying and selling securities, reducing investment risks, and enhancing returns. Secondly, financial data analysis is crucial for risk management. Financial markets are filled with various uncertainties and risks. Through the analysis of historical data and market trends, potential risks can be identified, and effective risk control measures can be implemented. Financial institutions and investors can optimize portfolio allocation using data analysis methods to achieve effective risk diversification and control. Additionally, financial data analysis also has a significant influence on market forecasting and policy decision-making. Analysis based on historical data can provide economists and policymakers with profound insights into the financial market, enabling them to formulate appropriate policies and measures to promote economic growth and financial stability. However, financial data analysis also faces challenges and difficulties, such as data quality, large data volume, and complexity. Therefore, this paper will focus on the application of the Brownian model in financial data analysis. As a classical stochastic process model, the Brownian model has been widely used in the field of finance and provides valuable references for financial data analysis. Through the study and application of the Brownian model [1], we can further improve the accuracy and effectiveness of financial data analysis and provide decision-makers with more reliable information and predictive capabilities.

There are various mathematical models used to study financial and stock-related issues, with commonly used ones including stochastic models and regression models.

The mean reversion model assumes that stock prices tend to revert towards their mean over a certain period of time. In other words, when stock prices deviate from their mean, there is a tendency for them to return back to the mean. This trend can be driven by buying or selling pressure in the market or investor reactions to the stock.

In the financial and stock domain, the application of mean reversion models is mainly reflected in the following two aspects: Mean reversion models can serve as decision support tools for formulating trading strategies. When stock prices deviate from their mean, investors can take corresponding buying or selling actions in the hope that the prices will revert back to the mean level. This strategy is usually based on statistical probabilities, where the probability of mean reversion increases as the price moves further away from the mean. Mean reversion models can be used for risk management purposes to help investors control portfolio volatility. By modeling the mean and variance of stock prices, investors can evaluate the level of risk based on current deviations and statistical analysis of historical data. This helps investors make appropriate adjustments to protect the value of their portfolio during periods of high market volatility. It is important to note that mean reversion models are not applicable to all stocks and market environments. Markets are dynamic, and stock prices may evolve due to various factors. Therefore, in practical applications, investors need to consider other factors such as market trends, company fundamentals, etc., in order to improve the accuracy and reliability of the model.

Another model is the stochastic volatility model. The stochastic volatility model is a mathematical model used to describe the volatility of financial stock prices. Its main characteristic is considering the time-varying nature of volatility and using stochastic processes to model price fluctuations [2]. In the financial and stock domain, the stochastic volatility model is widely used in risk management, option pricing, financial derivatives, and other areas. Firstly, the stochastic volatility model aids in risk management. By modeling the volatility of stock prices, investors can have a better understanding and assessment of market risk. The stochastic volatility model provides a framework for evaluating the potential range of stock price fluctuations under different market conditions. This enables investors to develop risk management strategies, such as determining stop-loss points or setting up protective derivative contracts. Secondly, the stochastic volatility model plays an important role in option pricing. The price of an option contract depends on the volatility of the underlying asset. The stochastic volatility model captures randomness and changes in volatility, providing a more accurate way to estimate option prices. Common stochastic volatility models, such as diffusion models and jump models, are used to calculate the theoretical value of options under different market conditions, providing important reference points for option trading. The stochastic volatility model also plays a crucial role in the pricing and risk management of financial derivatives. The value fluctuations and risk exposures of financial derivatives are closely related to the price volatility of the underlying assets. The stochastic volatility model provides a method for pricing and risk measurement of these derivatives [3]. By simulating price paths and calculating the value of derivatives, investors can have a better understanding and manage the risks associated with financial derivatives.

At the mathematical level, it is worthwhile to delve deeper into the derivation and extension of the Brownian model. A thorough understanding of the Brownian model, especially in terms of variable settings, is crucial for understanding the application scope of Brownian motion. Therefore, in this work, we will start from Newton's second law in mechanics and derive the form of the Brownian equation in detail. Due to the possible time dependence of the random variables in the Brownian equation, we will introduce stochastic terms and consider the influence of different time correlation lengths. Subsequently, we will study the change of individual stock prices over time using the Brownian equation (also known as the Langevin equation), with a particular emphasis on examining the dependence of stock prices on the parameters of the Brownian equation. When dealing with a large number of stock prices, we will employ a multitude of equations to evolve the changes in each stock price and calculate variables such as the mean and variance [4,5].

The paper is structured as follows: In the second section, we will present the derivation of the Brownian equation and introduce the stochastic terms. In the third section, we will solve the equations to obtain the time evolution graphs of individual stock prices and introduce a large number of stocks to calculate average prices. Finally, we will summarize the Brownian model.

2. Stochastic Model

In the stock market, the price of a stock is influenced by external factors. Here, let's assume that the price of a stock is denoted as p, and its rate of change over time is represented by F(p,t) . Therefore, the equation describing the change in stock price can be expressed as:

$$\frac{dp}{dp} = F(p,t)$$

In general, the rate of change of stock prices over time depends on both time and the current price of the stock. This equation is similar to the equations in Newtonian mechanics. The rate of change of stock prices reflects the influence of various factors in the market. If the entire market is under the influence of a stable macro policy, the stock prices will be affected by this major factor for a long period of time. However, at the same time, there are various factors and phenomena in society at each moment that can affect stock prices, and this effect is characterized by uncertainty and randomness. Such random phenomena, such as sudden news events, can only be introduced in the form of random variables. Therefore, we can parameterize the macro and micro factors as follows:

$$\mathrm{F(p,t) = -\Gamma p(t) + \theta(t)}$$

Note that theta(t) is a random variable, which can vary at any given moment, reflecting the impact of news events in society on the price of the stock. The variable Gamma represents the influence of overall macro policies. If there is no macro policy in place, and the stock price is entirely determined by random variables.

To capture an interesting phenomenon, namely the behavior of stock prices rising or falling depending on their original price, we introduce a dependence on the initial stock price. For example, if the stock price is already high, its growth rate will be limited. Conversely, if the price is initially low, the probability of further decrease is relatively smaller. This effect can be modeled by multiplying by p, resulting in a damping term.

An especially important property is the temporal correlation of random variables. If the random variable is completely independent and uncorrelated across different time points, it exhibits the property of white noise.

$$< \theta(t) > = 0$$

$$< \theta(t)\theta(t') >= A\delta(t - t')$$

It is important to note that the average value of the random variable across different time points is set to zero. The temporal correlation of the random variable is modeled by a correlation strength parameter, denoted as A. When numerically solving the equation and simulating the random variable at different time points, according to the Ito calculus, the random variables at time t and t' within a time interval Δt follow a Gaussian distribution with a width approximately equal to sqrt(A/dt).

We parameterize the macro policy as a "drag term." If this parameter varies with time, the fluctuations in stock prices also reflect overall changes. For example, if we consider a time period 10 < t < 12 where the policy stimulates stock prices, the Gamma term is no longer negative but positive. This can be expressed in the equation as follows:

$$\Gamma(t) = \Gamma_0 \quad (10 < t < 12)$$

At this point, we can observe a clear jump in the stock price. This is a feedback response to the Gamma variable. In general, we use the Langevin equation to study the price of individual stocks. Suppose we are studying N variables, and these N variables follow a Gaussian distribution. The time evolution of each variable can be described by the equations mentioned above. Then, at each time point, the average value of these N variables can be calculated.

$$< p(t_i) > = \sum_{j}^{N} p_j\,(t_i)/\,N$$

Here, i represents different time points, while j represents the jth stock (variable). We will now use the prices of 2000 stocks to study the average value of the variables.

3. Results and Analysis

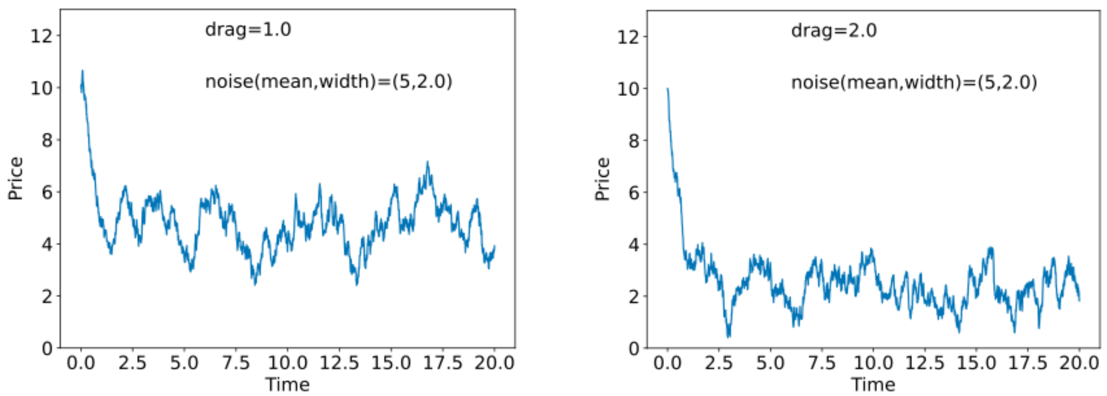

Figure 1: The time evolution of one variable. The initial value of the variable is set to be 10. The drag coefficient is $\Gamma=1$. The noise term is the white noise, satisfying the Gaussian distribution with the mean value 5 and the width 2.0.

Next, we will study the evolution of a variable over time, and the graph of a single variable is shown below. At the initial time, the variable has a value of 10, and then it rapidly decreases over time due to the drag term. The larger the drag term, or Gamma, the faster the variable decreases. After a certain time, the fluctuation in the curve becomes significant, which is caused by the contribution of the second term. For computational convenience, we take the average value of fluctuations as 5, and they follow a Gaussian distribution with a width of 2.0. Fig.1 illustrates that the evolution of the two variables differs when the drag term varies. This reflects the impact of macro policies on the variables.

It is important to emphasize that the variables are independent at each time point, resulting in fluctuations in the overall variable p. However, the overall average value follows a certain pattern. In Fig. 2, if we take the average value of the noise as 2, when the overall variable p evolves to a certain time, it will oscillate around 2. If the influence of noise is significant, it will be reflected in large fluctuations, with random numbers oscillating up and down intensely.

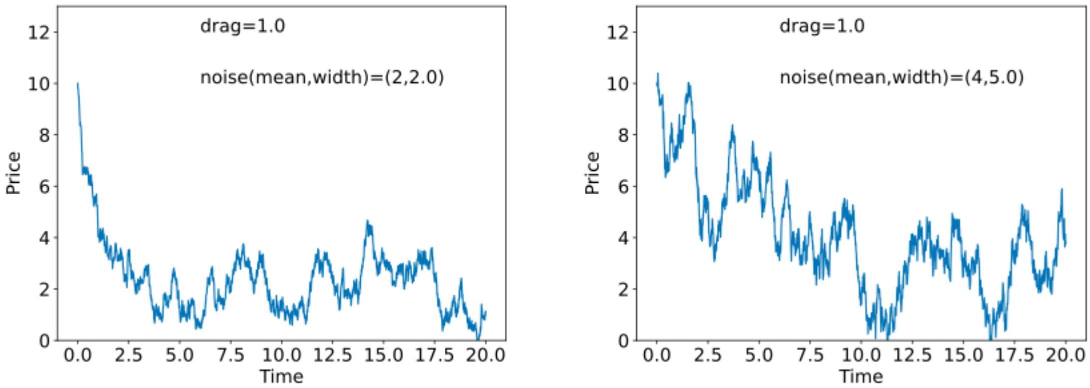

Figure 2: The time evolution of one stock's price. The initial value of the price is set to be 10. The drag coefficient is Gamma=1. The noise term is the white noise, satisfying the Gaussian distribution with the mean value 2 and 4. The width are 2 and 5.

In some case, there is positive factor acting on the random variable p. For example, in the time period 10<t<12, the drag term becomes positive, then p will be enhanced in this time period. Please see Fig. 3. As mentioned before, the drag term reflects the macroscopic factor. In realistic case, the drag term depends on time.

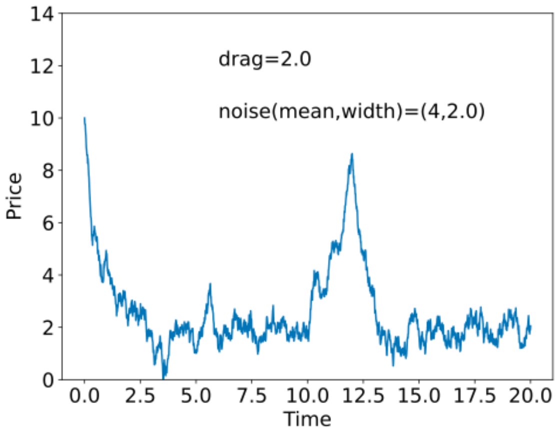

Figure 3: The time evolution of random variable p. In the time period, 10<t<12, the drag term becomes positive, where p increases with time. After this time period, the drag term reduces the variable.

The above figure is for one random variable. When we consider a large set of random variables, such as N=2000 variables, the mean value of these random variables change with time, plotted in Fig.4. In the time period 10<4<12, there is a positive factor increasing the mean value of p. In the short time period, t<2, variable is mainly reduced by the drag term as explained before. In the later stage, the mean value of p is equal to the mean value of the random term in the Langevin equation. This simulation can help to explain the dynamical evolution of random variables.

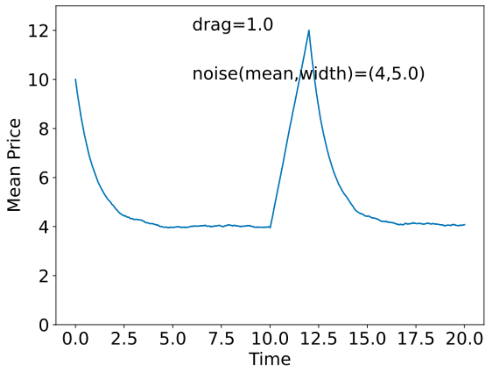

Figure 4: The time evolution of the mean value of a large number of random variables. The drag term is taken as 1, and the mean and width of the noise are taken as 4 and 5 respectively.

4. Summary

In conclusion, this study successfully employs the Langevin equation to simulate the evolution of a random variable over time. By numerically solving the equation and manipulating various parameters, we obtain valuable insights into the behavior of the variable under different conditions. The random variable is affected by drag and random terms. When employing different values of the drag term and the random term, the fluctuations of the variable is evidently different. This allows us to study random phenomena via the Langevin equation with the proper value. The mean value of the random variables is also calculated. After considering some positive contribution in the drag term, the mean value can be evidently enhanced. This mathematical model can be applied in aspects. This research contributes to the field of stochastic modeling and provides a framework for studying random variables in diverse applications such as finance, physics, and biology. Future studies can explore more complex scenarios and investigate the effects of additional factors on the evolution of random variables.

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