

Acoustic field separation with 2-layer microphone array

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Abstract: Interference noise seriously affects the recognition accuracy of the target acoustic field. To reconstruct acoustic field of the target sources in non-free acoustic field, a method of acoustic field separation and reconstruction with 2-layer microphone array is presented. With this method, spherical harmonics in different orders are superposed to describe acoustic pressure distribution for different sound sources, respectively. The coefficient vectors are obtained by matching the measured pressure with the mathematical model. As the coefficient vectors are not changed with the position of measurement planes, once these coefficients are specified, the acoustic pressure of the target sources are determined. The methodology is examined numerically in the acoustic field with two transversely oscillating rigid sphere. The results show that, when two sound sources on the both sides of the measurement arrays, the error of acoustic field separation is 7.36% for the frequency, this method can improve the accuracy of acoustic field recognition.

1. Introduction

Among the current acoustic field separation methods, the main theoretical basis is various near-field acoustic holography method [1-3]. Acoustic Wave separation method based on the Equivalent Source Method [4]. Acoustic wave separation method based on the Statistical Optimal Method [5, 6], Acoustic wave separation method based on the Boundary Element Method [7, 8]. Among the various acoustic wave separation methods, their characteristics are mainly determined by the principle of near-field acoustic holography on which they are based: for example, when selecting the array for the acoustic wave separation method based on two-dimensional Fourier transform, it needs to be based on the corresponding near-field acoustic holography. The requirements of the method select a regular sphere, cylinder or plane as the measurement array; In the process of implementing the acoustic field separation method based on the equivalent source method, it is necessary to arrange a virtual simple source near or inside the sound source according to the characteristics of the corresponding near-field acoustic holography method.

Wu [9, 10] proposed a near-field acoustic holography method based on the special solution of the Helmholtz equation in the spherical coordinate system, which is the Helmholtz Equation Least Squares method (HELs). This method describes the acoustic pressure distribution in the free field by the expression representing the outgoing wave component in the general solution of the Helmholtz equation, and optimizes the number of superposition items of the spherical harmonic

function in the outgoing wave component according to the least square method. This method is due to its theoretical simplicity and computational efficiency.

When there are sound sources on both sides of the array, this paper proposes an acoustic field separation method based on the theory of spherical wave superposition near-field acoustic holography. This method uses a 2-layer microphone array to collect the acoustic pressure information of the acoustic field, and uses the superposition of different orders of spherical harmonics on both sides of the array to describe the acoustic contribution of each sound source on the measurement array.

2. Theory of acoustic field separation

2.1. Near-field acoustic holography based on the HELS

The propagation of the acoustic wave in free field is similar to small disturbances in ideal fluid, it can be described by the wave equation. Transforming the wave equation from time domain to frequency domain by Fourier transform, we can get the expression of the Helmholtz equation as:

$$\nabla^2 p(x, y, z; \omega) + k^2 p(x, y, z; \omega) = 0 \quad (1)$$

where ∇^2 is Laplace operator, k is wave number.

In spherical coordinate system, it is assumed that the sound source is located at the origin, the Helmholtz equation can be rewritten as

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial p}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial p}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 p}{\partial \phi^2} - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0 \quad (2)$$

Using variable separation method, we obtain the solution at any arbitrary point in space for traveling wave, When there is only a single sound source in the analyzed space and it is a free field, that is, there is only outgoing wave in the acoustic field, then the acoustic pressure at each field point can be expressed as:

$$p(r, \phi, \theta; \omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^n \left[c_{nm} h_n^{(1)}(kr) Y_n^m(\theta, \phi) \right] \quad (3)$$

where $h_n^{(1)}(kr) Y_n^m(\theta, \phi)$ represents an outgoing wave. $h_n^{(1)}(kr)$ is Hankel function of the first kind. $Y_n^m(\theta, \phi)$ is spherical harmonic.

A holographic array with N measurement points is placed in the near field of the sound source, and the acoustic pressure at each measuring point in the acoustic field can be expressed as a matrix according to Eq.(3), as shown in Eq.(4):

$$\begin{bmatrix} p_{\text{meas},1} \\ p_{\text{meas},2} \\ \vdots \\ p_{\text{meas},N} \end{bmatrix} \cong \begin{bmatrix} \psi_{\text{out},11}^{(1)} & \cdots & \psi_{\text{out},1j}^{(1)} & \cdots & \psi_{\text{out},1M}^{(1)} \\ \psi_{\text{out},21}^{(1)} & \cdots & \psi_{\text{out},2j}^{(1)} & \cdots & \psi_{\text{out},2M}^{(1)} \\ \vdots & & \vdots & & \vdots \\ \psi_{\text{out},i1}^{(1)} & \cdots & \psi_{\text{out},ij}^{(1)} & \cdots & \psi_{\text{out},iM}^{(1)} \\ \vdots & & \vdots & & \vdots \\ \psi_{\text{out},N1}^{(1)} & \cdots & \psi_{\text{out},Nj}^{(1)} & \cdots & \psi_{\text{out},NM}^{(1)} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_M \end{bmatrix} \quad (4)$$

where $\psi_{\text{out},ij}^{(1)}(r, \phi, \theta) = h_n^{(1)}(kr) Y_n^m(\theta, \phi)$, M is the maximum number of expanded items. $[c_1 \ c_2 \ \cdots \ c_M]^T$ is the coefficient vector to be solve. This coefficient vector reflects the

distribution characteristics of the acoustic field and will not change with the change of the measurement array.

Eq.(4) can be abbreviated as :

$$\mathbf{P} = \Psi \mathbf{C} \quad (5)$$

Multiply the pseudo-inverse of the transfer matrix Ψ_M^\dagger on both sides of Eq.(5), Get the coefficient vector as:

$$\mathbf{C}_M = \Psi_M^\dagger \mathbf{P} \quad (6)$$

The selection of the number of superposition items in formula (3) will affect the accuracy of acoustic field reconstruction. In order to obtain the best reconstruction accuracy. According to the literature^[11, 12], the optimal number of superposition items is determined as M_{opti} . Then the optimal coefficient vector is:

$$\mathbf{C}_{opti} = \Psi_{opti}^\dagger \mathbf{P} \quad (7)$$

If we need to observe the acoustic field distribution at any position in the space. Then the acoustic field \mathbf{P}_{rec} distribution can be reconstructed according to the optimal coefficient vector as:

$$\mathbf{P}_{rec} = \Psi_{opti}' \mathbf{C}_{opti} \quad (8)$$

where Ψ_{opti}' is the expansion matrix corresponding to each measurement point on the reconstruction plane.

2.2. Acoustic field Separation Based on the HELS

As shown in Figure 1, there are two sound sources in space, namely, sound source S_1 and sound source S_2 . The two sound sources vibrate with small amplitude at the same frequency respectively. The vibration of the two remains stable, and the velocity field is a steady state field.

In order to obtain the acoustic pressure field distribution generated by the sound source S_1 alone, two parallel microphone arrays are arranged between the two sound sources, which are denoted as the array H_1 and the array H_2 , and are used to collect the acoustic pressure generated by the two sound sources.

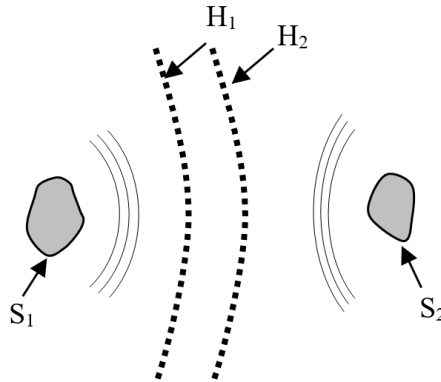


Figure 1: Schematic diagram of acoustic field layout

In the acoustic field shown in Fig. 1, since the vibration of the sound source is a small-amplitude linear vibration, the acoustic pressure responses generated by multiple sound sources have linear

additivity. Therefore, the acoustic pressure distribution on the array H₁ is the linear superposition of the independent acoustic field responses of the sound sources S₁ and S₂ on both sides of the array, namely:

$$\mathbf{P}_1(\mathbf{r}_{H_1}) = \mathbf{P}_1^1(\mathbf{r}_{H_1}) + \mathbf{P}_1^2(\mathbf{r}_{H_1}) \quad (9)$$

where $\mathbf{P}_1^1(\mathbf{r}_{H_1})$ is the acoustic pressure of the sound source S₁ alone at each measuring point on the H₁, $\mathbf{P}_1^2(\mathbf{r}_{H_1})$ is the acoustic pressure response of the sound source S₂ alone at each measuring point on the array H₂. \mathbf{r}_{H_1} is the coordinate position of each measuring point on the array H₁.

According to the expression method of acoustic field distribution described in Eq. (5), Eq. (9) can be written as:

$$\mathbf{P}_1(\mathbf{r}_{H_1}) = \Psi_1^1(\mathbf{r}_{H_1})\mathbf{C}_1^1 + \Psi_1^2(\mathbf{r}_{H_1})\mathbf{C}_1^2 \quad (10)$$

where $\Psi_1^1(\mathbf{r}_{H_1})$ and $\Psi_1^2(\mathbf{r}_{H_1})$ is the superposition matrix of different orders of spherical waves of each measurement point on the array H₁ in the local coordinate system where each sound source is located. \mathbf{C}_1^1 and \mathbf{C}_1^2 are the coefficient vectors corresponding to the array H₁ and the sound sources S₁ and S₂, respectively.

The acoustic pressure distribution on the array H₂ can also be written as a linear superposition of the independent responses of the sound sources S₁ and S₂ on both sides of the array, namely:

$$\mathbf{P}_2(\mathbf{r}_{H_2}) = \mathbf{P}_2^1(\mathbf{r}_{H_2}) + \mathbf{P}_2^2(\mathbf{r}_{H_2}) \quad (11)$$

where $\mathbf{P}_2^1(\mathbf{r}_{H_2})$ is the acoustic pressure of the sound source S₁ alone at each measuring point on the array H₂, $\mathbf{P}_2^2(\mathbf{r}_{H_2})$ is the acoustic pressure of the sound source S₂ alone at each measuring point on the array H₂, \mathbf{r}_{H_2} is the coordinate position of each measurement point on the array H₂. According to the expression method of sound field distribution described in Eq.(5), Eq.(11) can be written as:

$$\mathbf{P}_2(\mathbf{r}_{H_2}) = \Psi_2^1(\mathbf{r}_{H_2})\mathbf{C}_2^1 + \Psi_2^2(\mathbf{r}_{H_2})\mathbf{C}_2^2 \quad (12)$$

where $\Psi_2^1(\mathbf{r}_{H_2})$ and $\Psi_2^2(\mathbf{r}_{H_2})$ is the superposition matrix of spherical waves of different orders in the local coordinate system of each sound source on the array H₂. \mathbf{C}_2^1 and \mathbf{C}_2^2 are the coefficient vectors corresponding to the array H₂ and the sound sources S₁ and S₂, respectively.

In the process of describing the acoustic field, since the coefficient vector of the superposition matrix has nothing to do with the position of the array, it can be known that:

$$\mathbf{C}_1^1 = \mathbf{C}_2^1 \quad (13)$$

$$\mathbf{C}_1^2 = \mathbf{C}_2^2 \quad (14)$$

From Eq. (10) and Eq. (12), the acoustic pressure distribution of sound source S₁ alone on arrays H₁ and H₂ can be obtained as:

$$\mathbf{P}_1^1 = (\mathbf{E} - \mathbf{T})^+ (\mathbf{P}_1 - \Psi_1^2(\mathbf{r}_{H_1})\Psi_2^2(\mathbf{r}_{H_2})^+ \mathbf{P}_2) \quad (15)$$

$$\mathbf{P}_2^1 = \Psi_2^1(\mathbf{r}_{H_2})\Psi_1^1(\mathbf{r}_{H_1})^+ \mathbf{P}_{S_1} \quad (16)$$

where $\mathbf{T} = \Psi_1^2(\mathbf{r}_{H_1})\Psi_2^2(\mathbf{r}_{H_2})^+ \Psi_2^1(\mathbf{r}_{H_2})\Psi_1^1(\mathbf{r}_{H_1})^+$.

3. Numerical Experiments

There are sound sources on both sides of the array, the left sound source is the Z-direction vibration ball S_1 , and the right sound source is the Z-direction vibration ball S_2 . The vibrating ball S_1 on the right side of the array is the target sound source in the acoustic field, and the vibrating ball S_2 on the right side of the array is the interference sound source.

The parameters of the two sound sources are: the radius of the vibration ball S_1 is 0.055m, and the vibration speed is 0.04m/s; the radius of the vibration ball S_2 is 0.07m, and the vibration speed is 0.04m/s. The position of the sound source S_1 is at (0,0,0), The position of the sound source S_2 is selected at (0, -0.045m, 0.38m). The array planes H_1 and H_2 used to collect the acoustic pressure information of the acoustic field are regular square planes, the size of the two array planes and the number of measurement points are the same, and they are parallel to each other. The dimensions of the two arrays are both 0.2m×0.2m, and 8×8 measurement points are evenly distributed on each. The vertical distance from the array H_1 to the coordinate origin is $d_1=0.10$ m, and the vertical distance from the array H_2 to the coordinate origin is $d_2=0.12$ m. The density of the air is 1.29kg/m³, and the speed of acoustic waves in the air is taken as 340m/s. The vibration frequency of the sound source is randomly selected as 2.35kHz.

In order to quantitatively express the error between the separated acoustic pressure distribution and the directly measured acoustic pressure distribution, the formula for defining the separation error is:

$$\Delta = \frac{\|\mathbf{P}_{meas} - \mathbf{P}_{sepa}\|_2}{\|\mathbf{P}_{meas}\|_2} \times 100(\%) \quad (17)$$

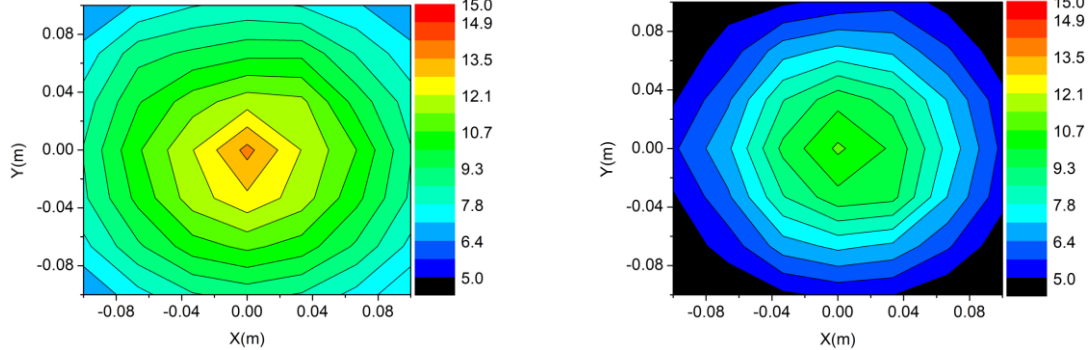
where \mathbf{P}_{meas} is the acoustic pressure vector measured directly on the reconstruction plane. \mathbf{P}_{sepa} is the acoustic pressure vector on the array obtained by the separation method.

In the proposed acoustic wave separation process, the acoustic pressure collected by the array H_1 and H_2 is taken as the input of the system, and then the acoustic pressure distribution of the sound source S_1 alone on the array plane H_1 is separated and reconstructed. Since the number of superposition terms of different spherical harmonics affects the separation accuracy of the acoustic field during the separation process.

In order to make the separation result calculated by simulation closer to the actual acoustic field distribution, 23dB random noise is added to the obtained acoustic pressure. The acoustic pressure data are collected at the measurement planes H_1 and H_2 at the same time, and this data is used as the input of the acoustic wave separation system,

After separation by the method described and the combination of the optimal number of superposition items selected. It can be seen from Fig.2(a) that since the position of the disturbing sound source on the other side of the measurement plane is biased towards the negative half axis of the Y axis. The centers of the acoustic field of the two vibrating balls are also shifted accordingly, and the acoustic pressure amplitude in the central area of the acoustic field is above 12Pa. This is a phenomenon caused by the linear superposition of acoustic pressure and the exponential decay of acoustic pressure with the distance. The relative distance between the two sound sources on the Y negative semi-axis on the array is smaller, and the attenuation is also smaller, so the overall amplitude of the superimposed acoustic field is too large. In order to obtain the acoustic pressure distribution of the target vibrating ball alone on the measurement plane H_1 in the superimposed acoustic field, the acoustic wave separation method is used to separate the measured superimposed acoustic field, and the acoustic pressure of the target vibrating ball sound source alone on the measurement plane H_1 is obtained. As shown in Fig.2 (b). The acoustic pressure distribution

obtained by separation can accurately reflect the real position of the target sound source in the acoustic field, but there is a certain error in the separation result in the central area of the measurement plane. Quantification by Eq. (17) shows that the overall separation error is 7.36%.



(a) Acoustic pressure on H_1 before separation (Pa) (b) Acoustic pressure of the target source on the H_1 after separation (Pa)

Figure 2: Comparison of the acoustic pressure on the H_1 before and after the acoustic field separation

In order to investigate the effect of vibration frequency on the separation effect, in the frequency band of 0.5 kHz~5kHz, we should keep all other parameters and settings unchanged and use the separation method to separate the acoustic field of each working condition.

The separation error at each measuring point on the H_1 at different frequencies, as shown in Fig 3. It can be seen from the figure that the direct measurement value and the true value when the target sound source exists alone, the deviation between the two is more than 75%. After separation, the relative error between the separated acoustic pressure and the target sound source is obtained. Separation errors at most frequencies are below 40%.

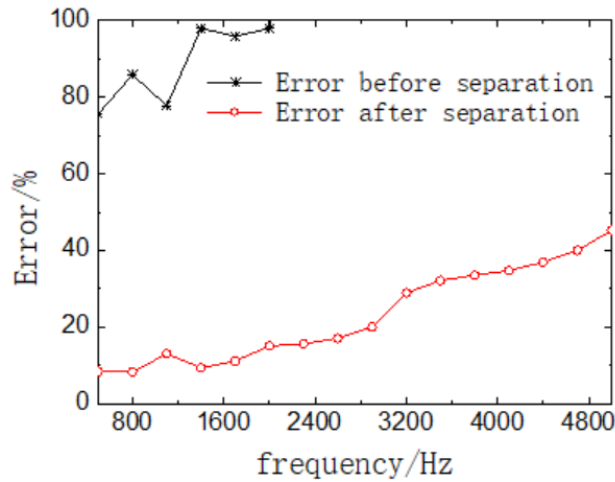


Figure 3: Separation error on H_1 at different frequencies

4. Conclusion

This method establishes local coordinate systems at the sound sources on both sides of the array, and uses the superposition of spherical harmonics of different orders to express the spatial distribution of the acoustic field in each local coordinate system. The main conclusions are as follows:

- (1) When the acoustic field is generated by two sound sources and there is a measurement error,

the acoustic field separation error is small, the separation error is only 7.36% at the frequency.

(2) When the acoustic field is generated by sound sources of different frequencies and there are measurement errors, the acoustic field separation error will increase significantly, below 40% at most frequencies. The method significantly reduces the influence of interfering sources.

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