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Optimized Pricing Mechanism and Design of Carbon Finance Structured Products

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Abstract: Carbon finance play an essential role in the promotion of carbon peaking and carbon neutrality, and one support for the development of carbon finance is the structured deposit launched by banks. This article first examines the pricing rationality of a carbon finance structured deposit by using risk neutrality pricing, GARCH model, Cholesky decomposition, BS Model, Monte Carlo simulation, geometric Brownian motion, Heston model and Merton jump-diffusion model, etc., parameters used for asset pricing are all estimated with reasonable basis. Moreover, this article also optimized its design from the perspectives of increasing market participants and risk diversification. Finally, several enlightenments are summarized and put forward.

1. Introduction

So far, there have been eight regional and one national trading centres for carbon emission allowance in China, and a great number of researches have been carried out based on carbon emission allowances, including asset pricing issues [1], allocation strategies [2], future price forcasting [3] and intermarket risk spillovers [4].

One support for the development of carbon finance is the structured deposit launched by banks. For instance, in Nov 2014, Industrial Bank (the first Equate Bank in China), Shenzhen Branch, combined a deposit with a carbon emission allowance; In May 2021, China CITIC Bank released the first structured deposit linked to a carbon neutral bond; In May 2021, Industrial Bank and Shanghai Clearing House issued the first structured deposit linked to a carbon neutral bond index.

This article aims to examine the pricing rationality of a certain carbon finance structured deposit mentioned above and optimize its design for better meeting diversified needs of investors.

2. Introduction of the selected product

We choose the carbon finance structured deposit launched by Industrial Bank in 2014 as the research object, whose purpose of issuance is encouraging enterprises to carry out carbon emissions trading and thus establish awareness of environmental responsibility. as the cash flow structure of the floating portion is similar to that of a European style call option, which is relatively simple and has room for improvement in both the rationality of asset pricing and meeting the needs of investors. The basic information is listed in Table 1.

Table 1: Basic Information of the Product.

Launcher	Industrial Bank
Amount	RMB 10 million
Term of deposit	One year, from 2014.11.26 to 2015.11.26
Underlying asset	Shenzhen carbon emission allowance
Expected annual return	4.1% (1.9% fixed and 2.2% expected floating)
Interest compensation	Additional payment of 1,000 tons of Shenzhen carbon emission allowances on the due date
Early termination rights	None (European-style option)
Earning structure	$1.9\% + 55\% * max\left(\left(\frac{S_T}{S_0} - 1\right), 0\right) \geq 1.9\%$ Participation rate: 55% S_T: Price on Nov 27, 2015 S_0: Price on Nov 27, 2014

3. Pricing of the Fixed Income Portion

The one-year deposit can be regarded as a zero-coupon bond: the principal and interest will be paid together only at maturity. The price of the fixed income portion is about RMB 9.8645 million.

1) F = Investment amount = RMB 10 million;
2) r_{fix} = Product fixed rate of return = 1.9%;
3) $r_{discount}$= Interest rate of one-year RMB fixed deposit of Industrial Bank in 2014 = 3.3%;
4) T = 1 year.

$$V_{fix} = \frac{F+F*r_{fix}}{(1+r_{discount})^T} = \frac{10+10*1.9\%}{(1+3.3\%)^1} \approx 9.8645 \quad (1)$$

4. Pricing of the Floating Portion (European Call Option)

4.1. Constant Volatility

Volatility is a key parameter for option pricing. At first, we assume that the volatility is constant in the whole period, so it can be easily estimated by using historical data. Regardless of the early-established stage with high volatility, we use the transaction records from January 2nd, 2014 to November 26th, 2014. After excluding missing values, a total number of 211 historical data are obtained, and the estimated value of annual volatility is 0.0492.

4.2. Time-varying Volatility

However, carbon emission allowances are not traded on a daily basis and their prices can experience sudden changes between consecutive trading days, resulting in high volatility and uncertainty. By visualizing the return series, it is evident that there exists a significant agglomeration effect of volatility. Hence, the former assumption of constant volatility is not reasonable, indicating that the volatility sequence may have an ARCH effect [5], and it is suitable to be fitted by the GARCH model [6]. The result of ADF test shows that the test statistic is $4.422 * 10^{-11} < 0.05$, indicating that the return series is stationary. However, the Ljung-Box test statistics of the residual term are all smaller than 0.05, rejecting the null hypothesis of white noise and proving the presence of an ARCH effect.

Generally, a GARCH (1,1) model is sufficient to describe the conditional heteroscedasticity of a sequence. The stabilized fitting parameters are shown as follows. After getting the estimated model, forecast the next daily volatility with historical data of the previous month and then annualize it. The estimated value of annual volatility is 0.0569 > 0.0492, indicating that the previous assumption of constant volatility will result in an undervaluation of the floating portion.

Besides, by repeating the above estimation on a daily basis, we can get the ARCH-fitted volatility series, as is shown in Figure 1. This curve can be further used in the next chapters.

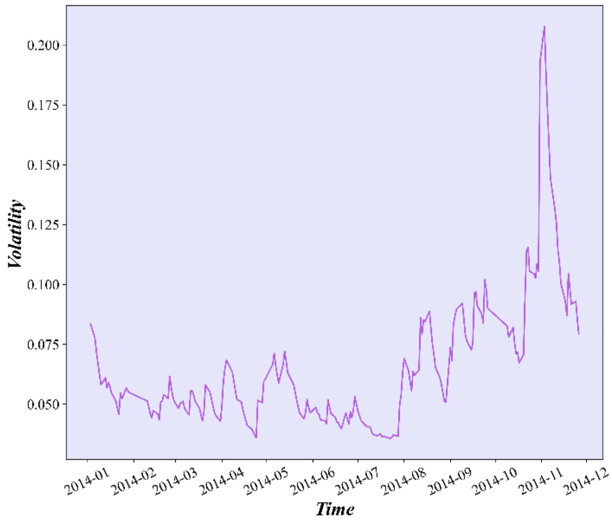

Figure 1: ARCH-fitted volatility of SZEA.

4.3. Pricing-based on BS Model

1) S_0 = initial price = the closing price of SCEA on Nov 27, 2014 = 39.7;

2) K = expiration exercise price, $E(K) = S_0$, set $K = S_0 = 39.7$;

3) r_f = risk-free rate of return = the one-year treasury bond yield announced by the central bank in 2014 = 3.6%;

4) T = one year;

5) $\sigma = 0.0569$, estimated by the former GARCH (1,1) model.

$$N(d_1) = N\left(\frac{\ln\left(\frac{S_0}{K}\right) + \left(r_f + \frac{\sigma^2}{2}\right) * T}{\sigma\sqrt{T}}\right) = 0.7457 \tag{2}$$

$$N(d_2) = N(d_1 - \sigma\sqrt{T}) = 0.7271 \tag{3}$$

$$C_0 = S_0 * N(d_1) - (Ke^{-rT}) * N(d_2) = 1.7585 \tag{4}$$

6) Expected return on the floating portion, where 55% is the participation rate in Table 1:

$$E(R_{float}) = E\left(55\% * max\left(\left(\frac{S_T}{S_0} - 1\right), 0\right)\right), 55\% * (E(S_T) - S_0) = S_0 * E(R_{float}) \quad (5)$$

And because $E(S_T) - S_0 = C_0 * e^{r_f T}$, hence $E(R_{float}) = \frac{55\% * C_0 * e^{r_f T}}{S_0}$.

7) F = Investment amount = RMB 10 million, then the price of the floating portion is about RMB 0.2438 million.

$$V_{float_1} = \frac{F * E(R_{float})}{(1 + r_f)^T} = \frac{10 * 55\% * 1.7585 * e^{0.036*1}}{(1 + 0.036) * 39.7} = 0.2438 \quad (6)$$

4.4. Pricing-Monte Carlo Simulation Based on Geometric Brownian Motion

The key advantage of Monte Carlo simulation [7] over B-S analytical solutions is its applicability to pricing various types of assets, including American options and option portfolios.

1) The formula for Monte Carlo simulation:

$$S_T = S_0 e^{\left(r - \frac{\sigma^2}{2}\right)T + \sigma\varepsilon\sqrt{T}} \quad (7)$$

2) Generate standard normal distribution randoms;

3) Simulate 20000 times to obtain 20000 different paths, as is shown in Figure 2;

4) Similarly, expected return and pricing of the floating portion:

$$E(R_{float}) = 55\% * \frac{1}{20000} * \sum_{i=1}^{20000} max\left(\left(\frac{S_{i_T}}{S_0} - 1\right), 0\right) \quad (8)$$

$$V_{float_2} = \frac{F * E(R_{float})}{(1 + r_f)^T} \approx 0.2430 \quad (9)$$

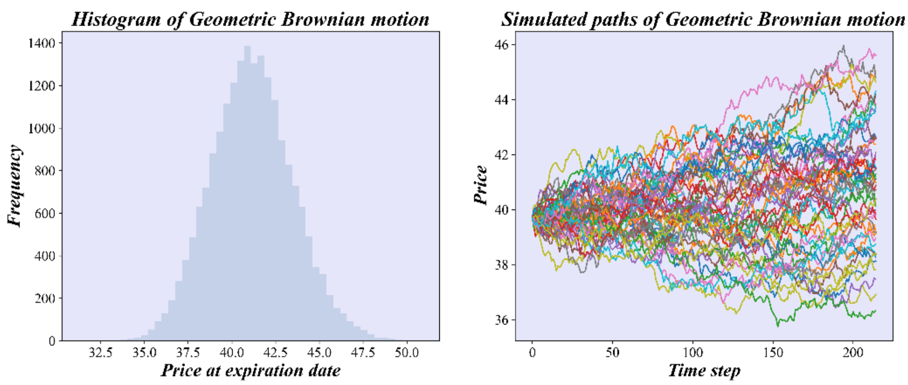

Figure 2: Geometric Brownian Motion.

4.5. Pricing-Monte Carlo Simulation Based on Heston Stochastic Volatility

1) Basic formula put forward by Heston [8], which is similar to geometric Brownian motion but assume a time-varying variance v that also follows a random process:

$$dS_t = \mu S_t dt + \sqrt{v} S_t dZ_{t1} \tag{10}$$

$$dv_t = k(\theta - v_t)dt + \delta\sqrt{v_t} dZ_{t2} \tag{11}$$

2) θ is the long-term average of v. This parameter can be estimated as the squared constant annual volatility in chapter 4.1;

3) k is the adjustment coefficient of θ. Referring to other models, this parameter can be set to 1 since $(\theta - v_t)$ is likely to be treated as excess return or premium;

4) δ is the volatility of v. This parameter can be estimated by using the series in Figure 1;

5) In reality, these parameters may all be immeasurable functions of S_t, so they are connected with each other by sophisticated distributions and can be estimated simultaneously. The approaches above just provide simplified ideas for parameter estimation;

6) Monte Carlo simulation formulas, where the two randoms correlate with each other:

$$S_T = S_0 e^{\left(r - \frac{max(v_T,0)}{2}\right)T + \sqrt{max(v_T,0)}\varepsilon_2\sqrt{T}} \tag{12}$$

$$v_T = v_0 + k(\theta - max(v_0, 0))T + \delta\sqrt{max(v_0, 0)}\varepsilon_1\sqrt{T} \tag{13}$$

7) Generate new independent randoms by Cholesky decomposition. Cholesky decomposition can be applied to positive definite matrices A, where $x^T Ax > 0$. After the decomposition, the lower triangle matrix L satisfies $A = LL^T$. If the correlation coefficient matrix of $\varepsilon_1, \varepsilon_2$: $\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$ is subjected to Cholesky decomposition, a lower triangular matrix $\begin{pmatrix} 1 & 0 \\ a & b \end{pmatrix}$ can be obtained. Hence, new independent randoms can be set as $w_1 = \varepsilon_1$ and $w_2 = a\varepsilon_1 + b\varepsilon_2$. The parameter ρ can be estimated by the closing price series and ARCH-fitted volatility series in Figure 1;

8) Similarly, using the new randoms w for Monte Carlo simulation. Simulate 20000 times to obtain 20000 different paths. The price of the floating portion is about 0.2532 million. It can be seen from Figure 3 that the variance v tends to approach its long-term mean level θ over time.

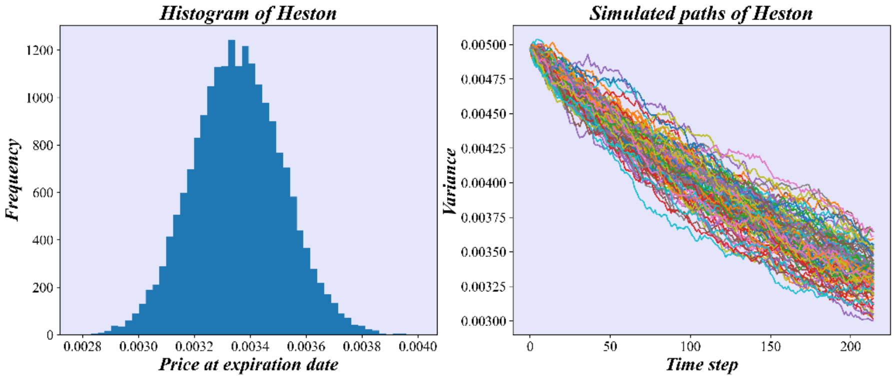

Figure 3: Heston-volatility.

4.6. Pricing-Monte Carlo Simulation Based on Merton Jump Diffusion

1) Put forward by Merton [9], for the fact that the prices of underlying assets may experience sudden jumps due to unexpected market extreme events;

2) Basic formulas are listed below. The total distribution is the combination of a discrete distribution Poisson and a continuous distribution;

3) P_t is a Poisson process with intensity λd_t, the parameter λ can be set as 0.2, which assumes

one extreme event occurs every 5 years (recall the worldwide financial crisis in 2008, the Chinese stock market crash in 2015 and the pandemic starts from 2020);

4) J represents the magnitude of the jump and follows a lognormal distribution, r_J is the correction term for the jumping drift, where μ_j is the minimum return and δ^2 is the highest volatility occurs during extreme events, which can also be estimated by the series in Figure 1.

$$dS_t = (r - r_J)S_t dt + \sigma S_t dZ_t + JS_t dP_t \tag{14}$$

$$ln(1+J) \sim N(ln(1+\mu_j) - \frac{\delta^2}{2}, \delta^2) \tag{15}$$

$$S_T = max(S_0, 0)e^{\left(r - r_J - \frac{\sigma^2}{2}\right)T + \sigma\varepsilon_1\sqrt{T} + \varepsilon_3(e^{\mu+\delta\varepsilon_2} - 1)} = max(S_0, 0)e^{\left(r - \lambda(e^{\mu+\frac{\delta^2}{2}} - 1) - \frac{\sigma^2}{2}\right)T + \sigma\varepsilon_1\sqrt{T} + \varepsilon_3(e^{\mu+\delta\varepsilon_2} - 1)} \tag{16}$$

5) Similarly, using three independent randoms for Monte Carlo simulation. Simulate 20000 times to obtain 20000 different paths, as is shown in Figure 4. The price of the floating portion is about 0.4801 million. It can be seen from Figure 4 that the paths fluctuate significantly after taking into account extreme scenarios.

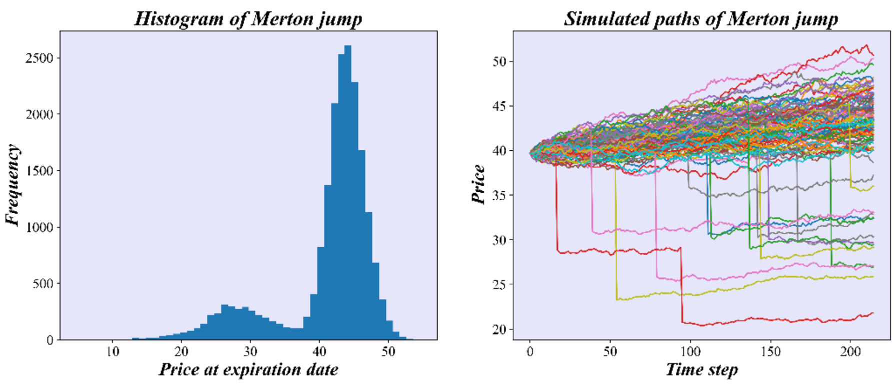

Figure 4: Merton Jump.

4.7. Pricing-Multiple Situations

Taking into account all the pricings provided by the four models, an expected pricing for the floating portion can be obtained. It is worth noting that since carbon market fluctuations are not as volatile as the stock market, we only give a small weight of 5% to the Merton jump diffusion model roughly equals to the significance level.

$$V_{float} = 95\% * \left(\frac{V_{float_1} + V_{float_2} + V_{float_3}}{3}\right) + 5\% * V_{float_4} \approx 0.2583 \tag{17}$$

5. Pricing of the Interest Compensation Portion

1000 tons of Shenzhen carbon emission rights quota will be given to investors for interest compensation, which is actually a futures contract that will be physically delivered upon expiry. The expected price of physical delivery futures at maturity can be the average level of the historical data since 2014.The price of the interest compensation portion is about RMB 0.0644 million.

$$V_{plus} = \frac{1000 * ES_T}{(1+r_f)^T} \div 1000000 \approx 0.0644 \tag{18}$$

6. Analysis of Pricing Rationality and Sensitivity

Firstly, in extreme cases of a 5% probability, there is no guarantee of returns. The estimated floating rate of return is calculated as follows, and it can be seen that the expectation of 2.2% in Table 1 is roughly reasonable.

$$E(R_{float}) = 5\% * 0 + 95\% * \frac{E(R_{float_1}) + E(R_{float_2}) + E(R_{float_3})}{3} = 2.4286\% \approx 2.2\% \quad (19)$$

Secondly, the total intrinsic value of this product can be obtained by adding up the three parts above, which is nearly equal to its launch price, indicating the pricing rationality. Although it seems that the product's fixed yield is quiet low (only 1.9%, even lower than the fixed deposit interest rate of 3.3%), there are also floating portion and carbon emission quotas for interest compensation, making the final value reach its reasonable level.

$$V = V_{fix} + V_{float} + V_{plus} = 10.1872 \approx 10 \quad (20)$$

Finally, the main parameters for sensitivity analysis including $r_{discount}$, r_f and σ. $r_{discount} \rightarrow V_{fix}$, $r_f \rightarrow V_{float}$ and V_{plus}, $\sigma \rightarrow V_{float}$. $r_{discount}$ (the one-year fixed deposit interest rate) is negatively related to the total value, while r_f and σ are positive influencing factors. By simple calculation, it is found that all the parameters have limited impacts on the price of the carbon finance deposit, indicating its low investment risk.

7. Thoughts on Optimizing the Design

7.1. Get More Market Participants Involved

As is mentioned above, a 1000 tons of carbon emission quota will be granted to participating entities upon expiration for interest compensation, so the participating entities are generally production enterprises with emission demands. If normal individual investors also want to involve in, perhaps the emission quota can be replaced by other equivalent tradable assets.

For instance, the green financial bond purchased by individuals and non-financial institutional customers in the counter market of the inter-bank trading market are almost risk-free. Therefore, a certain number of green financial bonds with a total face value equal to the terminal value of carbon emission quotas can be ideal substitutes, for it help expand the scope of market participants, which is beneficial for activating bond trading.

7.2. Construct Carbon Emission Allowances Portfolio

The underlying asset of a carbon finance structured deposit can also be a carbon emission allowances (CEA) portfolio, such as a combination of Shanghai CEA, Beijing CEA, Guangdong CEA, Hubei CEA, and Shenzhen CEA. Compared with a single CEA, the CEA portfolio performs better in risk diversification. The processes involved in the construction are as follows.

Firstly, determine the weights by using Markowitz investment portfolio theories and the same historical data as the original product. For the purpose of risk diversification, a minimum variance portfolio is required, and the calculation results turn out to be: 17.84% Shanghai CEA, 42.34% Beijing CEA, 0% Guangdong CEA, 22.36% of Hubei CEA and 17.45% Shenzhen CEA.

Secondly, use Monte Carlo simulation to generate random variables, correlate them according to the Jolisky-decomposed correlation coefficient matrix of the CEAs, and simulate the future asset price fluctuation paths of five CEAs with these random variables.

Finally, combine the paths using the pre-determined weights. Figure 5 presents the histogram and simulated paths of the CEA portfolio. Similarly, by adding up the fixed income portion and interest compensation, the total estimated value can be obtained, which is about 10.1474 million.

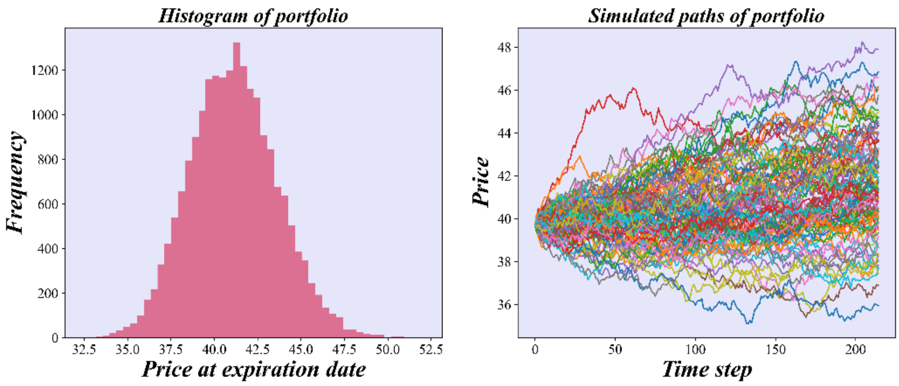

Figure 5: Geometric Brownian Motion-Portfolio Level.

8. Conclusion

By examining the pricing rationality of a carbon finance structured deposit and optimize its design, main enlightenments can be summarized. Firstly, the key for reasonable asset pricing is matching benefits and costs. As is shown in the case, the future cash flow of 1.9% fixed portion + floating portion + additional interest compensation may equal to or even exceed that of a 3.3% fixed asset. Secondly, parameters used for asset pricing must be estimated with reasonable basis, rather than solely on subjective judgement. Finally, developing carbon emission trading markets and encouraging financial institutions to design green financial products are significant and effective ways for China to achieve the targets of carbon peaking and carbon neutrality.

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