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Research on Sliding Mode and Backstepping Control of Missile Pitch Channal System

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Abstract: A new type of proportional plus sliding mode inversion controller is constructed and applied to solve the attack angle tracking problem of missile pitch channel; At the same time, in order to solve the differential explosion problem of backstepping control, a method combining an integral type sliding surface with an accurate differentiator is proposed to solve the derivative of the desired pitch angular rate signal; At last, a kind of integral type Lyapunov function is constructed to prove the stability and boundness of the closed-loop signal of the whole system; Finally, a digital simulation experiment is done to verify the correctness of the method.

1. Introduction

The control problem of the missile pitch channel is the main problem of the whole missile control system design, because the yaw channel is identical or similar to the pitch channel, while the control of the roll channel is relatively simple. Generally, PID control can meet the control task[1-5]. In addition to the traditional PID control, the current popular variable structure control has the advantage of good rapidity; The disadvantage is that its dynamic performance often appears flutter [6-9]. At the same time, inversion control is also loved by the majority of researchers, because its precise design idea and backward design idea are more suitable for combining with Lyapunov function method, and can rigorously analyze stability; However, its disadvantage is that the theoretical analysis of the differential solution of the desired signal is relatively complex, which will lead to the problem of differential explosion; In order to solve this problem, many scholars try to use the accurate differentiator method to calculate; We also propose a method based on the combination of sliding mode and accurate differentiator to solve the differential problem. At the same time, on the basis of inversion, we use a method combining two-layer integral sliding mode and error proportional feedback to solve the problem of angle of attack tracking, which is also very innovative. The digital simulation results also show the stability of the whole system.

2. Problem Description

A kind of second order system can be used to describe the linear model of supersonic missile pitch channel as following:

$$\dot{\alpha} = \omega_z - a_{34}\alpha - a_{35}\delta_z + f_1 \tag{1}$$

$$\dot{\omega}_z = a_{24}\alpha + a_{22}\omega_z + a_{25}\delta_z + f_2 \tag{2}$$

where a_{ij} is constantant air dynamic coefficient of missile which is caused by the outer shape of missile body, α is called attack angle of missile, ω_z is called the rotate speed of pitch angle , f_1 and f_2 are used to describe the outer disturbance of missile system.

The main task of controller design of missile system is to construct a backstepping control law δ_z by using tracking differentiator method such that the attack angle α can track the command angle α^d. Without loss of generality, we assume $\alpha^d = -3/57.3$,and the outer disturbance f_1 and f_2 can also be estimated by the control law δ_z.

3. Backstepping controller Design with Tracking Differentiator

First step, we define a new error variable as $e_\alpha = \alpha - \alpha^d$, then the first subsystem of missile system can be written as

$$\dot{e}_\alpha = \omega_z - a_{34}\alpha - a_{35}\delta_z + f_1 - \dot{\alpha}_d \tag{3}$$

And define an error sliding mode surface as

$$s_a = e_\alpha + c_1 \int e_\alpha dt + c_2\omega_z$$

Then the expect value of ω_z can be set as ω_z^d as follows:

$$\omega_z^d = a_{34}\alpha + a_{35}\delta_z - f_1 + \dot{\alpha}_d - k_{\alpha_1} e_\alpha - k_{\alpha 2} s_a \tag{4}$$

Since f_1 is unknown, we define a new variable as

$$\omega_{za}^d = a_{34}\alpha + a_{35}\delta_z + \dot{\alpha}_d - k_{\alpha_1} e_\alpha - k_{\alpha 2} s_a \tag{5}$$

Then

$$\omega_z^d = \omega_{za}^d - f_1, \dot{\omega}_z^d = \dot{\omega}_{za}^d - \dot{f}_1 \tag{6}$$

And the second step is to define a new error variable as $e_\omega = \omega - \omega_z^d$, then the second subsystem can be constructed as

$$\dot{e}_\omega = a_{24}\alpha + a_{22}\omega_z + a_{25}\delta_z + f_2 - \dot{\omega}_z^d \tag{7}$$

It can also be rewritten as

$$\dot{e}_\omega = a_{24}\alpha + a_{22}\omega_z + a_{25}\delta_z + f_2 + \dot{f}_1 - \dot{\omega}_{za}^d \tag{8}$$

And we choose a integral sliding mode surface as

$$s_b = e_\omega + c_3 \int e_\omega dt + c_4\omega_z$$

And we choose a integral type sliding mode surface for tracking differentiator as

$$s_c = z_1 + c_5 \int z_1 dt + c_6 x_2$$

And we build a sliding mode tracking differentiator to solve $\dot{\omega}_{za}^{d}$ as

$$\begin{aligned} & z_1 = x_1 - \omega_{za}^{d} \\ & \dot{x}_1 = x_2 \\ & \dot{x}_2 = R^2(-z_1 - k_c s_c - k_d \frac{s_c}{|s_c| + \varepsilon_a} - x_2 / R) \end{aligned} \tag{9}$$

Then the expect virtual control law can be designed as

$$\delta_{za} = \frac{1}{a_{25}}\{-a_{24}\alpha - a_{22}\omega_z - k_{a1}e_{\omega} - k_{a2}\frac{s_b}{|e_{\omega}| + \varepsilon_a} - k_{a3}s_b - f_2 - \dot{f}_1 + \dot{\omega}_{za}^{d}\} \tag{10}$$

Here we use the value $\dot{x}_1$ to approximate the value of $\dot{\omega}_{za}^{d}$ since $x_1 \to \omega_{za}^{d}$ as time goes to infinite. And assume $-f_2 - \dot{f}_1$ is bounded, then we can choose k_{a2} such that $|-f_2 - \dot{f}_1| \le k_{a2}$. And then we can design the final control law as

$$\delta_{za} = \frac{1}{a_{25}}\{-a_{24}\alpha - a_{22}\omega_z - k_{a1}e_{\omega} - k_{a2}\frac{s_b}{|e_{\omega}| + \varepsilon_a} - k_{a3}s_b + \dot{x}_1\} \tag{11}$$

Then third step of design is to choose a Lyapunov function as

$$V = \frac{1}{2}e_{\alpha}^2 + \frac{1}{2}e_{\omega}^2 \tag{12}$$

And its derivative can be solved as

$$\dot{V} \le -k_{\alpha 1}e_{\alpha}^2 - k_{a1}e_{\omega}^2 - k_{a3}e_{\omega}s_b \le -k_{\alpha 1}e_{\alpha}^2 - k_{a1}e_{\omega}^2 - k_{a3}e_{\omega}^2 - k_{a3}c_5 e_{\omega}\int e_{\omega}dt \tag{13}$$

And then we can choose a new integral type Lyapunov function as

$$V_b = V + \frac{1}{2}k_{a3}\left(\int e_{\omega}dt\right)^2 \tag{14}$$

And it derivative can be solved as

$$\dot{V}_b \le -k_{\alpha 1}e_{\alpha}^2 - k_{a1}e_{\omega}^2 - k_{a3}e_{\omega}^2 \tag{15}$$

So according to the Lyapunov stability theory, all signal of the the whole system is bounded and stable.

4. Numerical simulation experiment for example

According to the air wind hole experiments, all constant air coefficients of a type of missiles are as follows

$$a_{25} = -167.87; a_{35} = 0.243; a_{22} = -2.876; a_{24} = -193.65; a_{34} = 1.584$$

And we choose control parameters as

$$R = 10; k_{a1} = 18; k_{a2} = 25; k_{\alpha 1} = 45; k_{\alpha 2} = 15; k_{\alpha 3} = 8;$$

Then simulation results can be shown by figures 1-5.

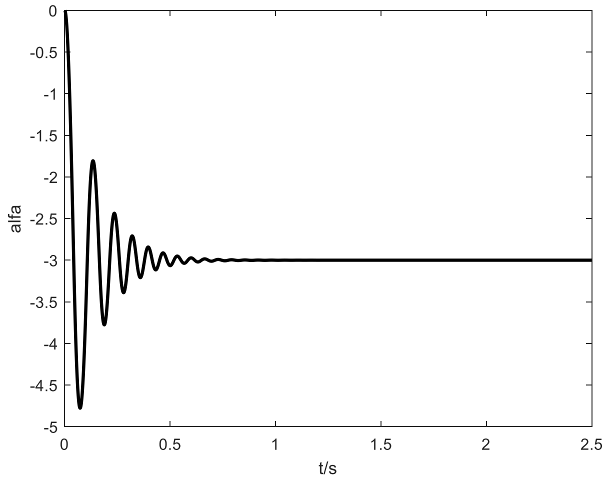

Figure 1 The attack angle of missile system

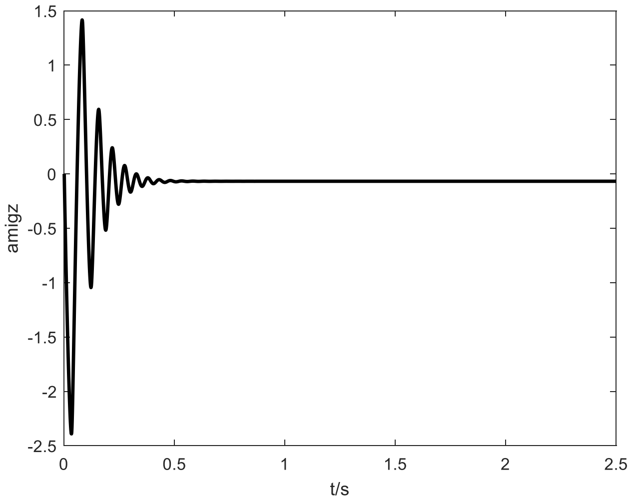

Figure 2 The angle of speed of missile system

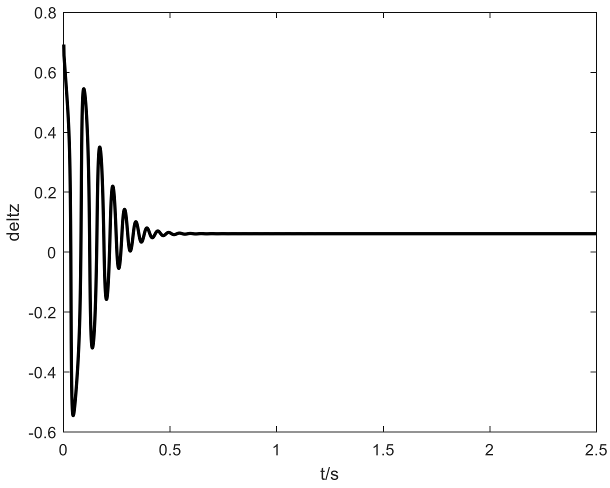

Figure 3 The actuator curve of missile system

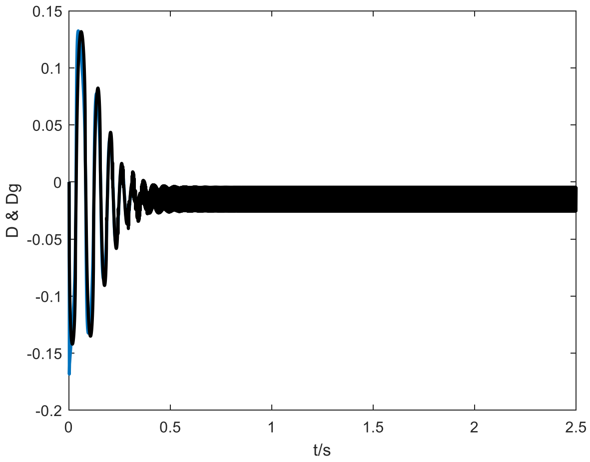

Figure 4 The estimation value of ω_{za}^{d}

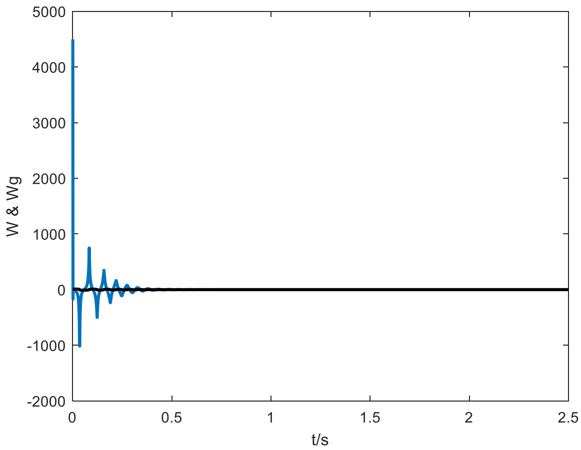

Figure 5 The curve of $\dot{\omega}_{za}^{d}$

It can be seen from Figure 1 to Figure 5 that the system oscillates severely at this time. We select the following parameters:

$$R=10; k_{a1}=18; k_{a2}=25; k_{\alpha 1}=25; k_{\alpha 2}=5; k_{\alpha 3}=18;$$

Then the simulation results are shown as following figure 6 -10:

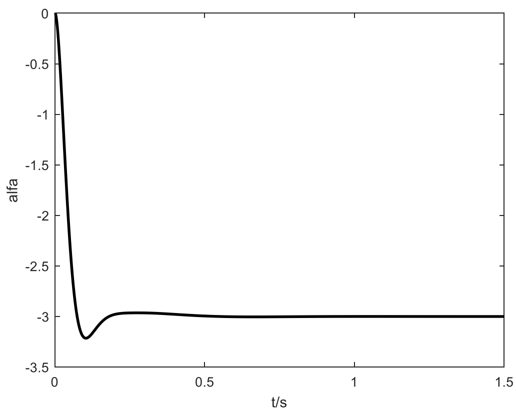

Figure 6 The attack angle of missile system

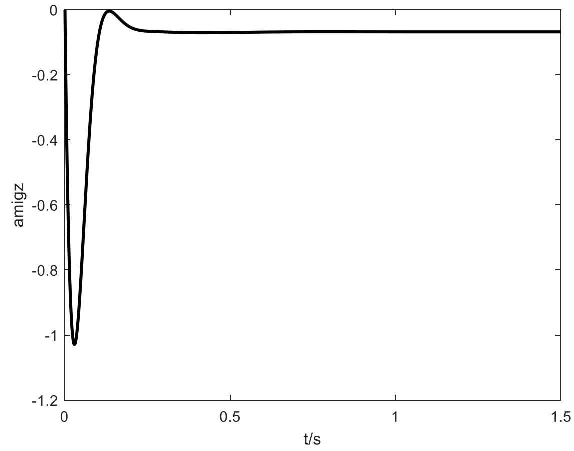

Figure 7 The angle of pitch speed

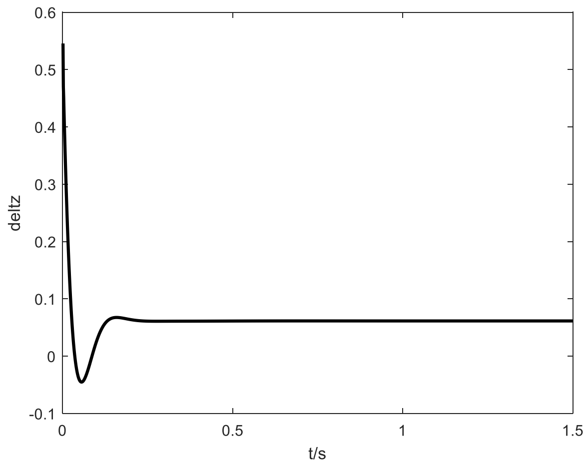

Figure 8 The actuator of missile system

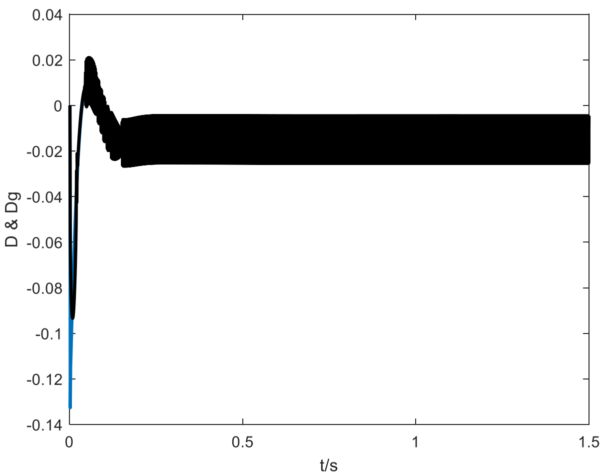

Figure 9 The estimation value of ω_{za}^{d}

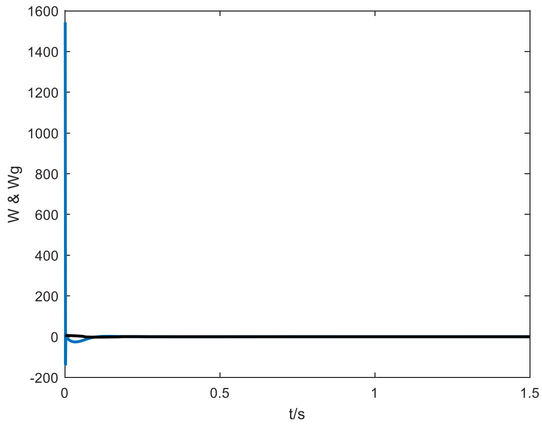

Figure 10 The curve of $\dot{\omega}_{za}^{d}$

It can be seen from Figure 6 to Figure 10 that the dynamic performance of the system has been greatly improved; The main manifestation is that the flutter is greatly weakened; The overshoot is reduced, and the flutter frequency is less. The rise time has decreased, but the overall rise time is still less than 0.15 seconds; Moreover, the adjustment time is greatly reduced from 0.6 seconds to 0.3 seconds. In general, the simulation results show that the proposed method is stable and valuable.

5. Conclusions

According to the angle of attack tracking task of the missile pitch channel control system, a kind of angle of attack stability controller combining backstepping and sliding mode method is designed. The error of sliding mode surface is designed according to the attack angle subsystem and the pitch angular speed subsystem respectively. In addition, according to the calculation requirements of the inversion differential, the sliding model accurate differentiator is introduced to solve the derivatives of the required variables, avoiding the differential explosion problem, so as to improve the dynamic performance of the entire system. Finally, the digital simulation experiment also shows the stability of the whole system.

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