

Prediction model of local scour depth of bridge piers based on LS-SVM

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Abstract: Local scour of pier foundation soil is one of the main causes of bridge failure. The scour mechanism around the pier foundation is complex. The current code mainly uses empirical formulas to predict the scour depth of the pier, and the prediction results are generally too discrete. In order to accurately predict the local scour depth of bridge piers, 337 sets of model scour test data were collected in this paper. The standardized method was used to process the data dimensionlessly, and the Pearson correlation analysis method was used to analyze the correlation of the experimental data. It is concluded that the pier diameter, water flow depth, water flow velocity, median particle size and particle size standard deviation are the main influencing factors of local scour depth of bridge piers. The sensitivity analysis method is used to analyze the sensitivity of the five parameters and analyze their influence on the local scour depth of the pier. A prediction model of local scour depth of bridge piers based on least squares support vector machine (LS-SVM) is proposed. The results show that the prediction results of the model are obviously better than the calculation results of the current specification. After the dimensionless treatment, the coefficient of determination of the prediction model is increased from 0.624 to 0.824. The predicted value of the local scour depth of the pier is in good agreement with the measured value, which can provide reference for bridge design and safe operation.

1. Introduction

Bridges are important infrastructures, and the failure of piers can lead to serious economic and social consequences. According to the survey of 823 bridge failures since 1950 in the United States, 60% of bridge failures are related to water flow, including pier erosion and river instability^[1]. In 2000-2019, China investigated the causes of 151 bridge collapses, and 97 bridge floods occurred, accounting for 64.24%^[2]. The annual economic loss caused by bridge erosion in New Zealand is about \$ 24 million^[3]. In addition, Europe is expected to invest about \$ 611 million annually in reducing the risk of pier erosion in 2040-2070^[4]. The local scour of piers is the main reason for the failure of piers on rivers. The scour depth of piers is an important factor in the design of bridge foundation depth. Overestimating or underestimating the scour depth will lead to excessive costs and serious bridge accidents. Therefore, in order to obtain a safe, economical and efficient bridge structure, it is very important to accurately predict the scour depth near the piers and abutments^[5-6].

Since the 1950s, some researchers at home and abroad have carried out experimental research and put forward the prediction formula of pier scour. Melville and Laursen et al^[7-10] carried out indoor model tests and proposed a prediction formula for scour depth under the premise of constant riverbed conditions^[11], constant depth and stable flow conditions. Froelich et al^[12-14] proposed a pier scour depth prediction formula based on the data collected from the pier scour site. The scour of bridge piers is affected by many factors, such as the form of foundation structure, flow characteristics, riverbed sediment movement and so on. The relationship between the local scour depth of the pier and the influencing factors is complex, and the relationship between some factors and the scour depth is nonlinear. The traditional regression method is used to regress the data collected from the field and indoor model tests to predict the scour depth, which has a significant error with the actual scour depth^[15].

Machine learning can solve the problem of complex nonlinear relationship between parameters. Machine learning has strong generalization and convenient application in data analysis. Compared with traditional methods, it has more obvious advantages and is gradually applied in the prediction of local scour depth of piers^[16-20]. Sharafi et al^[21] used polynomial function of support vector machine and artificial neural network (ANN), adaptive neuro-fuzzy inference system (ANFIS) and based on linear regression equation, the results obtained using polynomial function of support vector machine prediction model has higher accuracy and lower error prediction scour depth. Choi et al^[22] proposed that the SVM model is better than the ANFIS algorithm in predicting the scour depth. Sreedhara et al^[23] used support vector machine and particle swarm optimization algorithm (PSO-SVM) to predict the scour depth around the pier. The results show that PSO-SVM using RBF kernel function model to predict the scour depth around the pier is reliable and effective. LS-SVM is derived from the improvement of standard support vector, which has the characteristics of simple algorithm, easy implementation and fast calculation speed. The advantage of using LS-SVM is that it requires few user-defined parameters and faces the problem of local minima. It has the advantage of correct prediction without overfitting.

Based on the collected indoor model test data, the correlation analysis and sensitivity analysis of the data are carried out, and the main influencing factors affecting the local scour depth of the pier are obtained. The LS-SVM method is used to train and verify the data, and a high-precision prediction model of the local scour depth of the pier is established, and compared with the existing prediction formula of the local scour depth of the pier, which provides a reference for accurately predicting the scour depth of the pier and the protection design of the pier foundation.

2. Data characteristics and preprocessing

2.1 Data characteristics

In order to study the local scour depth of piers, the experimental data of indoor pier foundation scour model were collected. The total number of data was 337 groups of samples. The shapes of piers were all cylindrical and all were non-viscous riverbeds. The 337 sets of original data were screened, the abnormal data were eliminated, and 320 sets of data were retained as samples. It was found that the maximum scour depth of the pier was mainly affected by the diameter of the pier (D), water flow depth (H), water flow velocity (V), median particle size (D_{50}), particle size standard deviation (σ) and other factors. Table 1 shows the distribution of relevant parameters in the sample data, including minimum, maximum, average, standard deviation and coefficient of variation. Figure 1 shows the frequency histogram of the main parameters affecting the scour depth (y_0) of the pier in the sample data.

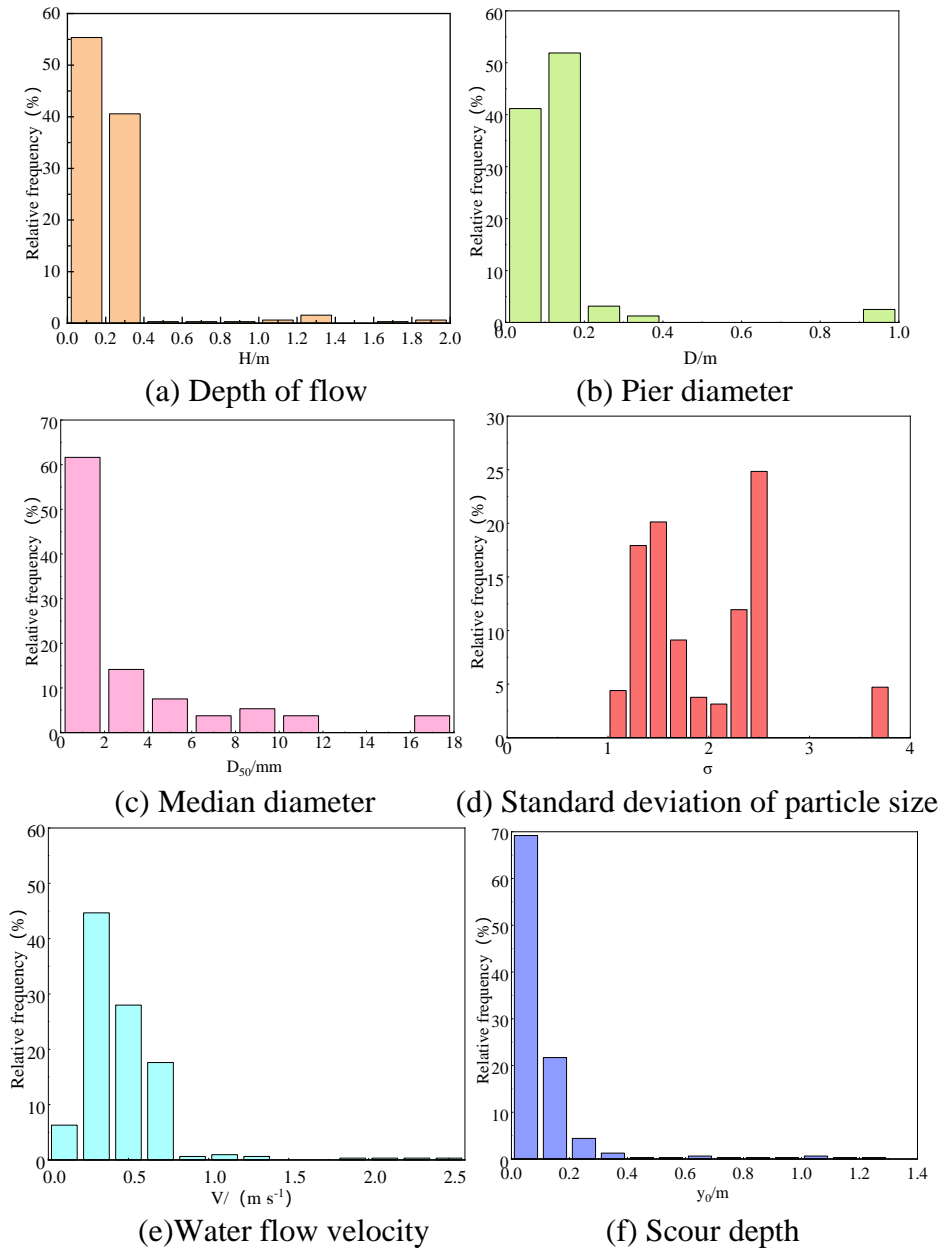


Figure 1: Histogram of frequency distribution of the main parameters of each sample

Table 1: Data parameter characteristics

indicators	D/m	H/m	V/(m/s)	D ₅₀ (mm)	σ
Max	0.91	1.90	2.48	16.90	3.70
Min	0.02	0.04	0.17	0.22	1.15
average	0.14	0.25	0.46	3.20	1.92
deviation	0.14	0.24	0.29	3.95	0.61
coefficient	0.96	0.98	0.61	1.24	0.32

Due to the complexity of water flow structure and sediment movement around the bridge pier, the dimension of different influencing factors is not uniform, which makes it more difficult to establish the model and the accuracy of the prediction results is not high. Therefore, the data needs to be dimensionless. Using the dimensionless standardization processing method, the data is transformed into a distribution with a mean value of 0 and a standard deviation of 1. The standardized processing

formula is as follows:

$$x_{new} = \frac{(x-u)}{\sigma} \quad (1)$$

x is the original data, u is the sample mean, σ is the sample standard deviation.

3. Analysis method and principle

3.1 Correlation analysis

Correlation analysis refers to the analysis of two or more correlated variable parameters to determine the degree of correlation between the two variable parameters. Pearson correlation analysis method was used to evaluate the Pearson correlation coefficient. Pearson correlation coefficient is a linear correlation coefficient, and it is also a method to measure vector similarity. The output range is from -1 to 1, where 0~1 represents positive correlation, -1~0 represents negative correlation, and 0 represents no correlation. The Pearson correlation coefficient is defined as follows:

$$R = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum(X_i - \bar{X})^2} \sqrt{\sum(Y_i - \bar{Y})^2}} \quad (2)$$

X_i, Y_i is the value of the sample, and \bar{X}, \bar{Y} is the average value of the sample. $0 < |R| < 1$ indicates that there are different degrees of linear correlation, $|R| \leq 0.5$ indicates low linear correlation, $0.5 < |R| \leq 0.8$ indicates significant correlation, $|R| > 0.8$ indicates high linear correlation.

3.2 The basic principle of LS-SVM

Support vector machine (SVM) is an intelligent algorithm based on statistical theory, which can efficiently deal with nonlinear classification and regression problems. Based on the principle of structural risk minimization, this method maps the input vector from low-dimensional space to high-dimensional feature space through nonlinear mapping, and finds the optimal regression hyperplane in this space to minimize the objective loss function to achieve the minimum regression error. Least squares support vector machine (LS-SVM) is one of the important methods to realize the prediction model of multi-input and multi-output system. LS-SVM method is used to transform the solution of optimization problem into the solution of linear equations^[22]. The mapping model of the data set can be expressed as:

$$y_i = w^T \psi(x_i) + b \quad (3)$$

y_i is the output value, $\psi(x_i)$ is the mapping function, w is the weight, b is the intercept. LS-SVM defines the objective function of the optimization problem as:

$$Min J(w, e) = \frac{1}{2} (w^T w) + \frac{c}{2} \sum_i^n e_i^2 \quad (4)$$

e_i is a relaxation variable, c is a regularization parameter and $c > 0$. The constraints are:

$$y_i = w^T \phi(x_i) + b + e_i \quad (5)$$

By introducing the Lagrange multiplier, the above constraint function is transformed into an unconstrained function in the form of:

$$L = \frac{1}{2} w^T w + \frac{c}{2} \sum_{i=1}^n e_i^2 - \sum_{i=1}^n a_i \{y_i [w^T \varphi(x_i) + b] - 1 + e_i\}, i = 1, 2, \dots, n \quad (6)$$

a_i is a Lagrange multiplier and $a_i \geq 0$. By solving the parameters a and b , the local scour depth prediction model based on LS-SVM is obtained:

$$Y(x) = \sum_{i=1}^n a_i K(x, x_i) + b \quad (7)$$

$K(x, x_i)$ is a kernel function, which is the inner product of high-dimensional feature space. The meridional basis function has a strong ability to map nonlinear samples to high dimensions, and has less numerical calculation. Therefore, the radial basis function (RBF) is selected as the kernel function of LS-SVM, namely:

$$K(x, x_i) = e^{-\frac{\|x - x_i\|^2}{\delta^2}} \quad (8)$$

δ represents the width of RBF. In the LS-SVM model based on RBF function, the parameters to be determined are kernel function δ and regularization parameter c . The generalization ability and learning ability of the model are greatly affected by δ and c .

3.3 LS-SVM model establishment

Based on the LS-SVM method, a prediction model of local scour depth of bridge piers is established. Let x_{11} be the diameter of the pier, x_{12} the depth of the water flow, x_{13} the velocity of the water flow, and x_{14} the median particle size. x_{15} is the standard deviation of particle size. The output of the prediction model is the local scour depth of the pier. The local scour depth prediction model of the pier is defined as:

$$y_0 = f(x_{11}, x_{12}, x_{13}, x_{14}, x_{15}) \quad (9)$$

f is a nonlinear mapping function, which represents the relationship between the key variables affecting the local scour depth of the pier and the local scour depth of the pier. A set of input and output data $(x_{11}, x_{12}, x_{13}, x_{14}, x_{15})$ is defined as a sample. The training set of the model is:

$T = \{(x_l, y_l)\}_{l=1}^n, x_l \in X \in R^n, y_l \in Y \in R^1, l = 1, 2, \dots, n$; n represents the number of training samples. x_l is the input vector of the model, and y_l is the output vector of the model. The selected test data is divided into training samples and test samples. The training samples account for 80 % and the test samples account for 20 %.

3.4 Evaluation indicators

In order to evaluate the effect of the proposed prediction model, the coefficient of determination (R²) and root mean square error (RMSE) are selected as the evaluation indexes of the prediction model. The theoretical calculation formula is as follows:

$$R^2 = 1 - \frac{\sum (y - \hat{y})^2}{\sum (y - \bar{y})^2} \quad (10)$$

y represents the true value of the data test, \hat{y} represents the predicted value, and \bar{y} represents the average value of the data. According to the value of R2 to judge the quality of the model, the range of R2 is between 0 and 1. The closer R2 is to 1, the more accurate the prediction of the model is. The value of R2 is greater than 0.6, indicating that the prediction model is reliable and effective^[15]. RMSE is another parameter to judge the prediction effect, which is the square root of the ratio of the square of the deviation between the predicted value and the real value to the number of observations n . The deviation between the predicted value and the true value is more sensitive to the predicted value in the data. The expression is as follows:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (11)$$

y_i is the true value of the test, \hat{y}_i is the predicted value, and the closer the value of RMSE is to 0, the better the prediction effect of the model.

4. Results and analysis

For the scour model test data collected in the literature, the Chinese standard formula and the American standard formula are used to calculate the scour depth of the pier, and the results are compared with the model test results. When predicting the local scour depth of piers in China, most of the formulas used are 65-1 revision and 65-2 in the standard 'Code for Hydrological Survey and Design of Highway Engineering (JTG C30-2002). When designing the buried depth of the foundation, one of the larger estimated values of the scour depth is generally used as the local scour depth of the pier foundation. The calculation formula is as follows:

$$\begin{cases} h_b = k_z k_{\eta_2} B_i^{0.6} h_p^{0.15} \left(\frac{v - v'_0}{v_0} \right) \dots \dots \dots v \leq v_0 \\ h_b = k_z k_{\eta_2} B_i^{0.6} h_p^{0.15} \left(\frac{v - v'_0}{v_0} \right) \dots \dots \dots v > v_0 \end{cases} \quad (12)$$

$$\begin{cases} h_b = k_z k_{\eta_1} B_i^{0.6} h_p^{0.15} (v - v'_0) \dots \dots \dots v \leq v_0 \\ h_b = k_z k_{\eta_1} B_i^{0.6} (v - v'_0) \left(\frac{v - v'_0}{v_0 - v'_0} \right)^{\eta_1} \dots \dots v > v_0 \end{cases} \quad (13)$$

The CSU equation in HEC-18 of AASHTO LRFD is used in the design of bridge engineering in the United States. The calculation formula is as follows:

$$\frac{y_s}{y_1} = 2.0 k_1 k_2 k_3 \left(\frac{\sigma}{y_1} \right)^{0.65} Fr^{0.43} \quad (14)$$

Figure 2 shows the predicted results of the collected data using the Chinese code formula and the American code formula. It can be seen that the predicted results of the local scour depth of the Chinese code are small, some are slightly higher than the measured values, and most are lower than the measured values, laying a safety hazard to the design of the bridge. The predicted results of the scour

depth of the pier in the American code formula are much higher than the measured values, which are conservative and easy to cause waste of resources and economy.

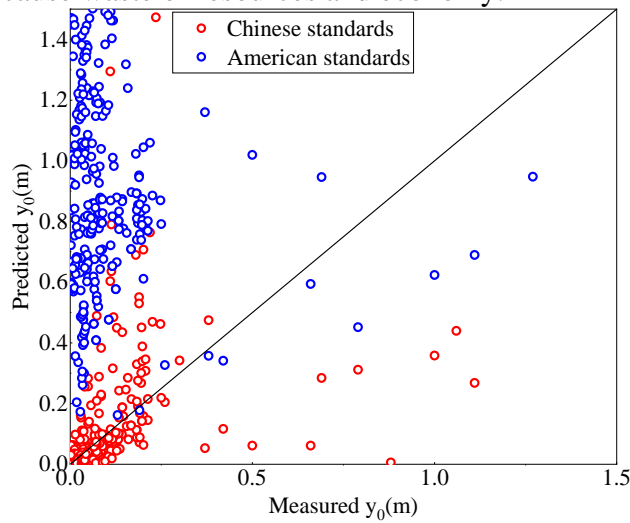


Figure 2: The canonical formula predicts the erosion depth result

Table 2: Correlation coefficient matrix of input parameters

parameter	diameter	Water flow depth	Water flow speed	Median particle size	Standard deviation of particle size	Local scouring depth
diameter	1	0.719**	0.090	-0.066	-0.087	0.856**
depth		1	-0.169**	-0.333**	0	0.630**
Water flow speed			1	0.837**	-0.269**	0.173
Median particle size				1	-0.375**	-0.0430
Standard deviation of particle size					1	-0.201
Local scouring depth						1

Note: **indicates that at the 0.01 level, the correlation is significant

Table 2 shows a matrix of Pearson's correlation coefficients between the five variables and the local erosion depth of the piers. It can be concluded that the local erosion depth of the pier is significantly positively correlated with the diameter of the pier, the depth of water flow and the water velocity, and the local erosion depth of the pier is significantly negatively correlated with the median particle size and the standard deviation of particle size. The local erosion depth of the pier increases with the increase of pier diameter, water depth and flow velocity, and the diameter of the pier has a significant correlation with the local erosion of the pier. The erosion depth of the pier decreased with the increase of the median particle size and the standard deviation of particle size, and showed a small correlation, specifically because the sample particle size value was large, and the particle size type was too small, and the influence was small compared with other factors, all showing a small correlation. Figure 3 shows the correlation between pier diameter, water flow depth, water velocity, median particle size and particle size standard deviation and local erosion depth of the pier, as well as the regression equation.

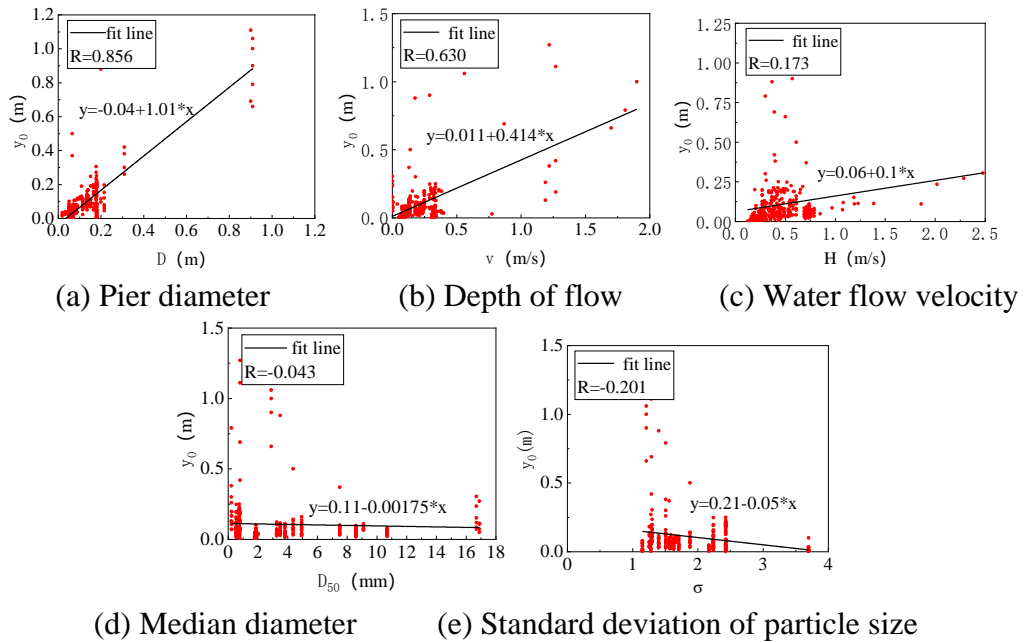


Figure 3: Correlation between parameters and pier erosion depth

Figure 4 is the result of direct simulation by selecting five important parameters in the original data, the R^2 of the training set is 0.7662, and the comparison point between the measured value and the predicted value is mostly around the 45° line, based on LS-S. The prediction of pier erosion depth of VM is better than that of the traditional canonical formula, but the prediction results are discrete, and the average of the prediction set R^2 is 0.707.

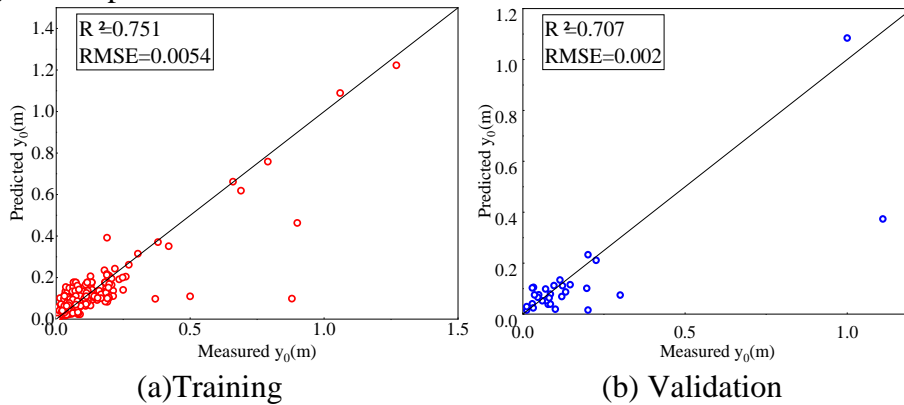


Figure 4: Raw data simulation results

The raw data were dimensionless, and the comparison points between the measured values and the predicted values are shown in Figure 5, the distribution of the comparison points is closer to the 45° line, and the result of the prediction set R^2 is 0.824, RMSE It is 0.176, which indicates that the training and prediction effect of the model has high accuracy, which can be used to predict the local erosion depth of bridge piers.

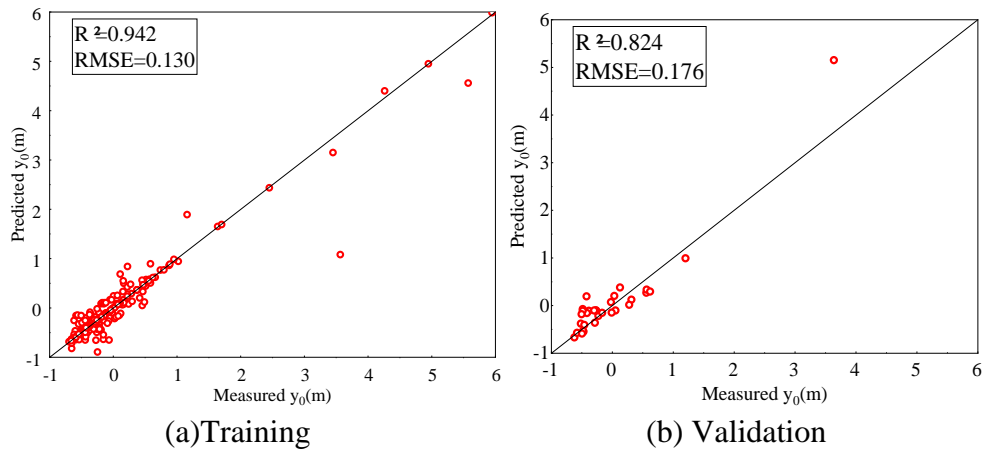


Figure 5: Simulation results after dimensionlessness

In order to study the importance of single variable in predicting the maximum erosion depth of piers, the diameter of the pier, the velocity of the water flow, the depth of the flow, the median particle size and the standard deviation of particle size were selected as five independent parameters to analyze their influence on the local erosion depth y_0 of the pier, and the sensitivity analysis of each parameter was carried out by considering the functional relationship of the local erosion depth of the pier. $y_0 = f(D, V, H, D_{50}, \sigma)$ Table 3 shows the models with different parameters, the models were trained separately, the correlation coefficients R^2 and RMSE were recorded, and the influence of each independent parameter on the local erosion depth of the pier was analyzed. It is clear from Table 3 that pier diameter D and median particle size have the greatest influence on the local erosion depth of the pier, and the results of the sensitivity analysis are consistent with the conclusions drawn by Mueller^[23]. The results also show that the DLM-SVM-based pier erosion prediction model is reliable, and the best combination of parameters is pier diameter, median particle size, water flow depth, particle size standard deviation and water flow velocity.

Table 3: Sensitivity analysis of independent parameters of local erosion depth of piers

Model type	raw data		Dimensionless	
	R^2	RMSE	R^2	RMSE
$y_0 = f(D, V, H, D_{50}, \sigma)$	0.707	0.0029	0.824	0.1758
$y_0 = f(V, H, D_{50}, \sigma)$	0.515	0.3860	0.669	0.2731
$y_0 = f(D, H, D_{50}, \sigma)$	0.634	0.2832	0.723	0.2399
$y_0 = f(D, V, D_{50}, \sigma)$	0.600	0.0042	0.735	0.1991
$y_0 = f(D, V, H, \sigma)$	0.577	0.3834	0.708	0.2382
$y_0 = f(D, V, H, D_{50})$	0.645	0.3520	0.751	0.1874

5. Conclusion

Aiming at the characteristics of the existing normative formula to predict the erosion depth of piers with low accuracy and many and complex external factors, based on the collected model test data as samples, the correlation analysis and sensitivity analysis methods were used to analyze the factors affecting the erosion depth of piers, and it was found that the correlation between pier diameter and local erosion depth of piers was the strongest, followed by water flow depth, and the correlation between median particle size and local erosion depth of piers was the smallest. In order to accurately predict the maximum depth of local erosion of bridge piers, the prediction model of local erosion depth of bridge piers based on the least squares support vector machine method is improved, and the

coefficient of determination of the prediction model is increased from 0.624 to 0.824 after dimensionless processing, and the prediction results show that the prediction accuracy of the model is high, and it is feasible and effective to use the model for the prediction of local erosion depth of bridge piers.

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