

PI α D β Controller with Neural Network in Active Magnetic Bearing

Zhongqiao Zheng

School of Electrical and Information Engineering, Changzhou Institute of Technology, Changzhou, Jiangsu, China

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Abstract: Active magnetic bearing control system has some characteristics of complexity, nonlinear and strong coupling, it is difficult to get better control effect by general PID controller. As a result, a new type of PI α D β controller with neural network is proposed. A mathematical model of active magnetic bearing is constructed. Construction of a fractional order controller with BP neural network is given. On the basis of working state of control system, five parameters of controller are adjusted to make active magnetic bearing system more accurate by self-learning and parallel processing capability of BP neural network. Experiment results indicate that when system parameters change, the system of PI α D β controller with neural network reaches steady state after 0.1s, when the external disturbance is suddenly added to system, the response time of PI α D β controller with neural network is 0.035s. The rapidity, stability, accuracy and anti-interference ability of PI α D β controller with neural network is obviously superior to other two kinds of control system.

1. Introduction

Working condition of an active magnetic bearing is complicated, some uncertain factors exist, it is difficult to control stability of the active magnetic bearing as in [1-3]. Performance of a controller affects directly whether the magnetic bearing can operate stably. Generally, there are two kinds of control methods, one is linear control of conventional PID and state feedback, the other is nonlinear control such as neural network and fuzzy control. For a control system with complex nonlinear and strong coupling, the traditional PID controller can't achieve better result. Therefore, the PI α D β controller with Neural Network is put forward, where. When, PI α D β controller becomes the traditional PID controller. There are more two adjustable parameters of PI α D β controller than traditional PID, so the parameter adjustment range becomes larger, which can more flexibly and precisely control the controlled object. BP neural network is a kind of algorithm with strong nonlinearity, adaptive, self-organizing and self-learning ability. The optimization method based on gradient descent is adopted, which makes the total error minimize by adjusting the weight coefficient as in [4-6].

In order to promote rapidity, robustness and accuracy of active magnetic bearing system and the parameter accuracy of PI α D β controller, a new kind of method is put forward that combines BP neural network model and PI α D β controller, it makes full use of the advantages of PI α D β controller and BP neural network, the dynamic characteristics of the active magnetic bearing is improved by adjusting parameter online as in [7-8].

2. Structure and Mathematical Model of an Active Magnetic Bearing Control System

An active magnetic bearing control system mainly includes radial magnetic bearings, axial magnetic bearings, displacement sensors, touchdown bearings, a controller, power amplifiers, a rotor and a stator, which is shown in Figure 1.

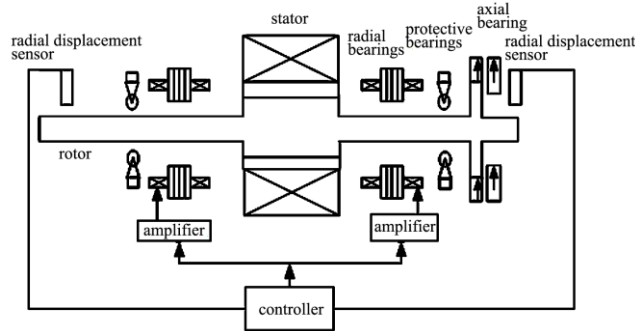


Figure 1: Structure of an active magnetic bearing

A control chart of the active magnetic bearing with single freedom is given as Figure 2.

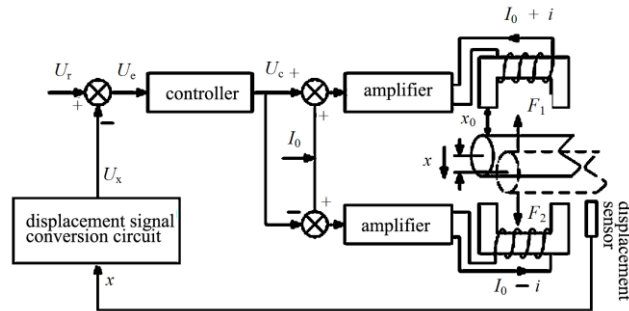


Figure 2: Control chart of active magnetic bearing with single freedom

Where, U_r is reference voltage value of input signal, U_x is voltage value of feedback signal, U_c is deviation value between input signal and output signal, U_c is output value of controller, I_0 is the reference value of bias current, F_1 is an electromagnetic force from above electromagnet, F_2 is an electromagnetic force from under electromagnet, x_0 is a distance between rotor and upper electromagnet when the rotor is in the balance position, x is the distance of the rotor from the balance center.

The mass of rotor is for M . When the rotor is in the balance position, it is in the middle of the upper and lower electromagnets. If only all force acting on rotor in the up and down directions is considered, when the rotor of active magnetic bearing with one degree of freedom is the balanced position between the upper and the lower magnet, the following assumptions are made.

(1) Reluctance of iron core and rotor is ignored, that is, the magnetic potential is only considered in air gap.

(2) Leakage flux of the winding is neglected and the magnetic force is uniformly distributed in the magnetic circuit.

(3) Eddy current effects and hysteresis of magnetic materials are not considered.

Assume that an interference signal appears at a certain moment and makes the rotor off-balance position, and the distance from the balance position is x , as shown in Figure 2. By adding a control current i , which makes the electromagnetic force from above electromagnet increase, meanwhile, the electromagnetic force from under electromagnet decreases, forming an upward resultant force, it makes the rotor return to the original balance position. The electromagnetic resultant force from two electromagnets is shown below.

$$F = F_1 - F_2 = \frac{\mu_0 SN^2}{4} \left[\frac{(I_0 + i)^2}{(x_0 + x)^2} - \frac{(I_0 - i)^2}{(x_0 - x)^2} \right] \quad (1)$$

In above formula, μ_0 is for vacuum permeability, whose unit is H/m, N is for the number of windings of solenoid, S is for a section area of single pole, whose unit is mm^2 . The resultant force on rotor is calculated as follows.

$$F = ma = m \frac{d^2 x}{dt^2} \quad (2)$$

Where, F is for the electromagnetic force on the rotor, mass of the rotor is expressed by m , the offset of rotor is expressed by x . The equation can be obtained as follows.

$$\frac{\mu_0 SN^2}{4} \left[\frac{(I_0 + i)^2}{(x_0 + x)^2} - \frac{(I_0 - i)^2}{(x_0 - x)^2} \right] = m \frac{d^2 x}{dt^2} \quad (3)$$

According to formula 3, the mathematical expression of the control system is a non-linear function related to the offset displacement and the control current. It is very laborious to solve it directly. In actual calculation, the non-linear mathematical expression is generally linearized and converted into a linear system for solution. Therefore, according to the Taylor formula, the electromagnetic force is linearized near the equilibrium position, and the converted equation is below.

$$F(i, x) \approx -k_i \cdot i + k_x \cdot x \quad (4)$$

In the formula 4,

$$k_i = \frac{\mu_0 SN^2 i}{x_0^2}, k_x = \frac{\mu_0 SN^2 i^2}{x_0^3} \quad (5)$$

k_i is a current coefficient, k_x is a displacement coefficient.

According to (2) and (4), the following formula can be deduced.

$$-k_i \cdot i_c + k_x \cdot x = m\ddot{x} \quad (6)$$

Laplace transform for formula (6), according to transfer function is given.

$$G(s) = \frac{X(s)}{I(s)} = \frac{-k_i}{ms^2 - k_x} \quad (7)$$

It is known from (7) that one characteristic root locates in right half plane of s plane, which makes the system unstable, so it is necessary to add a controller to make the system stable.

3. Control Algorithm

The PI α D β controller with BP network directly controls rotor, five parameters of the controller is

adjusted and output by BP neural network as in [9-11]. The optimization of the five parameters such as K_p , K_i , K_d , α , β are important in $PI^\alpha D^\beta$ controller, so control objective is to find the optimal combination and realize better control effect as in [12-15]. Construction of $PI^\alpha D^\beta$ controller with BP Neural Network is represented in Figure. 3, Input and output signal of controller is for e and u respectively.

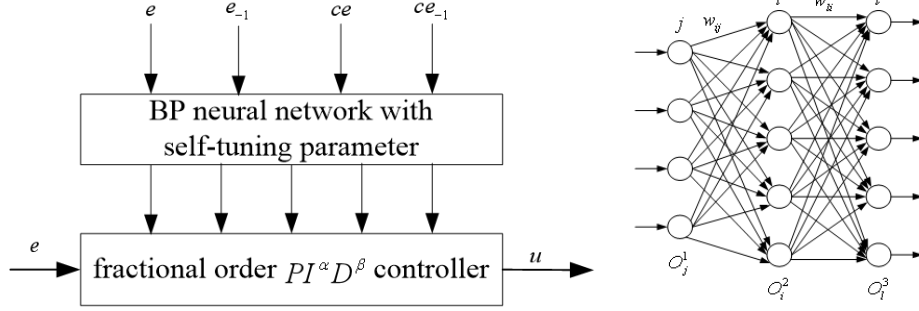


Figure 3: Construction of $PI^\alpha D^\beta$ controller with BP neural network

Construction of above Neural Network is for 4-5-5, the node of input layer is e , $e-1$, ce , $ce-1$, e is for an error at current moment, $e-1$ is for an error at previous moment, ce is for an error change rate at current $ce-1$ is for an error change rate at previous moment, five adjustable parameters of $PI^\alpha D^\beta$ controller are all nodes of output layer, namely K_p , K_i , K_d , α , β , the hidden layer consists of five nodes.

The input signal and output signal of j -th node in input layer is as follows.

$$X = (x_1, x_2, x_3, x_4) = (e, e_{-1}, ce, ce_{-1}) \quad (8)$$

$$O_j^1 = x_j(k), j = 1, 2, 3, 4 \quad (9)$$

Input signal and output signal from i -th node in hidden layer is as below.

$$net_i^2(k) = \sum_{j=1}^4 w_{ij}^2(k) O_j^1(k), i = 1, 2, 3, 4, 5 \quad (10)$$

$$O_j^2(k) = f(net_i^2(k)), j = 1, 2, 3, 4, 5 \quad (11)$$

The plus and minus symmetrical sigmoid function is used as an incentive function in hidden layer neurons, its expression is as follows.

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad (12)$$

Input signal and output signal from l -th in output layer neuron is calculated as below.

$$net_l^3(k) = \sum_{i=1}^5 w_{li}^3(k) O_i^2(k), l = 1, 2, 3, 4, 5 \quad (13)$$

$$O_l^3(k) = g(net_l^3(k)), l = 1, 2, 3, 4, 5 \quad (14)$$

$$O^3 = (O_1^3, O_2^3, O_3^3, O_4^3, O_5^3) = (K_p, K_i, K_d, \alpha, \beta) \quad (15)$$

Where, w_{ij} is a weight coefficient between input and hidden layer, w_{li} is a weight coefficient

between hidden and output layer.

Considering Kp , Ki , Kd isn't usually negative, and $0 \leq \alpha, \beta \leq 1$, so the excitation function of output layer neuron is realized by non-negative sigmoid function, its expression is as below.

$$g(x) = \frac{e^x}{e^x + e^{-x}} \quad (16)$$

Error evaluation index is calculated as formula (17).

$$e(k) = \frac{1}{2} [y_d(k) - y(k)]^2 \quad (17)$$

Where, y_d is for an ideal output signal. On the basis of the gradient descent method, a expression for weighting factor of every output layer is as follows.

$$\Delta w_{ii}^3(k) = -\eta \frac{\partial E(k)}{\partial w_{ii}^3(k)} + \lambda \Delta w_{ii}^3(k-1) \quad (18)$$

$$\frac{\partial E(k)}{\partial w_{ii}^3(k)} = \frac{\partial E(k)}{\partial y(k)} \cdot \frac{\partial y(k)}{\partial u(k)} \cdot \frac{\partial u(k)}{\partial O_i^3(k)} \cdot \frac{\partial O_i^3(k)}{\partial net_i^3(k)} \cdot \frac{\partial net_i^3(k)}{\partial w_{ii}^3(k)} \quad (19)$$

$$\frac{\partial E(k)}{\partial y(k)} = 2 \times \frac{1}{2} [y_d(k) - y(k)](-1) = -e(k) \quad (20)$$

$$\frac{\partial O_i^3(k)}{\partial net_i^3(k)} = \frac{\partial g[net_i^3(k)]}{\partial net_i^3(k)} = -2g[net_i^3(k)]\{1 - g[net_i^3(k)]\} \quad (21)$$

$$\frac{\partial net_i^3(k)}{\partial w_{ii}^3(k)} = O_i^2(k) \quad (22)$$

Where, η is for learning rate, λ is for inertial factor. $\partial y(k)/\partial u(k)$ is determined by $\text{sgn}[\partial y(k)/\partial u(k)]$, above partial derivative is substituted into (18), the correction formula for the weight coefficient from output layer to hidden layer is as formula (23).

$$\Delta w_{ii}^3(k) = \lambda \Delta w_{ii}^3(k-1) - \eta \delta_i^3 O_i^2(k) \quad (23)$$

Where

$$\delta_i^3 = e(k) \text{sgn} \left(\frac{\partial y(k)}{\partial u(k)} \right) \left(\frac{\partial u(k)}{\partial O_i^3(k)} \right) \{ 2g[net_i^3(k)] [1 - g[net_i^3(k)]] \} \quad (24)$$

When the error is back propagated from hidden layer to input layer, a correction formula of weight coefficient is calculated as follows.

$$\Delta w_{ij}^3(k) = -\eta \frac{\partial E(k)}{\partial w_{ij}^3(k)} + \lambda \Delta w_{ij}^3(k-1) \quad (25)$$

By (9) ~ (13), the following formula is deduced.

$$\begin{aligned} \frac{\partial E(k)}{\partial w_{ij}^3(k)} &= \sum_{l=1}^5 \frac{\partial E(k)}{\partial y(k)} \cdot \frac{\partial y(k)}{\partial u(k)} \cdot \frac{\partial u(k)}{\partial O_l^3(k)} \cdot \frac{\partial O_l^3(k)}{\partial net_l^3(k)} \cdot \frac{\partial net_l^3(k)}{\partial O_l^2(k)} \cdot \frac{\partial O_l^2(k)}{\partial net_l^2(k)} \cdot \frac{\partial net_l^2(k)}{\partial w_{ij}^2(k)} \\ &= \sum_{l=1}^5 \delta_l^3 \cdot \frac{\partial net_l^3(k)}{\partial O_l^2(k)} \cdot \frac{\partial O_l^2(k)}{\partial net_l^2(k)} \cdot \frac{\partial net_l^2(k)}{\partial w_{ij}^2(k)} \end{aligned} \quad (26)$$

$$\frac{\partial net_l^3(k)}{\partial O_l^2(k)} = w_{li}^3(k) \quad (27)$$

$$\frac{\partial O_l^2(k)}{\partial net_l^2(k)} = \frac{\partial f[net_l^2(k)]}{\partial net_l^2(k)} = 1 - f^2[net_l^2(k)] \quad (28)$$

$$\frac{\partial net_l^2(k)}{\partial w_{ij}^2(k)} = O_j^1(k) \quad (29)$$

(27) ~ (29) is substituted into (26), the following formula is gained.

$$\frac{\partial net_l^2(k)}{\partial w_{ij}^2(k)} = \sum_{l=1}^5 \delta_l^3 \cdot w_{li}^3(k) \cdot \{1 - f^2[net_l^2(k)]\} \cdot O_j^1(k) \quad (30)$$

(30) Is substituted into (25), the correction formula of weight coefficient in hidden layer is calculated therefore.

$$\Delta w_{ij}^2(k) = \lambda \Delta w_{ij}^2(k-1) - \eta \delta_i^2 O_j^1(k) \quad (31)$$

Where

$$\delta_i^2 = \{1 - f^2[net_i^2(k)]\} \sum_{l=1}^5 \delta_l^3 \cdot w_{li}^3(k), i = 1, 2, 3, 4, 5 \quad (32)$$

The inertia coefficient λ is for 0.9, learning rate η is for (0.01, 1].

The principle diagram of PI α D β controller with BP neural network is displayed as Figure. 4.

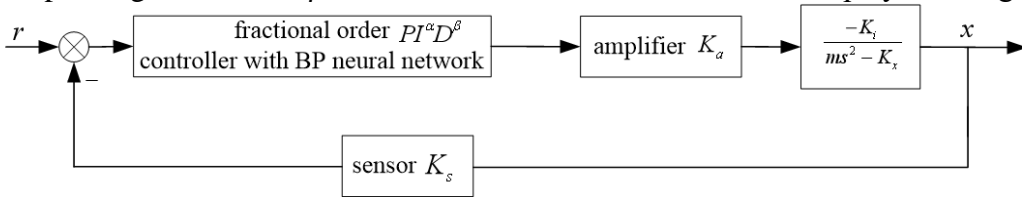


Figure 4: Principle diagram of PI α D β controller with BP Neural Network

Summary of algorithm is below.

- (1) Initialization. Ensure the amount of each layer neuron node, initialize the weight coefficient $w_{ij}^2(k)$ and $w_{li}^3(k)$, the initial learning rate η , inertial coefficient λ and $k=1$.
- (2) Calculate the error.
- (3) According to (7) ~ (15), calculate input signal and output signal of every network layer.
- (4) Calculate the output signal $u(k)$ of controller.
- (5) On the basis of formula (22) and (31), learning by neural network and self-adjusting weight

coefficient $w_{ij}^2(k)$ and $w_i^3(k)$, finally realizing adaptive adjustment of Kp , Ki , Kd , α , β .

(6) $k = k+1$, return to step (2), stop the loop when it's iterations reaches the maximum.

Input signal of BP neural network is error and error change rate, the output signal is five parameters of $PI\alpha D\beta$ controller.

4. Results and Discussion

The active magnetic bearing control system is simulated by MATLAB, when the relative error is greater than 0.05, the parameters are firstly adjusted by BP neural network, then the adjusted parameters are sent to controller, learning rate is for 0.28, inertial coefficient is for 0.9, input signal is unit step signal. When the rotor is at 0.25mm, an interference pulse signal is added at 500ms, its amplitude is 0.07 mm. The displacement response curve of electric spindle is given in Figure. 5, corresponding dynamic performance of the control system after disturbance is given as Figure. 6.

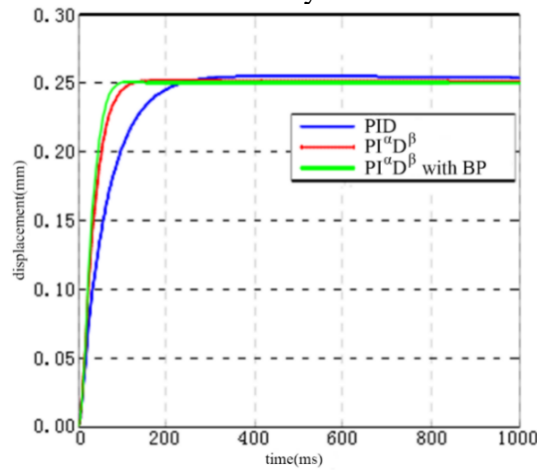


Figure 5: Displacement response curve of electric spindle

According to Figure. 5, when system parameters change, control system of $PI\alpha D\beta$ controller with neural network reaches steady state when it is 0.1s. The rapidity, stability and accuracy of system is obviously superior to other two kinds of control system.

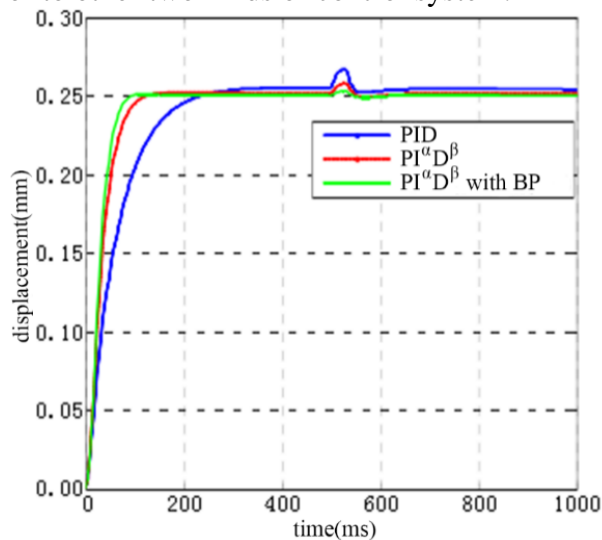


Figure 6: Comparison of anti-interference performance

According to Figure. 6, when the external disturbance is suddenly added to system, the response

time of $PI\alpha D\beta$ controller with neural network is 0.035s, it is superior to other two kinds of methods, its anti-interference ability is best. The detailed performance indicator from three controllers are displayed in Table 1.

Table 1: Performance indicator of system from three controllers

Type of controller	Performance index				
	Rise time (s)	Steady state time (s)	Over shoot	Steady state error (mm)	Anti-interference time (m)
conventional PID controller	0.15	0.25	0.25%	0.015	0.05
fractional order $PI\alpha D\beta$ controller	0.08	0.15	0.1%	0.01	0.04
fractional order $PI\alpha D\beta$ controller with BP neural network	0.06	0.1	0.05%	0.005	0.035

5. Conclusions

Because of the characteristic of nonlinear, uncertainty, open loop instability and disturbance signal in an active magnetic bearing control system, a $PI\alpha D\beta$ controller with BP neural network is designed. The designed controller is applied to the active magnetic bearing system and simulated by MATLAB, compared to a conventional PID controller, $PI\alpha D\beta$ controller and $PI\alpha D\beta$ controller with BP neural network, the designed controller can adjust parameters online, which makes the parameters of controller optimization and improves the stability and accuracy of the active magnetic bearing system, it can also suppress all interference signal more timely. From several simulation curve of control system with different controllers, $PI\alpha D\beta$ controller with BP neural network can significantly promote steady and dynamic performance of an active magnetic bearing control system.

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