

The Idiosyncratic Volatility Gap and Return Predictability

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Abstract: This paper studies the predictability of the idiosyncratic volatility GAP to the strategic returns of idiosyncratic volatility. Based on the data of A-share listed companies from 2001 to 2021, this paper uses single-factor and multi-factor models to empirically analyze the predictive effect of the idiosyncratic volatility GAP on idiosyncratic volatility strategies. The results show that the puzzle of idiosyncratic volatility does exist in China A-share market, and the idiosyncratic volatility GAP can positively predict the return of the idiosyncratic volatility strategy.

1. Introduction

As Jensen (1972) studies, there is a correlation between idiosyncratic volatility and stock returns, which challenges the traditional asset pricing model^[1]. Although many scholars have studied this anomaly, most of them are cross-sectional determinants of the idiosyncratic volatility, little is known about this time variation. In this paper, I document a new and significant determinant of variation in expected idiosyncratic volatility profits.

At the beginning of the month when a new idiosyncratic volatility portfolio is formed, an easily computed characteristic of the portfolio is the difference between high idiosyncratic volatility and low idiosyncratic volatility, which I term the idiosyncratic volatility GAP. The first part of this study focuses on the analysis of the forecast effect of idiosyncratic volatility GAP on the return of idiosyncratic volatility. After controlling the momentum factor, market premium, market value and book-to-market ratio, the predictive power of the idiosyncratic volatility GAP is remains significant economically and statistically. Under the condition of controlling these factors, the adjusted return of the idiosyncratic volatility strategy will decrease by 1.04% for a one-standard-deviation increase in the idiosyncratic volatility GAP. In addition, the idiosyncratic volatility GAP has significant predictive power for both the long and short strategies of the idiosyncratic volatility.

Finally, the effects of size and book-to-market ratio are introduced to ensure that the relationship between the idiosyncratic volatility GAP and the idiosyncratic volatility return obtained in this paper is robust.

2. Related Literature

The relationship between the idiosyncratic volatility and the expected future returns of stocks has been hotly debated. Some scholars argue that there is a positive correlation between the idiosyncratic volatility and the expected future returns of the stock. The standard CAPM model

assumes that all investors have timely and free access to sufficient market information. However, in reality, investors have asymmetric stock information and can only get information about a few stocks. The portfolio cannot completely disperse the non-systematic risks. Therefore, the idiosyncratic risks of stocks will also require corresponding compensation (Merton, 1987)^[2]. Fu(2009) argue that idiosyncratic volatility is time-varying, and uses the exponential GARCH model to estimate the expected idiosyncratic volatility. It finds that there is a significant positive correlation between the estimated conditional idiosyncratic volatility and the expected return. It also argue that the research results of Ang et al. can be explained to a large extent by the return reversal of some small stocks with high idiosyncratic volatility^[3]. Shi et al. (2012) argue that the undivided idiosyncratic risk can predict the market excess return, and there is a positive correlation between them due to the imperfect market and the existence of non-tradable human capital and corporate capital.

Some scholars argue that there is a negative correlation between the idiosyncratic volatility and the expected future returns of the stock. A key assumption of the standard CAPM model is cognate expectation, which assumes that all investors have the same estimate of the expected return and the probability distribution of return for all securities. However, in reality, the future is very uncertain and the predict is very difficult. Then, it is difficult to assume that everyone has made the same estimation on the return and risk of each security. In practice, the concept of uncertainty itself means that rational people may have different predictions. In view of this, assuming that there is a short selling restriction in the market, investors have different estimates of the return on investment in high-risk securities, so the demand for a particular security will come from the minority with the most optimistic expectations. Since heterogeneous beliefs are likely to increase with increasing risk, the expected return on high-risk securities is likely to be lower rather than higher (Miller, 1977)^[4]. Ang et al. (2006) used the Fama-French three-factor model to analyze the idiosyncratic volatility and found that the overall volatility risk premium was negative, and argue that this was due to the risk aversion factor which reduced the current consumption to increase precautionary savings when the uncertainty of future market returns was high. Further research finds that the total volatility risk in the stocks with high idiosyncratic volatility is very small. From this point, there should be a positive correlation between idiosyncratic volatility and the expected future earnings of the stocks. However, the research finds that the average return rate of the stocks with high idiosyncratic volatility is much lower. They call this phenomenon the puzzle of idiosyncratic volatility^[5]. Boehme et al. (2009) improved Merton's (1987) model, studied the cross-sectional effect of idiosyncratic volatility, and realized the mixed effect of belief dispersion and short selling constraint in Merton's model. It is found that the short selling restriction plays a decisive role in the relationship between the idiosyncratic volatility and the future returns of the stock. When there is a short selling restriction, the idiosyncratic risk is negatively correlated with the expected return, whereas when there is no short selling restriction, the two are positively correlated^[6]. Cao et al. (2010) decomposed volatility into long-term and short-term components. It is found that stocks with high long-term idiosyncratic risks have great future returns. In contrast, the short-term idiosyncratic risk component is negatively correlated with the stock return. The relationship between idiosyncratic volatility and the expected future returns of a stock depends on the dominance of the long-term and short-term components of the idiosyncratic risk^[7]. Li et al. (2014) applied CaoXuying's research to the China market and reached the same conclusion. Based on the prospect theory, Xie (2017) argues that Chinese stock investors are risk-biased, which results in the inverse relationship between idiosyncratic volatility and expected return basically holds regardless of whether the stock returns are in the profit or loss domain. Stambaugh et al. (2015) argues that buying is easier for many equity investors than shorting. They combine this arbitrage asymmetry with the arbitrage risk represented by the idiosyncratic volatility and find that the idiosyncratic volatility is positively

correlated with the average return in low-priced stocks and negatively correlated with the average return in high-priced stocks, and the rising investor sentiment weakens the positive correlation between low-priced stocks and strengthens the negative correlation between high-priced stocks, thus the negative correlation between the idiosyncratic volatility and the average return^[8]. Zhang and Ye (2019) argue that limited arbitrage is the reason why there is the puzzle of idiosyncratic volatility in the China market.

In view of the above theoretical analysis, it is found that there is no unified view on the relationship between idiosyncratic volatility and future returns of stocks. This paper studies the relationship between the idiosyncratic volatility GAP and expected return of idiosyncratic volatility by constructing the idiosyncratic volatility GAP.

3. Theoretical Analysis and Research Assumptions

This paper focuses on the research on idiosyncratic volatility, and uses Huang (2022) for reference to construct the idiosyncratic volatility GAP^[9]. Based on the data of A-share listed companies from 2001 to 2021, the paper studies the relationship between the idiosyncratic volatility GAP and the expected returns of idiosyncratic volatility strategy.

In the China A-share market, it is easier for many stock investors to buy than to short, and there is arbitrage asymmetry. For an undervalued stock, the more serious the mispricing, the lower the price relative to the intrinsic value of the stock, and therefore the higher the expected future return rate. Therefore, the idiosyncratic volatility is positively correlated with the future return rate; For an overvalued stock, the more serious the mispricing, the higher the price relative to the intrinsic value of the stock, and therefore the lower the expected future return rate, so the idiosyncratic volatility is negatively correlated with the future return rate; In an efficient market, mispricing can be eliminated by arbitrageurs. However, in reality, due to the existence of limited arbitrage, mispricing is difficult to eliminate. Due to the asymmetry of arbitrage, investors tend to arbitrage low-price stocks more fully, thus eliminating more pricing errors of undervalued stocks, resulting in the negative correlation between overvalued stock idiosyncratic volatility and future yield being stronger than the positive correlation between undervalued stock idiosyncratic volatility and future yield, thus generating the puzzle of idiosyncratic volatility. The GAP reflects the hypothetical performance of the idiosyncratic volatility strategy before the formation of the portfolio. The higher the idiosyncratic volatility is, the higher the arbitrage risk is. The more difficult it is to eliminate the mispricing. The negative correlation between overvaluation of the idiosyncratic volatility of the stock and the future yield is more obvious. Based on this, the following hypothesis is proposed.

Hypothesis: The puzzle of idiosyncratic volatility exists in China stock market, and there is a positive correlation between the idiosyncratic volatility GAP and the expected return of idiosyncratic volatility strategy.

4. Date and Methodology

The data in this paper are mainly collected from RESSET database. In order to verify the relevant viewpoints put forward in this paper, the period from 2001 to 2021 is selected as the research interval, studies the relationship between the idiosyncratic volatility GAP and expected return of idiosyncratic volatility.

The measure of idiosyncratic volatility is referenced by Ang et al. (2006). Firstly, the Fama-French three-factor model is regressed with monthly data of individual stocks, then the sample standard deviation of regression residuals is used to obtain the stock's idiosyncratic volatility for the month. In order to unify the units, this paper makes the series of standard deviations obtained monthly by multiplying the standard deviation by the square root of the trading day of the

current month. In this way, a measure of the stock's idiosyncratic volatility in the T-month can be obtained.

$$R_t = \alpha + \beta_{MKT}MKT_t + \beta_{SMB}SMB_t + \beta_{HML}HML_t + \varepsilon_t \quad (1)$$

$$IVOL_{i,t} = std(\varepsilon_{i,t}) \times \sqrt{T_{i,t}} \quad (2)$$

Among them, $IVOL_{i,t}$ represents idiosyncratic volatility, $std(\varepsilon_{i,t})$ represents the standard deviation of the residual error, $T_{i,t}$ represents the trading days of stock i in the t month.

In the T-month, stocks are sorted in ascending order by the magnitude of the idiosyncratic volatility, the idiosyncratic volatility GAP being defined as the difference between the 75th and 25th percentiles and recorded as IVG ; Defining the idiosyncratic volatility strategy as a portfolio of long the bottom 10% and short the top 10%, and defining its returns as LMW : The long strategy is defined as a portfolio of long the bottom 10% and short market portfolio, and the returns is defined as LMW^+ : The short strategy is defined as a portfolio of short the bottom 10% and long market portfolio, and its returns are defined as LMW^- . In addition, the return on idiosyncratic volatility adjusted by Fama and French (1993) is defined as the sum of the fitted values of α and ε_t in the full-cycle regression, and is recorded as LMW_α :

$$\begin{aligned} LMW_t &= \alpha + \beta_{MKT}MKT_t + \beta_{SMB}SMB_t + \beta_{HML}HML_t + \varepsilon_t \\ LMW_{\alpha,t} &= \alpha + \varepsilon_t \end{aligned} \quad (3)$$

Replace with LMW^+ and LMW^- respectively to obtain LMW_α^+ and LMW_α^- respectively.

In order to test the relationship between the idiosyncratic volatility GAP and returns of idiosyncratic volatility, this paper uses single-factor and multi-factor models to test hypothesis. In addition, in order to solve the problem that the residuals have heteroscedasticity or autocorrelation, the standard error with consistent autocorrelation and heteroscedasticity is used for adjustment (Newey&West.1986)^[10]. Set the model as follows:

$$LMW_t = \alpha + \beta_1 IVG_{t-1} + \beta_2 MoM_{t-1} + \beta_3 MKT_{t-1} + \beta_4 SMB_{t-1} + \beta_5 HML_{t-1} + \varepsilon_t \quad (4)$$

In model (4), IVG represents the idiosyncratic volatility GAP, MoM represents momentum factor, MKT represents market premium, SMB represents market value and HML represents book-to-market ratio. T represents time (months). LMW represents the explained variable "the returns of idiosyncratic volatility strategy". You can replace LMW with LMW_α , LMW_α^+ , and LMW_α^- in turn in the model. ε_t is a random perturbation term. After obtaining the relationship between the idiosyncratic volatility GAP and the idiosyncratic volatility returns, we tested the robustness of the forecast model.

5. Results

5.1 Descriptive Statistics and Correlation Test

As can be seen from Table 1, the average return of the idiosyncratic volatility strategy has reached 2.0%, which to some extent can explain the existence of the puzzle of idiosyncratic volatility in China's A-share market. In addition, the returns of the idiosyncratic volatility strategy constructed in the China A-share market did not show negative skewness, which indicates that the magnitude of the idiosyncratic volatility strategy crash was low or there was no crash. More importantly, IVG is positively correlated with the earnings of the idiosyncratic volatility strategy and the adjusted returns.

Table 1: Descriptive Statistics and Correlation Test

Panel A: Descriptive Statistics					
	LMW	LMW_{α}	LMW_{α}^{+}	LMW_{α}^{-}	IVG
Mean	0.020	0.020	0.009	0.011	0.045
SD	0.051	0.047	0.033	0.029	0.013
Skewness	0.626	0.629	2.635	-0.123	0.061
Kurtosis	4.278	3.330	17.585	0.960	0.113
Panel B: Correlation Test					
	LMW	LMW_{α}	LMW_{α}^{+}	LMW_{α}^{-}	
IVG	0.181***	0.244***	0.182***	0.197***	

Note: *** (**, *) are significant at 1% (5%, 10%) respectively.

5.2 The Regression Results

Table 2: Regression Analysis

		IVG	$Moment$	MKT	SMB	HML
LMW	(1)	0.732*** (3.24)				
	(2)		0.054 (0.64)	0.104*** (2.74)	0.248** (2.00)	0.003 (0.02)
	(3)	0.627*** (2.94)	0.036 (0.44)	0.102*** (2.85)	0.230* (1.94)	0.014 (0.09)
LMW_{α}	(1)	0.904*** (3.90)				
	(2)		0.043 (0.54)	0.151*** (3.80)	0.186* (1.67)	-0.014 (-0.11)
	(3)	0.803*** (3.73)	0.020 (0.26)	0.148*** (4.05)	0.163 (1.54)	0.000 (0.00)
LMW_{α}^{+}	(1)	0.464** (2.20)				
	(2)		0.029 (0.46)	0.070* (1.83)	0.156* (1.79)	0.025 (0.32)
	(3)	0.402** (2.24)	0.017 (0.29)	0.069* (1.86)	0.145* (1.73)	0.032 (0.42)
LMW_{α}^{-}	(1)	0.440*** (3.96)				
	(2)		0.015 (0.34)	0.081*** (3.30)	0.030 (0.62)	-0.039 (-0.61)
	(3)	0.401*** (3.55)	0.003 (0.07)	0.080*** (3.34)	0.018 (0.41)	-0.032 (-0.54)

Note: *** (**, *) are significant at 1% (5%, 10%) respectively.

Regression is performed according to model (4) and the results are shown in Table 2. The first column in Table 2 is the explained variable, and the (1) is the result of single-factor regression, and the explained variable is IVG ; The (2) is the result of multi-factor regression, the explanatory variables are momentum factor $Moment$, market premium MKT , market value SMB and book-to-market ratio HML ; The (3) is the result of multi-factor regression, the explanatory variables are IVG , momentum factor $Moment$, market premium MKT , market value SMB and book-to-market ratio HML . The results show that the past momentum factor ($Moment$), market value (SMB) and book-to-market ratio (HML) are not significant in predicting the returns of idiosyncratic volatility strategies, long and short strategies. It shows that the past momentum factor ($Moment$), market value (SMB) and book-to-market value ratio (HML) are not useful predictors of the returns of idiosyncratic volatility

strategy of China A-share market. On the contrary, the forecast of the returns of idiosyncratic volatility using *IVG* is significant at 1% significance level, regardless of whether other variables are added. This evidence indicates that *GAP* can significantly predict the returns of idiosyncratic volatility regardless of whether the other four variables are controlled.

5.3 Impact of Size and Book-to-Market Ratio

In this section, the effects of size and book-to-market ratio are introduced to ensure that the relationship between the idiosyncratic volatility *GAP* and the idiosyncratic volatility return obtained in this paper is robust.

Size factor—the total market value (*SMB*) of a company, and the size factor is selected as the total market value of the company. In the month of *T*, the shares are ranked in descending order of the total market value of the company, with the first half as the large-scale market value group (*B*) and the last half as the small-scale market value group (*S*). For the large-scale market value group and the small-scale market value group, the idiosyncratic volatility strategy is constructed and the returns are recorded as LMW_B and LMW_S respectively. Then the return on the scale-adjusted idiosyncratic volatility strategy is:

$$LMW_{SMB} = \frac{1}{2}(LMW_B + LMW_S)$$

Value factor—book-to-market value ratio (*HML*), which is used to calculate the book-to-market value ratio in Year *t-1*. The obtained calculation results are sorted in descending order, and divided into Group *H*, Group *M* and Group *L* of high, medium and low book-to-market value ratio according to the proportion of the top 30%, middle 40% and back 30%. The idiosyncratic volatility strategies are constructed for each group with yields of LMW_H , LMW_M and LMW_L respectively. The return on the idiosyncratic volatility strategy adjusted for book-to-market ratio would then be:

$$LMW_{HML} = \frac{1}{3}(LMW_H + LMW_M + LMW_L)$$

Using LMW_{SMB} and LMW_{HML} as dependent variables, the regression analysis was performed according to model (4). The results are shown in Table 3. They are basically consistent with the results in Table 2, and the idiosyncratic volatility *GAP* is an important predictor.

Table 3: Impact of Size and Book-to-Market Ratio

		<i>IVG</i>	<i>Moment</i>	<i>MKT</i>	<i>SMB</i>	<i>HML</i>
LMW_{SMB}	(1)	0.993*** (4.77)				
	(2)		0.021 (0.24)	0.095** (2.16)	0.238* (1.90)	-0.043 (-0.26)
	(3)	0.896*** (4.40)	-0.005 (-0.06)	0.091** (2.24)	0.213* (1.82)	-0.028 (-0.18)
LMW_{HML}	(1)	0.592** (2.52)				
	(2)		-0.014 (-0.14)	0.103* (1.64)	0.213* (1.70)	-0.053 (-0.33)
	(3)	0.499** (2.05)	-0.029 (-0.28)	0.101* (1.65)	0.199* (1.64)	-0.045 (-0.29)

Note: *** (**, *) are significant at 1% (5%, 10%) respectively.

6. Conclusion

This paper examines the impact and of the idiosyncratic volatility GAP as an important forecast variable on the expected return of idiosyncratic volatility. Through the in-sample research of China A-share market using single-factor and multi-factor models, it is found that GAP can significantly predict the expected return of idiosyncratic volatility in China's stock market. Finally, the mechanism of the idiosyncratic volatility GAP as an important forecast variable on the expected return of idiosyncratic volatility and the source of the idiosyncratic volatility GAP will be the next research direction.

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