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# *Pricing research on CSI 300 stock index options based on B-S model and GARCH model*

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***Keywords:*** CSI 300 stock index, B-S model, GARCH model, option pricing

***Abstract:*** The development of the financial market has always been accompanied by a variety of risks, and with the continuous improvement of the financial market and the increase of investor demand, the society's demand for risk avoidance tools is also increasing, and CSI 300 stock index options as a new type of financial derivatives can largely make up for the vacancy of China's options market, in risk control and price discovery can play a huge role. Therefore, it is very important to study its functions and predict the price. This paper uses the option pricing model B-S model as the pricing basis and combines the GARCH model to analyze the volatility of the CSI 300 Index yield, so as to complete the construction of the pricing model and predict the future trend of option prices and analyze the effectiveness of model pricing. Finally, according to the empirical results, some suggestions are provided on the accurate pricing of options, so as to promote the healthy development of the options market.

## 1. Introduction

China's futures market has greatly improved its comprehensive strength through reform and innovation in recent years, and its role in China's economic activities has become more and more prominent. The emergence of CSI 300 stock index options can help investors diversify price risks and avoid external uncertainty, thereby improving the efficiency of capital use. The emergence of derivatives also urges the progress of financial markets and the improvement of policies and regulations. The financial market has always been changeable, a variety of trading varieties emerged, as well as public expectations, government policies and other factors caused by frequent fluctuations in financial asset prices, have brought huge risks to the financial market, and options as a vital member of the risk management tool, through the theoretical research on the pricing of options can not only promote the innovation of financial theory, but also can be applied in real practice to reduce the risk of the financial market.

CSI 300 Stock Index Options21 February 23, 2019 listed on CICC, which is an option based on the CSI 300 Index as the underlying object, now more and more investors have joined it, it is on China's finance Market risk control, optimization of financial market structure, and improvement of financial market investment environment have made great contributions. With the discovery of a large number of studies, the return of stocks does not follow a strict normal distribution, and its variance will continue to change, and the GARCH model has a good fitting effect for such cases [1], this article applies historical volatility to the B-S model separately from the volatility predicted by the GARCH model , the corresponding fitting results are obtained, and then the results of the two are compared and analyzed to judge the fitting effect of the model.

## 2. Overview of the B-S model and its related theory

### 2.1 B-S model

At the beginning of the research on option pricing, because the analysis of investor preferences, expected returns of stocks and other factors could not be avoided in various theories at that time, problems such as inaccurate estimation occurred when selecting appropriate parameters, which was also an important reason that has been plaguing the promotion of option pricing models, but the B-S model solved these problems well and greatly promoted the development and progress of option pricing theory. Black and Scholes (1973) [2]. By assuming that the price of a stock conforms to a lognormal distribution under complete market conditions, both the price of the option and the price of the stock are perturbed by a random term. Investors can choose to short sell a certain percentage of stocks in the investment and buy an option to build a risk-free portfolio, and the risk-free return is the return of this portfolio per unit of time. This theoretical model solves the problem that investor preferences in option pricing cannot be accurately estimated. Therefore, Black and Scholes achieved a simple solution to option pricing in a risk-neutral world by constructing a risk-free portfolio[3].

Taking the non-dividend-free European call option price as an example, the option pricing formula is:

$$C = SN(d_1) - Ke^{-r(T-t)}N(d_2), \tag{1}$$

Thereinto:

$$d_1 = \frac{ln\left(\frac{S}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \tag{2}$$

$$d_2 = \frac{ln\left(\frac{S}{K}\right)+\left(r-\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}. \tag{3}$$

S is the current share price, K is the strike price of the option, T is the time of expiration, r is the risk-free yield, $\sigma$ and is the stock price volatility.

### 2.2 Volatility Model

#### 2.2.1 Historical volatility

Historical volatility is the simplest way to calculate volatility, which is estimated from the historical data of the price of the underlying asset, and the most common historical volatility estimation method is the standard deviation method. Under the assumption of geometric Brownian motion, volatility is constant and is the annualized standard deviation of the stock's logarithmic return.$\sigma$

Suppose the price on the nth day is $S_n$, then the logarithmic payoff of the underlying is:

$$\mu_n = ln\left(\frac{S_n}{S_{n-1}}\right) \tag{4}$$

$\sigma^2$ expressed as the variance rate of the nth day estimated based on the variables before the nth day, The unbiased estimate is:

$$\sigma^2 = \frac{\sum_{n=1}^{m}(\mu_n - \bar{\mu})^2}{m-1} \tag{5}$$

Thereinto $\bar{\mu} = \frac{1}{n}\sum \mu_n$, expressed as the average yield of n days.

### 2.2.2 ARCH model

Robert. Engel proposed the autoregressive conditional heteroscedasticity model, also known as the A RCH model, in Econometrics in 1982, which solved the problem caused by the constant variance of time series variables in traditional econometric research. This model proposed a volatility theory model for the first time, which believes that the variance of a time series is a time-related variable, not a constant, and the model obtains the variance by constructing a linear combination of the squares of the time series of the sample.

The ARCH model of the yield series is represented as

$$u_t = \sigma_t \varepsilon_t, \tag{6}$$

Formula, $\varepsilon_t \sim N(0,1)$, and $\varepsilon_t$ Independent homogeneous distribution. The standard deviation form is as follows.

$$\sigma_t^2 = a_0 + \sum_{i=1}^{m} a_i u_{t-i}^2, \tag{7}$$

establish $\prod_{t-1} = \sigma(u_{t-1}, u_{t-2}, \ldots)$ represents the space in which all information is generated before t-1,so

$$E(u_t^2 | \prod_{t-1}) = E(\sigma^2 \varepsilon_t^2 | \prod_{t-1}) = \sigma^2 E(\varepsilon_t^2 | \prod_{t-1}), \tag{8}$$

So the conditional variance is generated based on the past value of the return. For current volatility, the closer the data, the greater the impact, and the greater the corresponding weight.

### 2.2.3 GARCH model

If the concept of the A RCH model is generalized to the generalized ARCH model, that is, the G ARCH model, a G ARCH model can be expressed in the following form:

$$y_t = f(F_{t-1}) + u_t \tag{9}$$

$$u_t = \sigma_t \varepsilon_t \tag{10}$$

$$\sigma_t^2 = a_0 + \sum_{i=1}^{m} a_i u_{t-i}^2 + \sum_{j=1}^{n} b_j \sigma_{t-j}^2, \tag{11}$$

We call the above equation the GARCH(m,n) process, where the unconditional mean is 0 and its unconditional variance is:$u_t$

$$Eu_t^2 = \frac{a_0}{1-(\sum_{i=1}^{m} a_i + \sum_{j=1}^{n} b_j)} \tag{12}$$

In order to maintain the stability of the conditional variance and ensure that the variance is positive, the parameters of the GARCH model have non-negative and bounded limitations, namely: $a_0 > 0, a_i \geq 0 (i = 1,2, \ldots, n)$ *and* $b_j \geq 0 (j = 1,2, \ldots, m)$

Meantime, $\sum_{i=1}^{m} a_i + \sum_{j=1}^{n} b_j < 1$.

This model can explain the volatility aggregation of financial data and the phenomenon of thick tail, because the GARCH(1,1) model is simple in form and can solve the problem of heteroscedasticity in most financial data, so it is widely used [4].

The GARCH(1,1) model form is:

$$\sigma_t^2 = a_0 + a_1 u_{t-1}^2 + b_1 \sigma_{t-1}^2 \tag{13}$$

## 3. Empirical analysis of CSI 300 stock index options

### 3.1 Data selection and processing

In this paper, the CSI 300 Index is selected from January 04, 2002 to February 3 1, 2021 The daily closing price of the day, including a total of 4853 observations, is sourced from NetEase Finance. Write the sample sequence as $\{y_t\}$. The logarithmic return is calculated using the daily closing price of the CSI 300 stock index. Python was used to process the above data to obtain the following CSI 300 Index closing price trend and logarithmic yield chart, as shown in Figure 1:

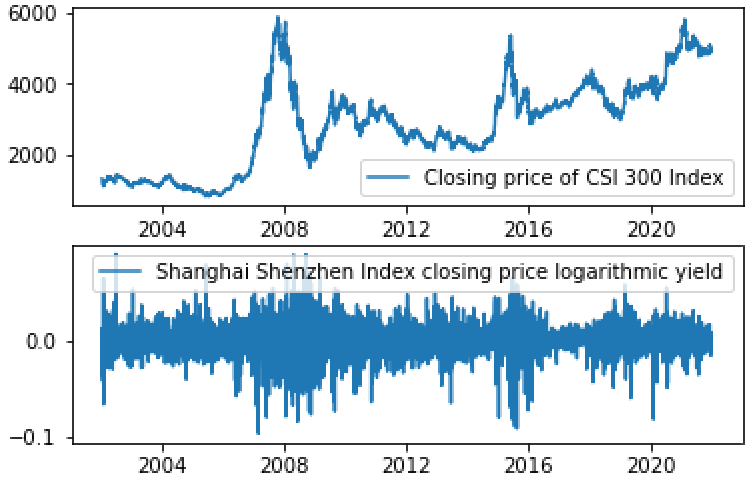


Figure 1 Timing diagram

From the logarithmic return graph of the CSI 3 00 index, it can be seen that there is a relatively obvious phenomenon of "fluctuation aggregation" in the sample series, from which we can simply judge that the daily return series of CSI 300 has an ARCH effect.

### 3.2 B-S model pricing under historical volatility

The sample data can be calculated using the historical volatility calculation formula using Python to obtain $\sigma_0$= 0.0118, since in the B-S model, variables such as volatility and time need to maintain a unified unit, so we need to find the annualized volatility, then

$$\sigma = \sigma_0 * \sqrt{250} = 0.18 \tag{14}$$

Take K=4900, $\sigma$ =r=0.0271, 0.18, and pass the B-S model to 2 January 4, 021 Simulated pricing of IO2112-C-4900 option contracts between February 1 and February 1 7, 2021, to obtain the actual price and simulated price data of the CSI 3 00 stock index option contract during this period. The comparison results in Figure 2 below.

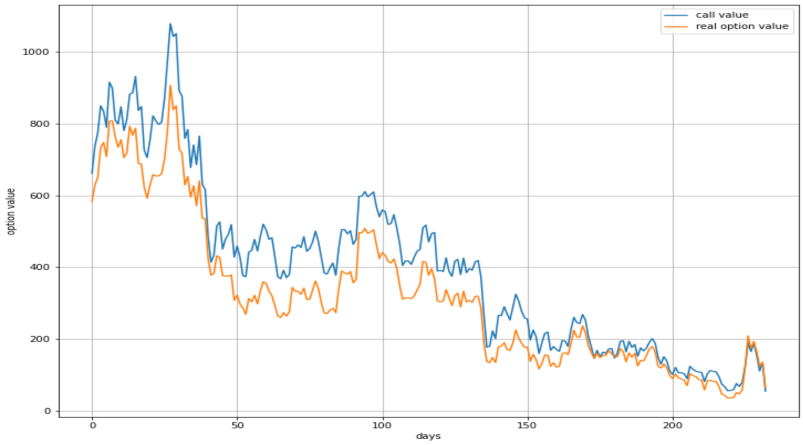


Figure 2 Price comparison under BSM model

## 3.3 Pricing Analysis of GARCH (1,1) Model

### 3.3.1ARCH model test and results

This article uses the CSI 300 Index from January 04, 2002 to February 3, 2021. The daily closing price of the day, using data from 0 0 0 0 0 January 2002 to 3 1 June 2021 using ARCH The model is analyzed, and the resulting model parameters predict volatility from July 1, 2021 to February 3, 2021.

Constant Mean - ARCH Model Results

| | | | |
|---|---|---|---|
| Dep. Variable: | close | R-squared: | 0.000 |
| Mean Model: | Constant Mean | Adj. R-squared: | 0.000 |
| Vol Model: | ARCH | Log-Likelihood: | 13198.4 |
| Distribution: | Normal | AIC: | -26390.7 |
| Method: | Maximum Likelihood | BIC: | -26371.3 |
| | | No. Observations: | 4852 |
| Date: | Tue, Nov 01 2022 | Df Residuals: | 4851 |
| Time: | 18:28:11 | Df Model: | 1 |

Mean Model

| | coef | std err | t | P>\|t\| | 95.0% Conf. Int. |
|---|---|---|---|---|---|
| mu | 1.0205e-04 | 2.296e-04 | 0.444 | 0.657 | [-3.480e-04,5.521e-04] |

Volatility Model

| | coef | std err | t | P>\|t\| | 95.0% Conf. Int. |
|---|---|---|---|---|---|
| omega | 2.1165e-04 | 8.926e-06 | 23.712 | 2.733e-124 | [1.942e-04,2.291e-04] |
| alpha[1] | 0.2201 | 3.489e-02 | 6.309 | 2.809e-10 | [ 0.152, 0.289] |

Table 1 ARCH model parameter estimation

Table 1 shows the parameter estimation results of the ARCH model, but in practical applications, there will be some problems in the analysis of financial data by the A RCH model, and the estimation of volatility by the ARCH model may be higher than the true value.

### 3.3.2 GARCH model test and results

In order to improve the ARCH model and prevent many parameters when the hysteresis end is too high, the GARCH model is used for analysis [5]. According to the test results of the G ARCH model, it can be seen that the establishment effect of the GARCH(1,1) model is the best.

Constant Mean - GARCH Model Results

| | | | |
|---|---|---|---|
| **Dep. Variable:** | close | **R-squared:** | 0.000 |
| **Mean Model:** | Constant Mean | **Adj. R-squared:** | 0.000 |
| **Vol Model:** | GARCH | **Log-Likelihood:** | 13715.8 |
| **Distribution:** | Normal | **AIC:** | -27423.5 |
| **Method:** | Maximum Likelihood | **BIC:** | -27397.6 |
| | | **No. Observations:** | 4852 |
| **Date:** | Tue, Nov 01 2022 | **Df Residuals:** | 4851 |
| **Time:** | 18:29:09 | **Df Model:** | 1 |

Mean Model

| | coef | std err | t | P>\|t\| | 95.0% Conf. Int. |
|---|---|---|---|---|---|
| **mu** | 3.2196e-04 | 1.343e-05 | 23.980 | 4.500e-127 | [2.956e-04,3.483e-04] |

Volatility Model

| | coef | std err | t | P>\|t\| | 95.0% Conf. Int. |
|---|---|---|---|---|---|
| **omega** | 5.3544e-06 | 5.505e-13 | 9.726e+06 | 0.000 | [5.354e-06,5.354e-06] |
| **alpha[1]** | 0.1000 | 1.069e-02 | 9.352 | 8.581e-21 | [7.904e-02, 0.121] |
| **beta[1]** | 0.8800 | 8.807e-03 | 99.919 | 0.000 | [ 0.863, 0.897] |

Table 2 GARCH model parameter estimation

According to Table 2, the model can be expressed as follows:

$$\sigma_t^2 = 0.00000535 + 0.1000u_{t-1}^2 + 0.8800\sigma_{t-1}^2 \tag{15}$$

Using Python software, the above results can be obtained to compare the volatility prediction estimation of the ARCH model with the GARCH model, as shown in Figure 3.

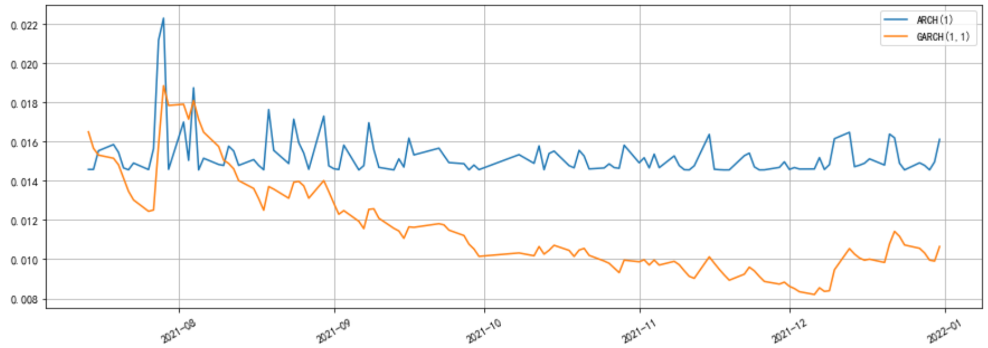


Figure 3 Comparison of volatility trend prediction between ARCH and GARCH models

The volatility predicted by the above two models through Python software is substituted into the B-S model to calculate the simulated option price of the CSI 300 stock index option IO2112-C-4900 contract from July 1, 2021 to December 17, 2021. At the same time, the simulated option price calculated by historical volatility is compared with the simulated price of the above two models to obtain Figure 4:

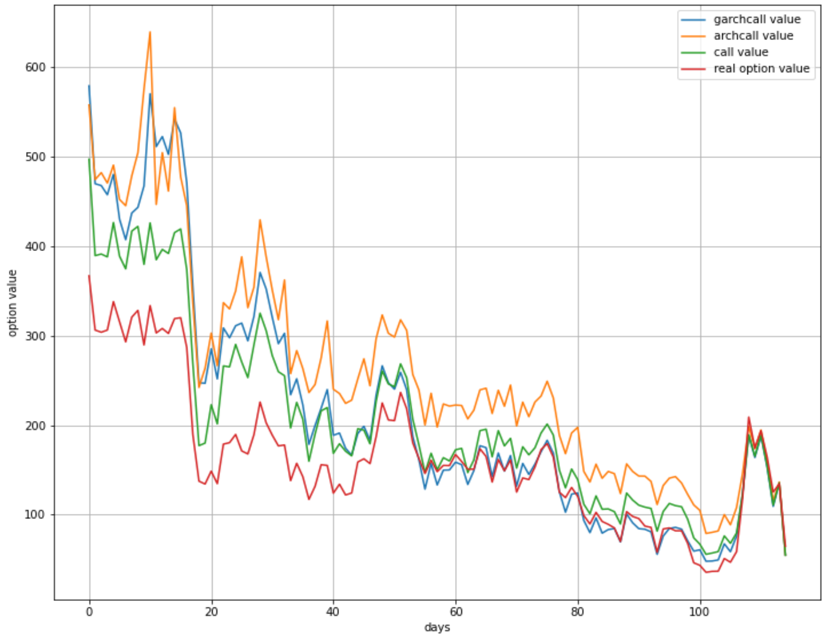


Figure 4 Comparison of model fitting results

It can be seen from the figure that the fitting of historical volatility on option prices is better than that of the ARCH model and G ARCH in the near future, but as the expiration date approaches, the fitting effect of the G ARCH model is closer and closer to the real price, which shows that GARCH The fitting effect of the model is better than that of the historical volatility model. Therefore, the establishment of GARCH option pricing volatility model based on B-S model can provide a theoretical method for option pricing and other derivatives pricing[6].

## 4. Summary

In this paper, the ARCH model and the G ARCH model are used to empirically simulate the return of the daily closing price of the CSI 300 stock index, and according to the test of return, it can be seen that the logarithmic return series belongs to a stationary time series, and the logarithmic return series also has "spike tailing" and aggregation phenomenon, so the sample series does not follow the normal distribution. With the characteristics of heteroscedasticity, it can be well fitted by using the GARCH (1,1) model, so as to predict the future trend of volatility, and then substitute the predicted volatility into the option pricing model to see the future price trend of the option price.

From the empirical effect of bringing three kinds of volatility into option pricing in the paper, the volatility calculated by the GARCH model is substituted into the B-S formula, and the deviation between the option price and the actual option price is the smallest, and the result is significantly better than directly using historical volatility through B-S formulas for pricing; In the real financial market, exchanges can only reasonably price options on the basis of option pricing theory and model, so as to develop various financial derivatives. Investors can also analyze reasonable option prices through the option pricing model to achieve investment transactions, in the real financial market, the pricing of financial derivatives is more difficult, the factors involved are very complex, whether to execute as scheduled and price changes, are difficult to predict, but also difficult to add to the pricing model to analyze, therefore, improve the pricing system of options is also the focus of scholars' research. In the stock market, its price is randomly wandering, it is difficult to predict the future price trend, but the volatility of stocks has obvious characteristics, easy to apply and analyze, and can reflect the expectation of future market changes and market conditions, therefore, the pricing of options generally starts from the perspective of volatility.

Most options are traded based on volatility, with the GARCH model at CSI 300 It plays an important role in the research on the pricing of stock index options, and the fitting effect of its volatility is good through the establishment of the model, which also plays an important role in forecasting, and also brings more investment basis for investors. When the implied volatility fluctuates from the model's predicted value, arbitrage occurs, eventually returning the implied volatility of the option to normal levels. Compared with the options market, the stock market has a certain degree of lag, so the volatility of options will also guide the stock market to gradually stabilize, thereby ensuring the stability of the index.

Many scholars have conducted in-depth research on the pricing method of CSI 300 stock index options

Monte Carlo simulation method, binary tree, Black-Scholes option pricing model are the main research focus of option pricing, combined with the current issuance of CSI 300 stock index options, the Black-Scholes option pricing formula is selected to determine its final price. Improving the option pricing system not only provides pricing suggestions for the issuance of innovative financial instruments in the securities market, but also plays a pre-emptive role for the innovation of the financial market.

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