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UAV planar passive pure orientation positioning under different conditions

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Abstract: UAVs should be kept as silent as possible during field operations and missions in order to conceal and prevent electrostatic interference. In this paper, with rigorous mathematical argumentative thinking, the planar passive pure orientation localization problem under error-free conditions is based on the triangulation localization model of basic analytic geometry. The triangulation model is solved by applying a combination of plane geometry and analytic geometry. The accurate planar triangulation model is obtained, and the target is located according to the determined observation points.

1. Introduction

In this paper, the relative position of the UAV swarm is adjusted to maintain the formation formation by the method of pure azimuth passive positioning, i.e., the formation signal machine and receiver, according to the received signal to extract the direction information, positioning to achieve UAV position adjustment. 10 UAVs form a circular formation and are located in the same plane [1]. The UAV numbered FY00 is located at the center of the circle, and the remaining 9 UAVs are evenly distributed around the circumference of the circle.

The UAV numbered FY00 and two other UAVs are used as signal sources, while the remaining UAVs with slightly deviated positions passively receive signals. If the position of the UAV transmitting the signal is not deviated and the number is known, the positioning model of the UAV passively receiving the signal is established.

A UAV with a slightly deviated position receives signals from UAVs numbered FY00 and FY01, and another UAV with an unknown number in the formation receives signals. The position of the UAVs with known signals is not deviated, and effective positioning of the UAVs is achieved.

As required by the formation, one UAV is located at the center of the circle and the other 9 UAVs are evenly distributed on the circumference of a circle with a radius of 100 m. If the position of the UAVs is known at the initial moment, the UAVs are located at the center of the circle. The position of the UAVs is known to be slightly deviated at the initial moment, and a reasonable adjustment plan for the UAV formation position is given. And use the known real data to give a specific UAV position adjustment scheme under the premise of pure azimuthal passive positioning.

2. Assumptions and notations

2.1 Assumptions [2]

Use the following assumptions.

1) Assume that the UAVs in the formation are always flying at the same altitude.

2) Assume that the relative position of each UAV in the formation to the rest of the UAVs remains constant in the no-operation state.

3) Assume that all UAVs in the formation can adjust their positions normally and autonomously without malfunction.

4) Assume that the deviation of UAV positions in the formation is small enough compared to the standard spacing.

5) Assume that the signals received by the UAVs are only from the other UAVs in the formation, without considering the interference from external sources.

2.2 Notations

The primary notations used in this paper are listed as Table 1.

Table 1: Notations

Symbols	Description
α_1, α_2	Observation angle of the UAV to be positioned
$\theta_0, \theta_1, \theta_j$	Signal source as abstract observation station observation angle
R	Characteristic linearity of the subsystem
d	Distance from the UAV to the origin
$\Delta\alpha$	Difference of observation angle
r_{nml}	Standard deviation from the standard position
$J(\alpha_1)$	Function to calculate the number of the UAV to be located
$K(\Delta\alpha)$	Function to calculate the number of the unknown signal source
r, φ	Coordinates of the total polar coordinate system
ρ, θ	Coordinates of the group of polar coordinate systems
σ	Standard deviation of the distribution of the error in the deviation of the UAV from the standard position

3. Model construction and solving

3.1 The position of the UAV transmitting the signal is not deviated and the number is known

Any corner coordinate (α_1, α_2) selected by the model on area $\mathbb{R}^2 \backslash \{(0,0)\}$ corresponds uniquely to a point (x, y) in the plane right angle coordinate system. This can be proved from both algebraic and geometric points of view [3].

From the geometric point of view, when the angle α_2 is determined, it follows from the sine theorem that there is

$$\frac{R}{\sin\alpha_2} = 2R_0 \tag{1}$$

The change in α_1 only causes the point P to move on the external circle O_1, as shown in Figure 1.

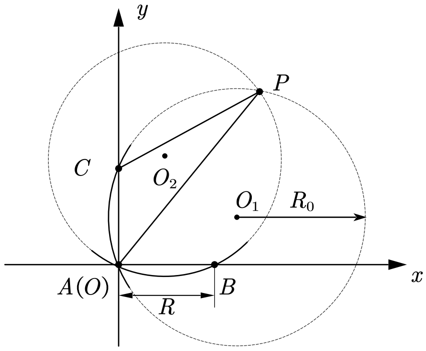


Figure 1: Coordinate completeness geometry schematic

There must exist another intersection point. That is, the mapping

$$\begin{aligned} \mathscr{F}\colon\ [-2\pi, 2\pi]\times[-2\pi, 2\pi] &\rightarrow \mathbb{R}^2\backslash\{(0,0)\} \\ (\alpha_1, \alpha_2) &\mapsto (x, y) \end{aligned} \tag{2}$$

is a single-valued mapping.

Conversely, when a point (x,y) is determined, the angle α_1, α_2 can be uniquely determined by connecting the lines, i.e., the mapping

$$\begin{aligned} \mathscr{F}^{-1}\colon\ \mathbb{R}^2\backslash\{(0,0)\} &\rightarrow [-2\pi, 2\pi]\times[-2\pi, 2\pi] \\ (x, y) &\mapsto (\alpha_1, \alpha_2) \end{aligned} \tag{3}$$

It is also a single-valued mapping. Thus, (α_1, α_2) and (x,y) can be uniquely determined from each other except for the origin O.

The invertibility of the mapping is proved algebraically, i.e., the specific functional form of the mapping is determined by means of analytic geometry. As shown in Figure 2.

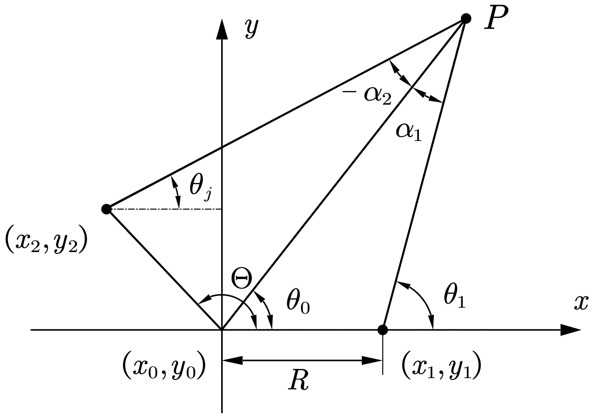


Figure 2: Three-point positioning geometry schematic

From the angle relationship, there are

$$\begin{aligned} \theta_1 &= \theta_0 + \alpha_1 \\ \theta_0 &= \theta_j - \alpha_2 \end{aligned} \tag{4}$$

From the sine theorem it have

$$\frac{d}{\sin\theta_1} = \frac{R}{\sin\alpha_1}$$
$$\frac{d}{\sin(\pi - \Theta + \theta_j)} = -\frac{R}{\sin\alpha_1} \tag{5}$$
$$\frac{\sin(\Theta - \theta_2)}{\sin\theta_1} = -\frac{\sin\alpha_2}{\sin\alpha_1}$$

When the observatory is determined, the angle θ can be extracted in a small amount around the standard angle θ_0, i.e.

$$\begin{aligned} \theta_0 &= \theta_{00} + \theta_0' \\ \theta_1 &= \theta_{10} + \theta_1' \\ \theta_2 &= \theta_{20} + \theta_2' \end{aligned} \tag{6}$$

This leads to a Taylor expansion of the nonlinear equations in the system of equations 5, which can be linearized as

$$\begin{aligned} \sin\theta_1 &\approx \sin\theta_{10} + \cos\theta_{10} \cdot \theta_1{}' \\ \sin(\Theta - \theta_2) &\approx \sin(\Theta - \theta_{20}) - \cos(\Theta - \theta_{20}) \cdot \theta_2{}' \end{aligned} \tag{7}$$

After simplification, it is

$$\frac{\sin(\Theta - \theta_{20}) - \cos(\Theta - \theta_{20}) \cdot \theta_2{}'}{\sin\theta_{10} + \cos\theta_{10} \cdot \theta_1{}'} = -\frac{\sin\alpha_2}{\sin\alpha_1} \tag{8}$$

The equation is collapsed into matrix form as

$$H_\theta \cdot A_\theta = Y_\theta \tag{9}$$

Among them

$$A_\theta = \begin{bmatrix} \theta_0{}' \\ \theta_1{}' \\ \theta_j{}' \end{bmatrix}, \quad H_\theta = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & \sin\alpha_2 \cos\theta_{10} & \sin\alpha_1 \cos(\Theta - \theta_{j0}) \end{bmatrix}$$
$$Y_\theta = \begin{bmatrix} \theta_{10} - \theta_{00} - \alpha_1 \\ \theta_{j0} - \theta_{00} - \alpha_2 \\ \sin\alpha_1 \cdot \sin(\Theta - \theta_{j0}) + \sin\alpha_2 \cdot \sin\theta_{10} \end{bmatrix} \tag{10}$$

From this, the angle θ can be solved, and for the angle θ_i, the slope equation in the Cartesian coordinate system can be listed as

$$\theta_i = \tan^{-1}\frac{y^* - y_i}{x^* - x_i} \tag{11}$$

Linearizable as

$$\begin{aligned} \sin\theta_i \cdot x^* - \sin\theta_i \cdot x_i &= \cos\theta_i \cdot y^* - \cos\theta_i \cdot y_i \\ \sin\theta_i \cdot x^* - \cos\theta_i \cdot y^* &= \sin\theta_i \cdot x_i - \cos\theta_i \cdot y_i \\ (i = 0, 1, j) \end{aligned} \tag{12}$$

Writing again in matrix form, there are

$$H_x X = Y_x \tag{13}$$

where each matrix is

$$X = \begin{bmatrix} x^* \\ y^* \end{bmatrix}, \quad H_x = \begin{bmatrix} \sin\theta_0 & -\cos\theta_0 \\ \sin\theta_1 & -\cos\theta_1 \\ \sin\theta_j & -\cos\theta_j \end{bmatrix}$$
$$Y_x = \begin{bmatrix} \sin\theta_0 \cdot x_0 - \cos\theta_0 \cdot y_0 \\ \sin\theta_1 \cdot x_1 - \cos\theta_1 \cdot y_1 \\ \sin\theta_2 \cdot x_j - \cos\theta_2 \cdot y_j \end{bmatrix} \tag{14}$$

It can be found that the coefficient matrix $H_x \in \mathbb{R}^{3\times 2}$ When there is an error, this equation does not have an exact solution, but its optimal solution can be obtained using different algorithms. In fact this is the basis for the cooperative correction of multiple errors in the later stages of the model.

3.2 Slightly deviated UAV positioning at a certain location

It is shown that in addition to FY00 and FY01, only one additional UAV in the formation needs to transmit signals to effectively locate the UAV with position deviation. From the geometric relationship, the intersection point of the line between the UAV and FY00 at the position to be measured is taken as the vertex and the angle α_1 obtained by making a ray to FY00 and FY01 as the vertex can determine the UAV's own number, thus enabling the determination of different signal source numbers under the standard geometric model. This results in the function.

$$\alpha_1 = \theta_1 - \theta_0 \tag{15}$$

$$J(\alpha_1) = \begin{cases} 2, & \frac{\alpha_1}{80°} \in (0.75, 1] \\ 3, & \frac{\alpha_1}{80°} \in (0.5, 0.75] \\ 4, & \frac{\alpha_1}{80°} \in (0.25, 0.5] \\ 5, & \frac{\alpha_1}{80°} \in (0, 0.25] \\ 6, & \frac{\alpha_1}{80°} \in (-0.25, 0] \\ 7, & \frac{\alpha_1}{80°} \in (-0.5, -0.25] \\ 8, & \frac{\alpha_1}{80°} \in (-0.75, -0.5] \\ 9, & \frac{\alpha_1}{80°} \in [-1, -0.75] \end{cases} \tag{16}$$

The return result is the number of the UAV to be located in the formation. Then specify

$$\Delta\alpha = \alpha_1 - \alpha_2 \tag{17}$$

Again, the function is obtained from the geometric relationship $K(\Delta\alpha)$

$$K(\Delta\alpha)=\begin{cases} 2, & \frac{\Delta\alpha}{150^\circ}\in\left(\frac{13}{15},1\right]\cup[-\frac{1}{5},-\frac{1}{15}] \\ 3, & \frac{\Delta\alpha}{150^\circ}\in\left(\frac{11}{15},\frac{13}{15}\right]\cup[-\frac{1}{3},-\frac{1}{5}) \\ 4, & \frac{\Delta\alpha}{150^\circ}\in\left(\frac{3}{5},\frac{11}{15}\right]\cup[-\frac{7}{15},-\frac{1}{3}) \\ 5, & \frac{\Delta\alpha}{150^\circ}\in\left(\frac{7}{15},\frac{3}{5}\right]\cup[-\frac{3}{5},-\frac{7}{15}) \\ 6, & \frac{\Delta\alpha}{150^\circ}\in\left(\frac{1}{3},\frac{7}{15}\right]\cup[-\frac{11}{15},-\frac{3}{5}) \\ 7, & \frac{\Delta\alpha}{150^\circ}\in\left(\frac{1}{5},\frac{1}{3}\right]\cup[-\frac{13}{15},-\frac{11}{15}) \\ 9, & \frac{\Delta\alpha}{150^\circ}\in[\frac{1}{15},\frac{1}{5}]\cup[-1,-\frac{13}{15}) \end{cases} \tag{18}$$

The return result is the number of the transmitting UAV with unknown number in the formation.

The problem is thus transformed into the above scenario, which allows for effective localization of the UAV position to be measured.

3.3 UAV positioning with slight deviation of the UAV's position at the initial moment

Adjustment strategy

1) For the circular type arrangement problem shown in the figure, we select FY00 as the coordinate origin, whose direction to FY01 is axis positive, and set this distance as.

2) In the initial state, FY00, FY01, FY04 and FY07 are selected for self-adjustment. Since each UAV has errors at the same time and has to be adjusted at the same time, it is a cooperative correction under multiple errors.

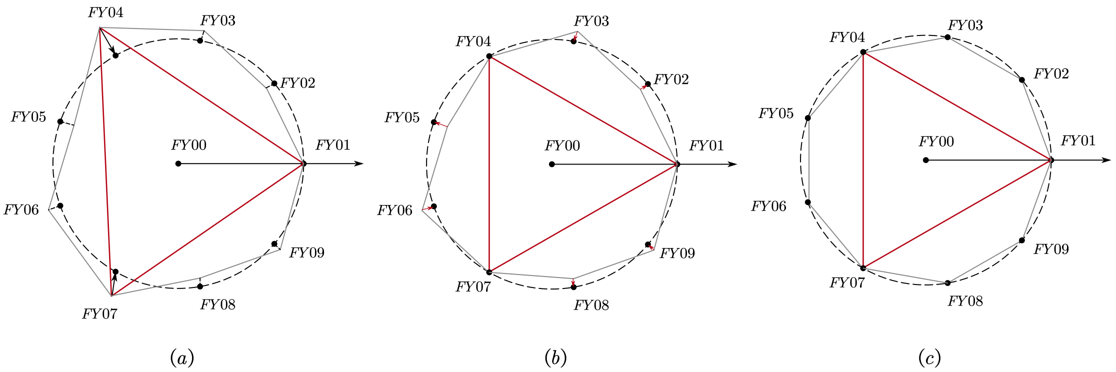
3) After they have been adjusted to their respective standard positions, all four act as signal sources at the same time, and the silent UAVs between each adjacent two sources are adjusted with the nearest three UAVs as the standard:.

a) Since each signal source UAV is located in the standard position, for FY02, use FY00, FY01 and FY04 as signal sources and apply the model to position itself.

b) Adjust itself to the correct position.

c) For the unadjusted UAV, repeat steps (a) and (b) until all adjustments are completed.

The flow schematic is shown in Figure 2



Co-correction algorithm under multiple errors

Figure 3: Diagram of circular arrangement adjustment process

For this multi-machine co-correction model, the key problem is that it cannot be solved geometrically accurate because the true coordinates are unknown (x^*, y^*), Referring to the current literature on multi-station localization methods, it can be found that some of them deal with the fusion of information, thus losing a certain amount of information [4], or they are not applicable to attitude adjustment under stationary formation due to the motion of the observatory[4,5], but the literature gives the possibility of using the CTLS algorithm when there is a certain error in the observation angle and the position of the observatory is precise. The optimal values are estimated and iterated to (x^*, y^*) obtain their optimal solutions.

For this problem, the essence is that the observation angle of the observatory is precise, while the position of the observatory is in error. In fact, if the position of the observatory is still considered accurate (i.e., it is the labeled position), and the deviation of the angle from its position in the standard position is an error, the problem is transformed into a problem of a standard observatory that observes the target with a certain error, and the optimal position of the target can be solved iteratively using the CTLS algorithm, and the position is corrected after each iteration until the preset accuracy is satisfied and the iteration is ended.

Reference CTLS method directly based on data θ_i ,Solved by the system of equations (13) (x^*, y^*), The "scintillation model" is proposed - although it is not easy to have a large number of source UAVs at the same moment, we can let the UAVs in the group of cooperative correction enter the electromagnetic silence sequentially with a certain strategy to receive the position information of the remaining sources, so we know the angle between any two sides in the group after a strategy cycle (defined as the scintillation cycle), and after specifying a positive direction, we can calculate the observation angle .

$$\theta_i = \tan^{-1} \frac{y^* - y_i}{x^* - x_i} \qquad (19)$$

In fact, since the observations of the UAV to be adjusted can be transformed into the observations of the abstract base station according to the transformation of a non-simultaneous linear system of equations, the error can be considered as a linear combination of the errors originating from the observations, since the sum of normally distributed random variables is also normally distributed, and it is sufficient to show that it obeys the normal distribution.

Take the standard position of the signal source as the origin, set the direction of the UAV to be adjusted pointing at the origin as the x-axis positive, in the plane of the UAV perpendicular to this direction as the y-axis positive, establish a plane right angle coordinate system, as shown in Figure 4.

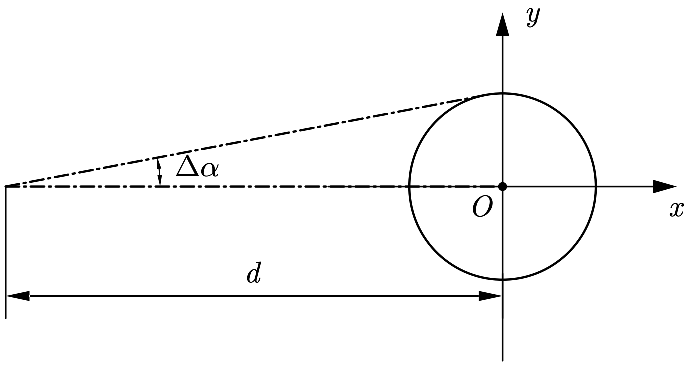


Figure 4: Schematic diagram of normality transfer

Since y obeys a normal distribution, and since the offset is assumed to be smaller than the standard spacing, there is

$$\tan\Delta\alpha \approx \frac{y}{d} \approx \Delta\alpha \qquad (20)$$

$$\Delta\theta \propto \Delta\alpha \propto y \qquad (21)$$

Since y obeys a normal distribution, $\Delta\theta$ obeys a normal distribution. Therefore, after abstracting the UAV signal source as the problem of observing the target of the observatory.

In particular, if we let $\Delta\alpha = \Delta\varphi$ be the change in polar angle in polar coordinates with another point as the origin, then the change in radius r in this polar coordinate system has

$$\Delta r = x \qquad (22)$$

Considering that the components of the coordinates (x,y) obey the normal distribution $N(0,\sigma)$ at the same time, it have

$$\begin{aligned} \Delta r &\sim N(0,\sigma) \\ \Delta\varphi &\sim N\left(0,\frac{\sigma}{d}\right) \end{aligned} \qquad (23)$$

CTLS algorithm

The noise vector is introduced to transform the passive localization problem into a constrained overall least squares problem. This constrained optimization problem can then be transformed into an unconstrained optimization problem by the Lagrange multiplier method. Following this, a real nonlinear function is obtained and solved using the Newton iterative method, ending the iteration when the tracking data (angle) falls in the acceptable interval.

4. Conclusion

In this paper, the UAV with the number FY00 and another 2 UAVs are used as signal sources, and the remaining UAVs with slightly deviated positions receive signals passively. If the position of the UAV transmitting the signal is not deviated and the number is known, the positioning model of the UAV passively receiving the signal is established. The problem that a UAV with a slightly deviated position receives signals from UAVs with numbers FY00 and FY01, and another receives signals from several UAVs in the formation with unknown numbers is solved. The problem of no deviation in the position of the UAVs known to be transmitting signals. As required by the formation, one UAV is located at the center of the circle and the other nine UAVs are evenly distributed on the circumference of the circle with a radius of 100 m. If the position of the UAVs is known at the initial moment, the UAVs will be located at the center. If the position of the UAVs is known to be slightly deviated at the initial moment, try to give a reasonable adjustment scheme for the position of the UAV formation. Using the known real data, a specific UAV position adjustment scheme is given under the premise of pure azimuthal passive positioning.

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