

# *A Mathematical Model for Analyzing and Identifying the Composition of Ancient Glass Objects and Its Application*

Chengwen Niu, Wencheng Wang, Nianwei Li

*College of Information Science and Engineering, Shandong Agricultural University, Taian, Shandong, 271017, China*

**Keywords:** Grey relational analysis, Hierarchical cluster method, Association, Chemical composition associations, Sensitivity analysis

**Abstract:** Aiming at the urgency of today's heritage conservation, a detection model based on gray correlation analysis and systematic clustering model is established using gray correlation analysis and systematic clustering algorithm, taking ancient glass relics as the research object. And in the paper, it is applied for the actual data and achieved good practical results. In addition, the model was optimized by using multiple linear regression to further improve the fit of the model with the actual data.

## 1. Introduction

The ancient glassware is a fragile cultural relic. Its main component is silicon dioxide ( $\text{SiO}_2$ ), and it can be divided into many types according to different characteristics for different protection. However, as time goes on, ancient glass products will be gradually weathered. During the weathering process, elements in the glass and elements in the natural environment will exchange with each other, leading to changes in the proportion of their components, affecting the analysis and judgment of their classification, and adversely affecting the protection of cultural relics. This paper introduces how to use the existing data of ancient glass relics for statistical analysis and correlation analysis, and use clustering algorithm [2] to analyze and identify the components of glass products.

## 2. Model Preparation and Idea Description

### 2.1 Data Pre-Processing

Firstly, the given data are cleaned to check whether there are reasonable missing items in the measured data, and for the reasonable missing items, no further testing and processing is required. The data with the proportion of each component of the glass summed up between 85% and 105% were considered as valid data. For each component of the same test data is recorded as (1).

The data that are not in this range are deleted, and the part of the color that is empty is recorded as color 1, color 2, color 3, and color 4 from top to bottom, and the other empty values are filled with "0" to facilitate the subsequent calculation.

## 2.2 Model Assumptions

(1) For the same decoration, type, color, but different degrees of weathering of cultural relics, can be considered caused by external factors.

(2) Do not consider the effect of time on the change of glass type as well as chemical composition content before and after weathering.

## 2.3 Symbol Description (Table 1)

Table 1: Symbol Description

Symbol	Definition
$a_i$	The i-th component content
$F_{ij}$	Component j of differentiated group i data
$W_{ij}$	The jth component of the ith group of data without differentiation
$Q_1$	Weight of all data for high potassium glass
$Q_2$	Weight of all data of lead-barium glass A
$Q_3$	Weight of all data of lead-barium glass C
$Y_{ij}$	Weights of all data for standard data
$B_{ij}$	Content of component j of group i of the predicted data
$C_{ij}$	Treatment value of the jth component of the ith group obtained
$H_i$	obtained from the summation of $C_{ij}$ fixed i
$q_{i,j}$	The weight of the jth component of the ith group

## 2.4 Relationship between Surface Weathering of Artifacts and Their Glass Type, Decoration and Color

The surface weathering of glass artifacts was analyzed in relation to their glass type, decoration and color, and they were classified according to Annex Table 2. The glass artifacts were firstly classified into two categories: high potassium and lead-barium, followed by classification according to ornamentation, and finally according to color. After classification it is easy to find that high potassium B blue-green type are weathered, high potassium AC two categories have not been no weathering, lead barium A in part of the light blue, dark blue appear no weathering phenomenon, the rest are weathered, lead barium C in green, light green, dark green and purple among, no weathering phenomenon, the rest are weathered. This is to make a preliminary judgment, for high potassium type, all examples of A and C are unweathered type, B are all weathered type, preliminary judgment for high potassium type, all A and C will not be weathered, while for B, all B will be weathered. Then for the lead barium type, it only contains two types of ornamentation, A and C. For the A type, for the same color, such as light blue appears both weathering and no weathering two types, by the model assumption (3), it is considered that it is caused by the external environment, for different colors, such as black, blue-green, color 1, color 2 alone appears weathering, dark blue alone appears no weathering, because the sample is small, do not make a positive judgment, the same reason For type C, light green, dark green, and purple appear weathering and no weathering together, and for blue green, light blue, color 2, and color 4 appear

weathering alone, and green appears no weathering alone, making the same explanation as type A. The above are the results of the analysis of the data set, while organizing them into Table 2.

Table 2 : Relationship between Glass Type, Ornamentation, Color and Weathering

Glass Type	Ornamentation	Color	Whether weathering
High Potassium	B	blue-green	Weathered
	A, C	blue-green, light blue, dark blue	Unweathered
Lead Barium	A	black, blue-green, light blue, color 1, color 3	Weathered
		light blue, dark blue	Unweathered
	C	blue-green, light blue, light green, dark green, violet, color 2, color 4	Weathered
		green, light green, dark green, purple	Unweathered

Then it was further analyzed using gray correlation analysis [1], where the correlation coefficient represents the value of the degree of correlation on the corresponding dimension of whether that subsequence ornament, type, and color differentiates the parent sequence. The Table 3 shows results [2].

Table 3 : Ranking of Evaluation Items and Correlation Results

Relevance results		
Evaluation items	Relevance	Rank
Type	0.86	1
Ornamentation	0.795	2
Color	0.702	3

The above correlation coefficient results were weighted to obtain the correlation value, and the correlation value was used to rank the three evaluation objects; the correlation value ranged from 0 to 1, and the larger the value was, the stronger the correlation with the “reference value” (weathering or not), and the higher the evaluation. As can be seen from the Table 3, for the three evaluation items, the type was rated the highest (correlation: 0.86), followed by ornamentation (correlation: 0.795). This method is approximately the same as the preliminary judgment.

## 2.5 Analysis of Statistical Patterns for the Presence or Absence of Weathering on the Type Surfaces of Different Glasses

The chemical composition components in the data set were analyzed to find out the statistical laws and to make predictions.

First, the contents of the dataset were classified according to the order of differentiation or not, type, ornamentation, and color according to the analysis method in 1.4, and the classified data were renumbered based on the original numbering, and all weathering points were put together, and all non-weathering points were put together. For special severely differentiated points, they were also put together separately [3].

For the high potassium types, all Bs are differentiated types and all As and Cs are unweathered types, so there is no need to subdivide the high potassium types for comparison between A, B and C. The standard results before and after differentiation are obtained by direct comparison of the same types.

Secondly, we started to consider the lead-barium types. Since for different types of A and C, each contains its own differentiated and undifferentiated types, the A and C ornaments were considered in categories, but for the different colors, no more detailed division was made. For the different

cases that occur individually in A or C for different detection points, the following decisions are made: for the severely differentiated point, it is treated separately because the data are too abnormal; for the unweathered point that occurs among differentiated individuals, it is considered to correspond to the data of the unweathered individuals and this is treated as unweathered.

The mean and median were put into a table and their standard deviation and variance were calculated to obtain the following Table 4.

Table 4 : Descriptive Statistical Analysis of Changes in Chemical Composition Content Before and after Weathering of C Lead-Barium Glass with Ornamentation

Variable Name	Weathering	Sample size	Maximum value	Minimum value	Mean	Standard deviation	Median	Variance	Kurtosis	Skewness
SiO <sub>2</sub>	Before weathering	7	75.51	31.94	59.73	13.69227206	65.91	187.47831	1.2767735	-1.247761
	After weathering	22	50.61	3.72	24.196364	10.52867148	23.445	110.85292	0.8725905	0.27565268
Na <sub>2</sub> O	Before weathering	7	2.71	0	0.3871429	0.948302458	0	0.8992776	7	2.64575131
	After weathering	22	2.31	0	0.3240909	0.718158371	0	0.5157514	3.1560532	2.10105797
K <sub>2</sub> O	Before weathering	7	0.23	0	0.0842857	0.099836601	0	0.0099673	-2.221168	0.55252782
	After weathering	22	0.44	0	0.0954545	0.150354952	0	0.0226066	0.0009273	1.25510139
CaO	Before weathering	7	1.6	0.38	0.0842857	0.411160627	0.64	0.1690531	0.4235778	1.093425
	After weathering	22	6.4	0	2.4931818	1.761258296	2.07	3.1020308	-0.596842	0.59403455
MgO	Before weathering	7	1	0	0.0842857	0.42791855	2.35	0.1831143	-0.713642	1.24988904
	After weathering	22	2.73	0	0.5295455	0.700587268	0	0.4908225	2.7013488	1.52057795
Al <sub>2</sub> O <sub>3</sub>	Before weathering	7	3.11	1.44	2.1457143	0.61390686	0	0.3768816	-1.646728	0.19573804
	After weathering	22	5.25	0.45	2.2236364	1.230589458	2.04	1.5143504	-0.024863	0.62757139
Fe <sub>2</sub> O <sub>3</sub>	Before weathering	7	4.59	0	0.8228571	1.57468105	0.47	2.4796204	5.9427386	2.41659976
	After weathering	22	1.79	0	0.43	0.5590983	0.21	0.3125909	0.4036096	1.26861242
CuO	Before weathering	7	8.46	0.11	1.93	2.822086361	19.76	7.9641714	4.6064495	2.14600068
	After weathering	22	10.57	0	2.4859091	2.897385434	1.25	8.3948424	3.4524402	1.99436956
PbO	Before weathering	7	32.92	16.16	21.974286	6.114469623	5.68	37.386739	-0.610579	0.97701866
	After weathering	22	70.21	25.39	44.269091	11.65138342	44.435	135.75474	-0.441349	0.31762559
BaO	Before weathering	7	26.23	3.42	8.8642857	7.449619084	0.35	55.496824	4.9505856	2.16430059
	After weathering	22	35.45	0	12.758182	10.12696699	9.35	102.55546	0.3437368	1.17071673
P <sub>2</sub> O <sub>5</sub>	Before weathering	7	1.62	0.13	0.6128571	0.596554734	0	0.3558776	-0.817131	1.13299492
	After weathering	22	14.13	0	4.8390909	4.231611459	3.915	17.906536	-0.622882	0.58340568
SrO	Before weathering	7	0.91	0	0.2171429	0.312625485	0	0.0977347	3.1951749	1.77575913
	After weathering	22	1.12	0	0.4240909	0.270330759	0.425	0.0730787	0.7505841	0.51997251
SnO <sub>2</sub>	Before weathering	7	0	0	0	0	0	0	0	0
	After weathering	22	0.47	0	0.0213636	0.097900481	0	0.0095845	22	4.69041576
SO <sub>2</sub>	Before weathering	7	3.66	0	0.5228571	1.280733208	0	1.6402776	7	2.64575131
	After weathering	22	15.95	0	1.6145455	4.43873828	0	19.702398	7.6432406	2.95247874

### 3. Model Development and Solution

#### 3.1 Prediction Model of the Percentage of Chemical Composition Before Weathering

The mean values in the measured data were analyzed and the prediction model was made as follows.

$$\frac{F_{ij}}{W_{ij}} = \frac{\bar{F}_{ij}}{\bar{W}_{ij}} \quad (1)$$

From this prediction model can be made for the location artifacts as Table 5 chemical composition prediction.

Table 5: Predicted Chemical Composition of Some Artifacts

Artifact number	Ornament	Type	SiO <sub>2</sub>	Na <sub>2</sub> O	K <sub>2</sub> O	CaO	MgO	Al <sub>2</sub> O <sub>3</sub>	Fe <sub>2</sub> O <sub>3</sub>	CuO
7	B	High Potassium	66.08	0.00	0.00	5.62	0.00	6.05	1.16	4.71
8	C	Lead Barium	11.38	0.00	0.00	1.02	0.00	1.07	0.00	2.44
23	A	Lead Barium	85.15	111.00	0.00	0.25	1.00	1.34	0.00	3.26

In the table shown above

i=1 refers to the high potassium type.

i=2 refers to lead-barium type A ornamentation.

i=3 refers to C ornamentation of lead-barium type.

i=4 refers to the heavily differentiated C ornament of the lead-barium type.

### 3.2 Systematic Clustering Model for Glass Types [2]

According to the dataset, the general classification law of high potassium glass and lead-barium glass can be derived as follows: high potassium glass has a high silica content and has a high potassium oxide content, for B, although the potassium oxide content is low, it is considered that it is lower due to weathering, and high potassium glass has a low barium oxide and lead oxide content, while for lead-barium glass, which is exactly the same as high potassium. On the contrary, the content of potassium oxide is low, while the content of both barium oxide and lead oxide is high, which is the most basic classification rule for high potassium glass and lead-barium glass [4].

On the basis of subclassification into high potassium glass and lead-barium glass, the high potassium glass was divided into pre-weathering and post-weathering, and the pre-weathering was recorded as class 1 and the post-weathering was recorded as class 2; the lead-barium glass was divided into four classes, A and C pre-weathering and post-weathering, and the A pre-weathering was recorded as class 3, the A post-weathering was recorded as class 4, the C pre-weathering was recorded as class 5, and the C post-weathering was recorded as class 6.

The analysis was carried out using cluster analysis [2], and the content of partial oxides was used as a division criterion to classify this.

The weights of the components of the high-potassium glass are given in Equation (2).

$$Q_1 = (q_{1,1}, q_{1,2}, q_{1,3}, \dots, q_{1,14}) \quad (2)$$

The weights of each component of lead barium glass A are as in equation (3).

$$Q_2 = (q_{2,1}, q_{2,2}, q_{2,3}, \dots, q_{2,14}) \quad (3)$$

The weights of each component of lead barium glass C are as in equation (4)

$$Q_3 = (q_{3,1}, q_{3,2}, q_{3,3}, \dots, q_{3,14}) \quad (4)$$

The data in high potassium glass are weighted as in Equation (5)

$$a_i \times q_{1,i} \quad (5)$$

The weighted data are clustered by selecting the appropriate chemical composition.

Weighting the data in lead-barium glass A

$$a_i \times q_{2,i} \quad (6)$$

Clustering the weighted data by selecting the appropriate chemical composition.

Weighting the data in lead barium glass C

$$a_i \times q_{3,i} \quad (7)$$

Select the appropriate chemical components from the weighted data to cluster them.

The Table 6 can be obtained by comparing the predicted data with the actual data.

Table 6: Comparison of The First Forecast and Actual Data

	Data volume of prediction data	Prediction data	Data volume of actual data	Actual data	Fit
No weathering	7	25 Unweathered 、 55、 32、 35、 33、 37、 31	8	25 Unweathered 、 55、 32、 35、 33、 37、 24、 31	88%

Weathering	22	56、57、11、 part 1 of 43、 part2 of 43、 part 1 of 51、 part2 of 51、 52、 54、 54 severe weathering 、 41、 34、 36、 38、 39、 8、 8 severe weathering 、 26、 26 severe weathering 、 40、 58、 24	21	56、 57、 11、 part 1 of 43、 part 2 of 43、 part 1 of 51、 part 2 of 51、 52、 54、 54 severe weathering 、 41、 34、 36、 38、 39、 8、 08 severe weathering 、 26、 26 severe weathering 、 40、 58	105%
------------	----	---	----	--	------

From Table 6, it can be obtained that most of the detection points meet the data, there are still a small number of detection points with anomalies, resulting in a reduced fit, which will be treated as abnormal data and deleted, and cluster analysis will be performed again to obtain the following Table 7.

Table 7: Comparison of The Second Forecast and Actual Data

	Data volume of prediction data	Prediction data	Data volume of actual data	Actual data	Fit
No weathering	7	25 Unweathered、 55、 32、 35、 33、 37、 31	7	25 Unweathered、 55、 32、 35、 33、 37、 31	100%
Weathering	21	56、 57、 11、 part 1 of 43、 part 2 of 43、 part1 of 51、 part 2 of 51、 52、 54、 54 severe weathering、 41、 34、 36、 38、 39、 8、 08 severe weathering、 26、 26 severe weathering、 40、 58	21	56、 57、 11、 part 1 of 43、 part 2 of 43、 part 1 of 51、 part 2 of 51、 52、 54、 54 Unweathered、 41、 34、 36、 38、 39、 8、 08 Unweathered、 26、 26 Unweathered、 40、 58	100%

From the above table, we find that the fit is 100 percent. The coordinates of the centroid of the clusters at this point are taken as the criteria for the class (Table 8).

Table 8: Coordinates of Clustering Centroids

Clustering type	Type 6	Type 5	Clustering type	Type 6	Type 5
Central value (SiO <sub>2</sub> )	22.9386	62.3971	Central value (CuO)	2.550952	0.8814
Central value (Na <sub>2</sub> O)	0.2295	0.7171	Central value (PbO)	44.8581	22.3686
Central value (K <sub>2</sub> O)	0.1000	0.0843	Central value (BaO)	13.04905	6.0671
Central value (CaO)	2.5819	0.8186	Central value (P <sub>2</sub> O <sub>5</sub> )	5.060476	0.6200
Central value (MgO)	0.5548	0.2700	Central value (SrO)	0.434762	0.1157
Central value (Al <sub>2</sub> O <sub>3</sub> )	2.2390	2.1900	Central value (SnO <sub>2</sub> )	0.022381	0.0000
Central value (Fe <sub>2</sub> O <sub>3</sub> )	0.3767	1.0443	Central value (SO <sub>2</sub> )	1.691429	0.5229

Rational interpretation: the classification is first divided according to the type of glass, then the grain of the type of lead barium, and finally the division of weathering and non-weathering, which is divided into six categories, and according to the central value, it can be concluded that for each category there is a unique indicator, and it is obvious that the classification result is reasonable [5].

Sensitivity analysis: randomly selected detection data are clustered again with the standard values of all categories, and its sensitivity is detected by virtue of whether they can be classified into one category with the corresponding standard data category, knowing the classification of the randomly selected detection data.

Finally, based on the detection, it is concluded that their sensitivity is 100%.

### 3.3 Application of the Systematic Clustering Model for Glass Types

The data in Annex Table 4 are denoted as  $Y_{ij}$ . The standard data derived from Problem 2 are denoted as  $B_{ij}$ .

$Q_{4,j}(q_{4,1}, q_{4,2}, q_{4,3}, q_{4,14}, \dots)$  is used as the weight of the standard data.

$$\sum C_{i,j} = \sum Q_{4,j}(Y_{ij}-B_{ij}) \quad (8)$$

$$\sum H_i = \sum C_{ij} \quad (9)$$

The  $i$  when taking  $\min(H_i)$  is the classification of the requested Annex Table 4.

The following Table 9 shows all the data.

Table 9: Difference value of different cultural relic types according to classification standards

	A1	A2	A3	A4	A5	A6	A7	A8
Class 1	0.693	4.110	5.651	5.212	3.264	3.547	3.240	4.783
Class 2	1.683	5.501	6.047	5.466	2.987	0.215	0.453	3.967
Class 3	1.608	3.234	3.701	2.401	1.758	2.082	1.675	1.300
Class 4	3.048	1.704	1.639	1.972	4.614	3.768	3.458	3.466
Class 5	1.585	3.074	3.470	2.778	1.767	1.598	1.629	1.338
Class 6	3.425	2.162	2.266	2.885	5.190	3.591	3.737	3.113

From the above table taking the smallest difference value (value of  $H$ ) as the prediction type of the artifact can be obtained from the new table.

Table 10: Prediction results of cultural relics types

Artifact Number	Class where the minimum value is located	Forecast Type
A1	Class 1	High potassium glass before weathering
A2	Class 4	Lead barium glass A after weathering
A3	Class 4	Lead barium glass A after weathering
A4	Class 4	Lead barium glass A after weathering
A5	Class 3	Lead barium glass A before weathering
A6	Class 2	High potassium glass before weathering
A7	Class 2	High potassium glass before weathering
A8	Class 5	Lead barium glass C before weathering

Sensitivity analysis.

For the obtained prediction results (Table 10), it was judged whether they met the classification criteria.

All the abnormal data in Table 3 of the Appendix were removed and divided according to six categories, and then the data in each category were compared with the standard data according to the method of question three prediction to find  $H$ . The  $\max H$  (as in Appendix III) was taken and compared with the  $\min H$  in the predicted data, and if the  $\min H$  was approximately the same as the  $\max H$ , the predicted data were considered reasonable, and it was found that all the predicted data were reasonable, and Based on the value of  $\min H$ , it can be judged that the predictions of A1, A6, and A7 are more reasonable, while the predicted data of A2, A3, A4, and A5 are less satisfactory.

### 3.4 Association Model for Different Chemical Components

Assuming that a class all satisfies normal distribution, in order to find out the relationship of each chemical component in each class, correlation analysis was performed for each of its classes

using Pearson correlation coefficient, and the direct correlation relationship of chemical components in each class and its correlation coefficient heat map were derived, and those greater than 0.5 were considered to have higher correlation.

From the correlation coefficient heat map, it can be seen that.

High potassium non-weathering types: calcium oxide has high correlation with potassium oxide, barium oxide with lead oxide, phosphorus pentoxide with magnesium oxide, aluminum oxide, and iron oxide, respectively, and strontium oxide with magnesium oxide, aluminum oxide, iron oxide, barium oxide, and phosphorus pentoxide, respectively.

High potassium weathering types.

Potassium oxide has a high correlation with silica, magnesium oxide with calcium oxide, alumina with calcium oxide, magnesium oxide, iron oxide with potassium oxide, and phosphorus pentoxide with copper oxide, respectively.

Lead barium A non-weathering types.

Calcium oxide has high correlation with silica, aluminum oxide with calcium oxide, iron oxide with potassium oxide, barium oxide with copper oxide, and phosphorus pentoxide with potassium oxide and iron oxide, respectively, and strontium oxide with lead oxide.

Lead and barium A weathering types.

Potassium oxide has a high correlation with silica, magnesium oxide with silica, aluminum oxide with silica, potassium oxide and magnesium oxide, respectively, iron oxide with silica, magnesium oxide and aluminum oxide, respectively, lead oxide with potassium oxide, phosphorus pentoxide with calcium oxide, strontium oxide with barium oxide.

Lead and barium C unweathered types.

Potassium oxide with silica, aluminum oxide with magnesium oxide and calcium oxide, respectively, iron oxide with aluminum oxide and calcium oxide, lead oxide with nana oxide, barium oxide with copper oxide, phosphorus pentoxide with copper oxide, iron oxide, aluminum oxide and calcium oxide, strontium oxide with phosphorus pentoxide, iron oxide, copper oxide, aluminum oxide and calcium oxide, respectively, sulfur dioxide with strontium oxide, phosphorus pentoxide, barium oxide, copper oxide and potassium oxide have high correlations.

Lead and barium C weathering types.

MgO and CaO, Al<sub>2</sub>O<sub>3</sub> and MgO, FeO and Al<sub>2</sub>O<sub>3</sub>, MgO and CaO respectively, BaO and CuO, P<sub>5</sub>O<sub>2</sub> and MgO and Al<sub>2</sub>O<sub>3</sub> respectively, Strontium Oxide and Al<sub>2</sub>O<sub>3</sub>, SO<sub>2</sub> and B<sub>2</sub>O have high correlation.

### **3.5 Differential Judgments of Correlations of Chemical Composition between Different Categories**

According to the above correlation analysis for each category of artifacts, it is easy to conclude that each category has its own correlation of chemical composition, and even if it is the same high potassium type, the difference between weathered and unweathered makes a big difference in the correlation of each chemical composition, and for most of the correlation between chemical compositions, all classifications present weak correlation characteristics. However, if the classification is divided into weathering and non-weathering types, it is found that for the high potassium weathering type and lead-barium A weathering type and lead-barium C weathering type, the correlations of many groups of chemical components show a consistent state, while for the high potassium non-weathering type and lead-barium A non-weathering type and lead-barium C non-weathering type, the correlations of many groups of chemical components show a consistent state, but there are still some data with special phenomena, which are considered to be due to the difference between high potassium and lead-barium, as well as the difference in ornamentation.



This leads to the following conclusions.

For different classifications, each category has its own chemical composition correlations.

For the same broad category of weathered and unweathered, there is not the same chemical composition correlation, rather, great inconsistency occurs.

For the same weathering type or no-weathering type, there are multiple groups of chemical composition correlations that appear consistent. There are some chemical components correlations appearing inconsistent, which are thought to be caused by their types or ornamentation.

## **4. Advantages and Disadvantages of the Model**

### **4.1 Advantages of the Model**

The prediction model of the percentage of chemical components before weathering yields the relationship between weathering and type, ornamentation, and color. After the calculation, the relationship between the content of each chemical composition of the three broad categories can be derived, and finally the mean value can be used to find the prediction of weathering points before weathering.

By using the glass type system clustering model, the glass is divided into high potassium glass and lead-barium glass, while the lead-barium glass is divided into grain, and finally the three categories are divided into pre-differentiation and post-differentiation, which increases the number of categories and gives more options for different classifications of predicted data. The model was used directly for analysis under the premise of high accuracy of the glass type system clustering model. Based on the standard data derived earlier and its weights, the data to be predicted were tested one by one, and it was found that the errors were small and in accordance with the desired results.

### **4.2 Model Deficiency**

The prediction model of the percentage of chemical composition before weathering may be relatively low in the accuracy of the prediction, due to the choice of using only the mean value calculation, ignoring other influencing factors, resulting in prediction deviation, in the case of not considering the median and variance, and may even have a large error.

The glass type system clustering model reclassification does not consider the effect of color on the chemical composition content, which may lead to variations in the individual chemical composition content due to different colors. At the same time, the classification of high potassium glass only into pre-dissociation and post-dissociation, and not the classification of high potassium glass into pre-dissociation ornamentation, resulted in a reduced number of classifications, which may lead to the appearance of errors. For the classification results, only predictions were made, and no validation and testing were performed. Since the authenticity of the predicted data is not known, it cannot be determined whether it is validated correctly. Also, only the broad category to which it belongs was validated, and its color was not differentiated. Directly judging that its component distribution is of the normal distribution type may lead to a large error in the data.

### **4.3 Application of the Model**

According to the weathering before the chemical composition of the prediction model, combined with a variety of glass artifacts, whether the surface of the weathering and its relationship between the type of glass, decoration and color analysis and then combined with the type of glass, thus deriving the surface of the artifact samples with or without the weathering of chemical composition

content of the statistical law. The weathering point is then used to derive the test data, which can predict the chemical composition content before weathering.

According to the systematic clustering model of glass types, combined with the classification rules of various glass artifacts and the selection of the appropriate chemical composition of each category for its subclasses, can be derived from the specific type of glass artifacts classification methods and classification results.

## 5. Conclusion

Ancient glass objects are extremely fragile artifacts. This paper has presented, how to analyze the relationship between surface weathering of glass artifacts and their glass type, decoration and color using mathematical modeling methods, as well as the process of analyzing the statistical laws for the presence or absence of weathering on the surface of different glass types.

This paper also establishes a prediction model for the percentage of chemical composition before weathering and a systematic clustering model for glass types, and makes a practical application of the model, successfully predicting the weathering of several unknown ancient glass artifacts with an optimized fit of 100 percent, indicating that the model has a high application value.

## References

- [1] Azzeh M, Neagu D, Cowling P I. *Fuzzy grey relational analysis for software effort estimation [M]*. Kluwer Academic Publishers, 2010.
- [2] Scientific Platform Serving for Statistics Professional 2021. SPSSPRO. (Version 1.0.11)[Online Application Software]. Retrieved from <https://www.spsspro.com>.
- [3] Saroj, Kavita. Review: study on simple k mean and modified K mean clustering technique [J]. *International Journal of Computer Science Engineering and Technology*, 2016, 6(7):279-281.
- [4] Helmut Kopka, Patrick W. Daly, *Guide to LaTeX (4th Edition) [M]*. Addison Wesley/Pearson Publishers, 2005.
- [5] Mark M. Meerschaert, *Academic Press [M]*. Brooks Cole.