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Resale or Wholesale? Consider the Sales Model Choice of the Social Responsibility of the E-commerce Platform

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Abstract: To help e-commerce supply chain enterprises choose appropriate online sales models, and study how e-commerce platforms undertake corporate social responsibility to maximize profits. For the three-tier e-commerce supply chain composed of suppliers, logistics enterprises and e-commerce platforms, under the wholesale and resale online sales channels, build decentralized decision-making models and centralized decision-making models led by suppliers and e-commerce platforms respectively, and study the impact of e-commerce platforms' social responsibility on supply chain decision-making and profits. The research shows that: (1) The social responsibility of e-commerce platforms will lead to the increase of wholesale prices and retail prices of goods, which will help to increase the profits of the whole e-commerce supply chain and its members under the wholesale model, help suppliers and logistics enterprises to make profits under the resale model, but will lead to the loss of the profits of e-commerce platforms themselves. (2) Under the resale channel, the increase of the commission ratio of e-commerce platforms will reduce the profits of suppliers and logistics enterprises, and improve the profits of e-commerce platforms. However, under the leadership of suppliers, when the elasticity coefficient of e-commerce platform social responsibility is large, the excessive commission ratio will lead to the reduction of e-commerce platform profits. (3) When suppliers and e-commerce platforms dominate respectively, when the commission exceeds a certain critical value, the profits of suppliers and logistics enterprises under resale channels will be lower than those of wholesale channels, while the profits of e-commerce platforms will exceed those of wholesale channels.

1. Introduction

Under the digital empowerment and innovation-driven role of the Internet platforms, e-commerce is undergoing deep changes [1]. The market has become more complex and the competition among enterprises is becoming more brutal, and suppliers begin to build online sales channel [2]. In addition to online and offline sales channels, there are two representative models of wholesale and resale in the choice of online sales model for supply today [3]. For e-commerce supply chain enterprises, how to choose the right sales model to obtain more profits has become an important issue.

More and more scholars have started to study the issue related to the selection of sales models of

supply chain enterprises. The magnitude of sales cost in different sales models is the key factor that influences supplier's decision making [4]. When the product cost is small, suppliers are more willing to choose more sales models [5]. Scholars also concluded that factors such as service cost [6], advertising cost [7], platform transaction fee and number of customers all have an impact on the choice of sales model of suppliers [8,9]. In recent years, the research on the two online sales models, wholesale and resale, have attracted the attention of scholars. The service efficiency advantage, service competition intensity and the price competition intensity of the goods sold of e-commerce platform, can affect the choice of the two channels [10]. When the channel acceptance difference of consumers is large enough, the self-operation of the e-commerce platform can bring additional benefits to the e-commerce platform, but it will always cause losses to the manufacturers [11]. When the online channel has a negative impact on the demand of the offline channel, the agence model should be adopted; conversely, the resale model should be used [12]. Scholars examined the pricing and decision-making issues of the two online sales models from a micro perspective, and found that the commission rate [13], the consumer transfer rate [14], Service efficiency of e-commerce platforms have an impact on the sales decisions [15].

The performance of social responsibility of supply chain enterprises has an important impact on the profits of supply chain enterprises [16]. Throughout the whole process of online sales process, consumers pay special attention to the protection of their own rights and interests [17] and the concern about whether they are green and pollution-free When buying goods, and whether the sellers provide sales services in compliance, etc. reflecting the fulfillment of social responsibility behavior of the e-commerce supply chain is increasing day by day. The social responsibility of suppliers and the social responsibility of retailers in the supply chain will have an important impact on the pricing decision of the e-commerce supply chain [18,19]. So as an important part of e-commerce supply chain, does the performance of social responsibility of e-commerce platform affect the pricing and decision making of e-commerce supply chain? It is an important question with practical significance. In order to deliver the goods to consumers intact, the suppliers' online sales channels cannot be separated from the cooperation with platforms and logistics enterprises. Compared with the traditional transaction mode, e-commerce platforms not only reduces the transaction costs for both buyers and sellers, but also brings great convenience to the bilateral users. The sales channels brought by the e-commerce platform enable suppliers to have more opportunities to save costs and obtain a larger and finer market [20]. For online sales, logistics enterprises also play a pivotal role [21]. In today's situation of the normal epidemic, logistics enterprises have become a key factor directly affecting whether online sales can be carried out smoothly [22].

This paper explores the choice of wholesale and resale sales mode of suppliers, and the logistics enterprises as the third level with the supplier and e-commerce platform to constitute a level three-tier supply chain model, distinguishing between the resale and wholesale modes and the cooperative relationship between the e-commerce platform and the supplier as the leading enterprises, and considering the impact of the e-commerce platform's fulfillment of social responsibility on the profit and decision of the supply chain, so as to provide a theoretical reference for supply chain enterprises.

2. Model Construction

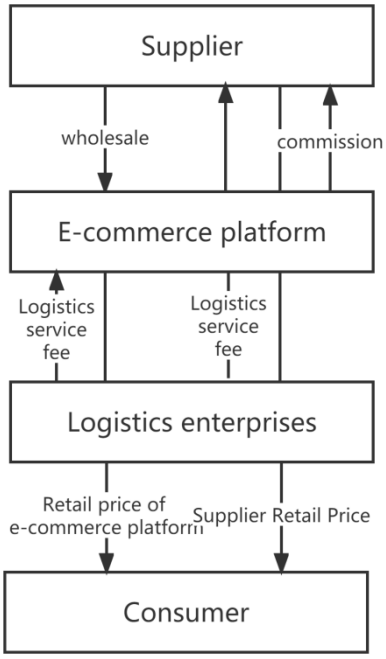

Figure 1: Three-level supply chain system under wholesale and resale mode

Table 1: Three-level supply chain system under wholesale and resale mode

Symbol	Description
π_j^i	Profit earned by enterprise j under model i, where $i=a,b,c,d,e,f$, $j=1,2,3$
p_1^i	Retail prices of suppliers under mode i, where $i=a,b,c,d,e,f$
p_2^i	Retail price of e-commerce platform under model i, where $i=a,b,c,d,e,f$
s^i	The social responsibility input of e-commerce platform under model i, where $i=a,b,c,d,e,f$
ρ	The percentage of commission charged by e-commerce platforms
w_1^i	Supplier wholesale price in mode i, where $i=a,b,c,d,e,f$
w_2^i	Logistics service pricing under Model i, where $i=a,b,c,d,e,f$

In this paper, we mainly consider the three-tier supply chain consisting of suppliers, e-commerce platforms and logistics enterprises (Figure 1). Among them, the higher the level of logistics service, the higher the service cost paid by logistics enterprises. According to Liu et al.'s study [23], it is assumed that the social responsibility cost function of e-commerce platform is $\frac{1}{2}ks^2$, where A is the input coefficient of corporate social responsibility cost and $k>0$, and Consumers' consumption intention is influenced by the price of goods and the amount of social responsibility input of e-commerce platform, assuming that the market demand is $f(p,s)=M-p+\alpha s$, where M is

the market volume, the price elasticity coefficient is 1, α is the social responsibility elasticity coefficient of e-commerce platform for market demand, p is the price of goods, and S is the amount of social responsibility input of e-commerce platform. It is assumed that the production cost of the supplier is 0, which does not affect the decision result. In order to ensure that the values of each decision in the model are positive, it is assumed that the parameters should satisfy $k-\rho\alpha^2>0$, $2k-\alpha^2>0$, $8k-3\rho\alpha^2>0$, $\rho<\frac{2k}{5\alpha^2}$.

2.1. Model of Resale

In the resale model (Table 1), the supplier sells the goods at retail prices p_1 through the e-commerce platform and pays a ρ proportional commission to the e-commerce platform. The supplier delivers the goods to the consumer through a logistics company and pay w_2 to the logistics company for the logistics services.

The model is divided into two decision-making situations: centralized and decentralized. Under centralized decision making, each member of the supply chain cooperates with the goal of maximizing the overall benefit of the supply chain. Under the decentralized decision, the profit of each enterprise in the supply chain led by different enterprises is different. According to the different dominant enterprises of e-commerce platform leading and supplier leading are constructed respectively.

According to the above assumptions,

The profit function of the supplier is:

$$\pi_1=(p_1-w_2-\rho p_1)(M-p_1+\alpha s) \tag{1}$$

The profit function of the e-commerce platform is:

$$\pi_2=\rho p_1(M-p_1+\alpha s)-ks^2/2 \tag{2}$$

The profit function of logistics enterprise is:

$$\pi_3=w_2(M-p_1+\alpha s) \tag{3}$$

The total profit function of the supply chain is:

$$\pi=p_1(M-p_1+\alpha s)-ks^2/2 \tag{4}$$

2.1.1. Centralized Decision-making

Under the centralized decision, each member of the supply chain aims at maximizing the overall benefit of the supply chain and cooperates with each other.

Calculate the first-order partial derivative of p_1^a and s^a in equation (4) respectively, set the first-order partial derivative equal to zero, and the system of joint cubic equations is solved to obtain the optimal selling price of the product under the centralized decision as:

$$p_1^a=\frac{kM}{2k-\alpha^2} \tag{5}$$

The optimal service level of e-commerce platform is:

$$s^a = \frac{\alpha M}{2k - \alpha^2} \tag{6}$$

Calculate the second partial derivative of p_1^a and s^a in equation (4), respectively, and obtain $\begin{bmatrix} -2 & \alpha \\ \alpha & -k \end{bmatrix}$ according to the Hesse matrix:

$2k - \alpha^2 > 0$, $-2 < 0$, so we get a maximum at this point.

By substituting equations (5) and (6) into equation (4), the overall optimal profit of the supply chain under wholesale channel under centralized decision-making can be obtained as follows:

$$\pi^a = \frac{kM^2}{4k - 2\alpha^2}$$

2.1.2. Decentralized Decision-making Led by E-commerce Platform

Under decentralized decision-making, the profits of each enterprise in the supply chain dominated by different enterprises are different. The following two decentralized decision mode are constructed respectively according to the different leading enterprises.

The e-commerce platform first determines the commission and social responsibility input level, and the supplier, as the weak enterprise in the supply chain, determines the retail price of the commodity according to the decision of the e-commerce platform. Logistics enterprises in order to maintain a long-term cooperative relationship with suppliers, finally determine the charging standard.

Set n^b as the supplier's markup, so $p_1^b - \rho p_1^b = w_2^b + n^b$, get:

$$p_1^b = \frac{w_2^b + n^b}{1 - \rho} \tag{7}$$

By substituting equation (7) into equation (3), we can get:

$$\pi_3^b = w_2^b (M + \alpha s^b - \frac{w_2^b + n^b}{1 - \rho}) \tag{8}$$

Find the first derivative of w_2^b in equation (8), and set the first derivative function as 0, we can get:

$$w_2^{b*} = \frac{(1 - \rho)(M + \alpha s^b) - n^b}{2} \tag{9}$$

Take the second derivative with respect to w_2^b in equation (8), get $\frac{-2}{1 - \rho} < 0$ so obtain the maximum value at this point.

Substitute equations (7) and (9) into equations (1), and find the first derivative with respect to n, and set the first derivative function as 0, get:

$$n^{b*} = \frac{(1 - \rho)(M + \alpha s^b)}{2} \tag{10}$$

The second derivative of π in equation (1) yields $\frac{1}{\rho-1}<0$, so the maximum is obtained at this point.

Formula (7), (9) and (10) are substituted into Formula (2), and the first derivative of s^b in equation (2) is calculated, and the first derivative function is set as 0, to obtain the optimal service level of e-commerce platform:

$$s^b=\frac{3\rho\alpha M}{8k-3\rho\alpha^2} \tag{11}$$

The second derivative of s^b in equation (2) yields $3\rho\alpha^2-8k<0$, so the maximum is obtained at this point.

By substituting equation (11) into Equation (10), the supplier's optimal markup can be obtained as follows:

$$n^b=\frac{4(1-\rho)kM}{8k-3\rho\alpha^2} \tag{12}$$

By substituting equations (11) and (12) into equations (9), the optimal pricing of logistics enterprises can be obtained as follows:

$$w_2^b=\frac{2(1-\rho)kM}{8k-3\rho\alpha^2} \tag{13}$$

By substituting equations (12) and (13) into equations (7), the optimal retail price of the supplier can be obtained as follows:

$$p_1^b=\frac{6kM}{8k-3\rho\alpha^2} \tag{14}$$

Equations (11), (13) and (14) are substituted into equations (1), (2) and (3) to obtain the maximum profits of suppliers, e-commerce platforms, logistics enterprises and the whole supply chain at this time:

$$\pi_1^b=\frac{8(1-\rho)k^2M^2}{(8k-3\rho\alpha^2)^2}$$

$$\pi_2^b=\frac{12\rho k^2M^2-9\rho^2k\alpha^2M^2}{2(8k-3\rho\alpha^2)^2}$$

$$\pi_3^b=\frac{4(1-\rho)k^2M^2}{(8k-3\rho\alpha^2)^2}$$

$$\pi^b=\pi_1^b+\pi_2^b+\pi_3^b$$

2.1.3. Decentralized Decision Making under Vendor Leadership

The supplier, as the dominant company in the supply chain, first decides the sales price, followed by the logistics companies that cooperate to determine the logistics pricing. Finally the e-commerce

platform, which is in a weak position, sets the commission and platform service level.

Set n^c as the supplier's markup, so $p_1^c - \rho p_1^c = w_2^c + n^c$, get:

$$p_1^c = \frac{w_2^c + n^c}{1-\rho} \tag{15}$$

By substituting equation (15) into equation (2), Find the first derivative of n^c in equation (2), and set the first derivative function as 0, we can get:

$$s^{c*} = \frac{\rho\alpha(w_2^c + n^c)}{(1-\rho)k} \tag{16}$$

Take the second derivative with respect to s^c in equation (2), get $-k < 0$, so obtain the maximum value at this point.

Substituting equations (15) and (16) into Equation (3), and calculating the first derivative of w_2^c in Equation (3), and setting the first derivative as 0, we can get:

$$w_2^{c*} = \frac{(1-\rho)kM - (k-\rho\alpha^2)n^c}{2(k-\rho\alpha^2)} \tag{17}$$

The second derivative of w_2^c in equation (1) yields $\frac{-2(k-\rho\alpha^2)}{(1-\rho)k} < 0$, so the maximum is obtained at this point.

Formula (15), (16) and (17) are substituted into Formula (1), and the first derivative of n^c in equation (1) is calculated, and the first derivative function is set as 0, to obtain the optimal service level of e-commerce platform:

$$n^c = \frac{(1-\rho)kM}{2(k-\rho\alpha^2)} \tag{18}$$

The second derivative of n^c in equation (1) yields $\frac{k-\alpha^2\rho}{k(-1+\rho)} < 0$, so the maximum is obtained at this point.

By substituting equations (18) into equations (17), the optimal pricing of logistics enterprises can be obtained as follows:

$$w_2^c = \frac{(1-\rho)kM}{4(k-\rho\alpha^2)} \tag{19}$$

By substituting equations (18) and (19) into equations (15) and (16), the optimal retail price of the supplier the optimal service level of the e-commerce platform can be obtained as follows:

$$p_1^c = \frac{3kM}{4(k-\rho\alpha^2)} \tag{20}$$

$$s^c = \frac{3\rho\alpha M}{4(k-\rho\alpha^2)} \tag{21}$$

Equations (19), (20) and (21) are substituted into equations (1), (2) and (3) to obtain the maximum

profits of suppliers, e-commerce platforms, logistics enterprises and the whole supply chain at this time:

$$\pi_1^c = \frac{(1-\rho)kM^2}{8(k-\rho\alpha^2)}$$

$$\pi_2^c = \frac{6\rho k^2 M^2 - 15\rho^2 k\alpha^2 M^2}{32(k-\rho\alpha^2)^2}$$

$$\pi_3^c = \frac{(1-\rho)kM^2}{16(k-\rho\alpha^2)}$$

$$\pi^c = \pi_1^c + \pi_2^c + \pi_3^c$$

Proposition 1: p_1^a, p_1^b, p_1^c, s_1^a, s_1^b, s_1^c, w_2^b and w_2^c, is positively correlated with α (see Appendix for propositional proofs).

From Proposition 1, it is clear that supplier retail price, e-commerce platform socially responsible input level and logistics pricing increase with the increase of e-commerce platform socially responsible elasticity coefficient under centralized versus decentralized decision making. The increase in the social responsibility elasticity coefficient of e-commerce platforms implies that consumers are more likely to purchase products from e-commerce platforms with high levels of socially responsible inputs and are willing to pay higher prices for such products. Therefore, e-commerce platforms will increase the level of socially responsible investment in e-commerce platforms in order to meet consumers' needs. The increase in retail price of supplier makes logistics companies set higher logistics pricing to obtain more profit.

Proposition 2: π_1^b, π_1^c, π_3^b, and π_3^c is positively correlated with α, and π_2^b and π_2^c is negatively correlated with α.

Proposition 2 shows that the profits of suppliers and logistics enterprises increase with the increase of social responsibility elasticity coefficient of e-commerce platform. The increase of social responsibility elasticity coefficient of e-commerce platform means that consumers are more inclined to buy products purchased by e-commerce platform with high level of social responsibility input, and e-commerce platform will increase the level of social responsibility input of e-commerce platform in order to meet consumers' needs, which helps to expand market demand, and the increase of retail price and logistics pricing of suppliers makes the profit of suppliers and logistics enterprises increase, because the commission of e-commerce platform under resale channel is lower The increase in supplier retail prices and logistics pricing increases the profits of suppliers and logistics companies, and the lower commissions of e-commerce platforms under the resale channel makes retailers' profits suffer.

Proposition 3: p_1^b, p_1^c, s_1^b, and s_1^c is positively correlated with ρ, w_2^b and w_2^c is negatively correlated with ρ.

From Proposition 3, it can be seen that the retail price of suppliers and the level of socially responsible input of e-commerce platform under centralized and decentralized decision increases with the increase of commission ratio of e-commerce platform, and the logistics pricing decreases with the increase of commission ratio of e-commerce platform. The increase of commission ratio of e-commerce platform makes the suppliers increase their retail price to ensure their own profit, and the cooperative logistics companies choose to reduce the logistics pricing in order to maintain the cooperative relationship because the suppliers' profit is damaged. The increase of commission ratio of

e-commerce platform makes e-commerce platform have more money to invest in social responsibility cost.

Proposition 4: π_2^b is positively correlated with ρ, π_2^b is positively correlated with ρ when $\rho < \frac{k}{4\alpha^2}$, and is negatively correlated with ρ when $\rho > \frac{k}{4\alpha^2}$ π_1^c, π_3^b and π_3^c is negatively correlated with ρ.

From Proposition 4, it can be seen that the profits of suppliers and logistics companies decrease with the increase of commission ratio of e-commerce platform under decentralized decision making. Under e-commerce platform domination, the profit of e-commerce platform increases with the increase of e-commerce platform commission ratio. Under the supplier-led decision, the profit of e-commerce platform increases with the increase of commission ratio of e-commerce platform when the commission ratio of e-commerce platform is less than a certain value, and the profit of e-commerce platform decreases with the increase of commission ratio of e-commerce platform when the commission ratio of e-commerce platform is greater than a certain value. The increase of commission ratio of e-commerce platform reduces the retail price of suppliers and logistics pricing, which leads to the decrease of profit of suppliers and logistics enterprises. When the e-commerce platform is dominant, the increase in commission percentage enables the e-commerce platform to earn more profit. When the supplier dominates, the increase of commission ratio makes the supplier's retail price increase, and the excessive increase of retail price leads to the decrease of commodity demand, which makes the profit of e-commerce platform decrease when the commission ratio exceeds a certain value.

Proposition 5: $\pi_1^b < \pi_1^c$, $\pi_2^b > \pi_2^c$, $\pi_3^b < \pi_3^c$.

From Proposition 5, it can be seen that suppliers and e-commerce platforms can gain more profit under decentralized decision making in the resale model when each dominates. Logistics companies can gain more profits when the e-commerce platform dominates.

2.2. Wholesale Model

In the wholesale model, the supplier wholesales the goods to the e-commerce platform at wholesale price w_1. The e-commerce platform sells the goods at retail price b p_2 and delivers the goods to the consumers through the logistics enterprise, paying the logistics enterprise w_2 for the pricing of logistics services.

The profit function of the supplier is:

$$\pi_1 = w_1(M - p_2 + \alpha_1 s) \tag{22}$$

The profit function of the e-commerce platform is:

$$\pi_2 = (p_2 - w_1 - w_2)(M - p_2 + \alpha s) - ks^2/2 \tag{23}$$

The profit function of logistics enterprise is:

$$\pi_3 = w_2(M - p_2 + \alpha s) \tag{24}$$

The total profit function of the supply chain is:

$$\pi = p_2(M - p_2 + \alpha s) - ks^2/2 \tag{25}$$

The model is divided into two types of decision-making situations: centralized and decentralized. Under centralized decision making, each member of the supply chain cooperates with the goal of maximizing the overall benefit of the supply chain. Under the decentralized decision, the profit of each enterprise in the supply chain led by different enterprises is different. According to the different dominant enterprises, two decentralized decision mode are constructed, namely, e-commerce platform-dominated and supplier-dominated.

2.2.1. Centralized Decision-making

Under the centralized decision, each member of the supply chain aims at maximizing the overall benefit of the supply chain and cooperates with each other.

Calculate the first-order partial derivative of p_2^d and s^d in equation (4) respectively, set the first-order partial derivative function equal to zero, and the system of joint cubic equations is solved to obtain the optimal selling price of the product under the centralized decision as:

$$p_2^d = \frac{kM}{2k-\alpha^2} \tag{26}$$

The optimal service level of e-commerce platform is:

$$s^d = \frac{\alpha M}{2k-\alpha^2} \tag{27}$$

Calculate the second partial derivative of p_2^d and s^d of equation (4) respectively, and obtain according to the Hesse matrix $\begin{bmatrix} -2 & \alpha \\ \alpha & -k \end{bmatrix}$, get:

$2k-\alpha^2>0$, $-2<0$, so we get a maximum at this point.

By substituting equations (26) and (27) into Equation (25), the overall optimal profit of the supply chain under wholesale channel under centralized decision-making can be obtained as follows:

$$\pi^d = \frac{kM^2}{4k-2\alpha^2}$$

2.2.2. Decentralized Decision-making Led by E-commerce Platform

Under decentralized decision-making, the profits of each enterprise in the supply chain dominated by different enterprises are different. The following two decentralized decision mode are constructed respectively according to the different leading enterprises.

The e-commerce platform first determines the commission and service level, and the supplier, as the weak enterprise in the supply chain, determines the retail price of the commodity according to the decision of the e-commerce platform. Logistics enterprises in order to maintain a long-term cooperative relationship with suppliers, finally determine the charging standard.

Set π as the supplier's markup, so $p_2^e - w_1^e - w_2^e = m^e$, get:

$$p_2^e = w_1^e + w_2^e + m^e \tag{28}$$

By substituting equation (28) into equation (24), we can get:

$$\pi_3^e = w_2^e(M + \alpha s^e - w_1^e - w_2^e - m^e) \tag{29}$$

Find the first derivative of w_2^e in equation (29), and set the first derivative function as 0, we can get:

$$w_2^{e*} = \frac{M + \alpha s^e - w_1^e - m^e}{2} \tag{30}$$

Take the second derivative with respect to w_2^e in equation (29), get $-2 < 0$ so obtain the maximum value at this point.

Substitute equations (28) and (30) into equations (22), and find the first derivative with respect to n, and set the first derivative function as 0, get:

$$w_1^{e*} = \frac{M + \alpha s^e - m^e}{2} \tag{31}$$

The second derivative of w_1^e in equation (22) yields $\frac{1}{\rho - 1} < 0$, so the maximum is obtained at this point.

Formula (28), (30) and (31) are substituted into Formula (22), and the first derivative of s^e and m^e in equation (22) is calculated, and the first derivative function is set as 0, to obtain the optimal service level of e-commerce platform and the best markup on e-commerce platform:

$$s^e = \frac{\alpha M}{8k - \alpha^2} \tag{32}$$

$$m^e = \frac{4kM}{8k - \alpha^2} \tag{33}$$

The second derivative of s^e and m^e in equation (22), According to the Hesse matrix $\begin{bmatrix} -\frac{1}{2} & \frac{\alpha}{4} \\ \frac{\alpha}{4} & -k \end{bmatrix}$, yields $\frac{k}{2} - \frac{\alpha^2}{16} > 0$, $-\frac{1}{2} < 0$, so the maximum is obtained at this point.

By substituting equation (32) and (33) into Equation (31), the supplier's wholesale price is:

$$w_1^e = \frac{2kM}{8k - \alpha^2} \tag{34}$$

By substituting equations (32) and (33) into equations (34), the optimal pricing of logistics enterprises can be obtained as follows:

$$w_2^e = \frac{kM}{8k - \alpha^2} \tag{35}$$

By substituting equations (33) and (34) into equations (28), the optimal retail price of the supplier can be obtained as follows:

$$p_2^e = \frac{7kM}{8k-\alpha^2} \tag{36}$$

Equations (32), (34), (35) and (36) are substituted into equations (21), (22) and (23) to obtain the maximum profits of suppliers, e-commerce platforms, logistics enterprises and the whole supply chain at this time:

$$\pi_1^e = \frac{2k^2M^2}{(8k-\alpha^2)^2}$$

$$\pi_2^e = \frac{kM^2}{16k-2\alpha^2}$$

$$\pi_3^e = \frac{k^2M^2}{(8k-\alpha^2)^2}$$

$$\pi^e = \pi_1^e + \pi_2^e + \pi_3^e$$

2.2.3. Decentralized Decision Making under Vendor Leadership

As the leading enterprise in the supply chain, the supplier first decides the selling price, and then the logistics partner determines the logistics pricing. Finally, the e-commerce platform in a weak position sets the commission and platform service level.

Set n as the supplier's markup, so $p_2^f - w_1^f - w_2^f = m^f$, get:

$$p_2^f = w_1^f + w_2^f + m^f \tag{37}$$

By substituting equation (37) into equation (23), find the first derivative of s^f and m in equation (22), and set the first derivative function as 0, we can get:

$$s^{f*} = \frac{\alpha(M - w_1^f - w_2^f)}{2k-\alpha^2} \tag{38}$$

$$m^{f*} = \frac{k(M - w_1^f - w_2^f)}{2k-\alpha^2} \tag{39}$$

Take the second derivative with respect to s^f and m in equation (23), According to the Hesse matrix $\begin{bmatrix} -2 & \alpha \\ \alpha & -k \end{bmatrix}$, get $2k-\alpha^2 > 0$, $-2 < 0$, so obtain the maximum value at this point.

Substitute equations (37), (38) and (39) into equations (24), and find the first derivative of n in equation (24), and set the first derivative function as 0, get:

$$w_2^{f*} = \frac{M - w_1^f}{2} \tag{40}$$

The second derivative of w_2^f in equation (24) yields $\frac{-2k}{2k-\alpha^2} < 0$, so the maximum is obtained at this point.

Formula (37), (38), (39) and (40) are substituted into Formula (22), and the first derivative of w_1^f in equation (22) is calculated, and the first derivative function is set as 0, to obtain the best wholesale price of suppliers is:

$$w_1^f = \frac{M}{2} \tag{41}$$

By substituting equation (41) into Equation (40), the optimal pricing of logistics enterprises can be obtained as follows:

$$w_2^f = \frac{M}{4} \tag{42}$$

By substituting equations (41) and (42) into equations (38) and (39), the optimal service level and markup of e-commerce platform can be obtained as follows:

$$s^f = \frac{\alpha M}{8k - 4\alpha^2} \tag{43}$$

$$m^f = \frac{kM}{8k - 4\alpha^2} \tag{44}$$

Equation (41), (42) and (44) are substituted into Equation (37) to obtain the optimal sales price of e-commerce platform:

$$p_2^f = \frac{M\left(7k - 3\alpha^2\right)}{8k - 4\alpha^2} \tag{45}$$

Equations (41), (42), (43) and (45) are substituted into equations (22), (23) and (24) to obtain the maximum profits of suppliers, e-commerce platforms, logistics enterprises and the supply chain as a whole:

$$\pi_1^f = \frac{kM^2}{16k - 8\alpha^2}$$

$$\pi_2^f = \frac{kM^2}{64k - 32\alpha^2}$$

$$\pi_3^f = \frac{kM^2}{32k - 16\alpha^2}$$

$$\pi^f = \pi_1^f + \pi_2^f + \pi_3^f$$

Proposition 6: p_1^d, p_1^e, p_1^f, s_1^d, s_1^e, s_1^f, w_2^e, w_2^f, w_1^e and w_1^f is positively correlated with α.

From Proposition 1, it is clear that supplier retail price, e-commerce platform socially responsible input level, logistics pricing, and e-commerce platform retail price increase with the increase of e-commerce platform social responsibility elasticity coefficient under centralized versus decentralized decision making. The increase in the social responsibility elasticity coefficient of e-commerce platforms implies that consumers are more likely to purchase products from e-commerce

platforms with high levels of socially responsible inputs and are willing to pay higher prices for such products. Therefore, e-commerce platforms will increase the level of socially responsible investment in order to meet the needs of consumers. The cost investment makes e-commerce platforms set higher e-commerce platform retail prices.

Proposition 7: π_1^b, π_1^c, π_2^b, π_2^c, π_3^b, π_3^c is positively correlated with α.

Proposition 6 shows that the profits of suppliers, e-commerce platforms and logistics companies increase with the increase of social responsibility elasticity coefficient of e-commerce platforms under decentralized decision making. The increase of social responsibility elasticity coefficient of e-commerce platform means that consumers are more inclined to buy products purchased by e-commerce platform with high level of social responsibility input, and e-commerce platform will increase the level of social responsibility input of e-commerce platform to meet consumers' needs and expand market demand, and the increase of retail price of e-commerce platform, wholesale price of suppliers and logistics pricing makes the profit of e-commerce platform, suppliers and logistics enterprises increase.

Proposition 8: $\pi_1^e < \pi_1^f$, $\pi_2^e > \pi_2^f$, $\pi_3^e < \pi_3^f$.

From Proposition 8, it can be seen that suppliers and e-commerce platforms can gain more profits under decentralized decision making in the wholesale model when each dominates. Logistics companies can gain more profits when the e-commerce platform dominates.

3. The Comparison between Models

By comparing the optimal decisions of different models, the following conclusions can be drawn (see Appendix for proof):

Conclusion 1: when $\rho < \frac{2\left(-64k^2+28k\alpha^2-\alpha^4+\sqrt{2}\sqrt{2048k^4-1792k^3\alpha^2+672k^2\alpha^4-100k\alpha^6+5\alpha^8}\right)}{9\alpha^4}$, $\pi_1^e - \pi_1^b < 0$, when $\frac{2\left(-64k^2+28k\alpha^2-\alpha^4+\sqrt{2}\sqrt{2048k^4-1792k^3\alpha^2+672k^2\alpha^4-100k\alpha^6+5\alpha^8}\right)}{9\alpha^4} < \rho < \frac{2k}{5\alpha^2}$, $\pi_1^e - \pi_1^b > 0$. when $\alpha > \frac{2\sqrt{k}}{\sqrt{5}}$, $\pi_1^f - \pi_1^c < 0$; when $\rho < \frac{1}{2}$, $\pi_1^f - \pi_1^c < 0$; when $\frac{1}{2} < \rho < \frac{2k}{5\alpha^2}$, $\pi_1^f - \pi_1^c > 0$.

Conclusion 1 shows that under e-commerce platform domination, the supplier's profit under the resale channel is greater than the profit under the wholesale channel when the e-commerce platform commission ratio is less than a certain value, and the supplier's profit under the resale channel is less than the profit under the wholesale channel when the e-commerce platform commission ratio is greater than a certain value. Under supplier-led, when $\alpha < \frac{2\sqrt{k}}{\sqrt{5}}$, the supplier's profit under the resale channel is greater than the profit under the wholesale channel when the e-commerce platform commission ratio is less than a certain value, and the supplier's profit under the resale channel is less than the profit under the wholesale channel when the e-commerce platform commission ratio is greater than a certain value. When $\alpha > \frac{2\sqrt{k}}{\sqrt{5}}$, the supplier's profit under the resale channel is greater than the profit under the wholesale channel. The supplier's profit under the resale channel is greater than the profit under the wholesale channel. When the e-commerce platform commission ratio is too large, supplier profits are impaired, resulting in lower supplier profits under the resale channel. The

profit under the resale channel is always larger than the profit under the wholesale channel when the social responsibility elasticity coefficient of the e-commerce platform is low when the supplier dominates.

Conclusion 2: when $\rho < \frac{8k+3\alpha^2-\sqrt{64k^2-80k\alpha^2+9\alpha^4}}{12\alpha^2}$, $\pi_2^e-\pi_2^b>0$, when $\frac{8k+3\alpha^2-\sqrt{64k^2-80k\alpha^2+9\alpha^4}}{12\alpha^2}<\rho<\frac{2k}{5\alpha^2}$, $\pi_2^e-\pi_2^b<0$ .when $\frac{kM^2\left[k^2(1-12\rho)-14\alpha^4\rho^2+2k\alpha^2\rho(2+15\rho)\right]}{32(2k-\alpha^2)(k-\alpha^2\rho)^2}>0$, $\pi_2^f-\pi_2^c>0$; when $\frac{kM^2\left[k^2(1-12\rho)-14\alpha^4\rho^2+2k\alpha^2\rho(2+15\rho)\right]}{32(2k-\alpha^2)(k-\alpha^2\rho)^2}<0$, $\pi_2^f-\pi_2^c<0$.

Conclusion 2 shows that under the dominant e-commerce platform, the profit of the e-commerce platform under the resale channel is smaller than the profit under the wholesale channel when the commission ratio of the e-commerce platform is less than a certain value, and the profit of the supplier under the resale channel is larger than the profit under the wholesale channel when the commission ratio of the e-commerce platform is larger than a certain value. Under the supplier-dominated channel, the supplier's profit.

Conclusion 3: when $\pi_3^e-\pi_3^b<0$ and $\alpha>\frac{2\sqrt{k}}{\sqrt{5}}$ $\pi_3^f-\pi_3^c<0$; when $\alpha<\frac{2\sqrt{k}}{\sqrt{5}}$, when $\rho<\frac{1}{2}$, $\pi_3^f-\pi_3^c<0$, when $\frac{1}{2}<\rho<\frac{2k}{5\alpha^2}$, $\pi_3^f-\pi_3^c>0$.

Conclusion 3 illustrates that logistics firms' profits under the resale channel are greater than those under the wholesale channel when the e-commerce platform dominates. When $\alpha<\frac{2\sqrt{k}}{\sqrt{5}}$, Logistics firms' profits under the resale channel are greater than those under the wholesale channel when the e-commerce platform commission ratio is less than a certain value, and logistics firms' profits under the resale channel are less than those under the wholesale channel when the e-commerce platform commission ratio is greater than a certain value. When $\alpha>\frac{2\sqrt{k}}{\sqrt{5}}$, Logistics firms' profits under the resale channel are greater than those under the wholesale channel when the profit under the resale channel is greater than the profit under the wholesale channel. When the commission ratio of e-commerce platform is too high, the profits of logistics enterprises cooperating with suppliers are damaged, resulting in lower profits of logistics enterprises under the resale channel.

4. Numerical Analysis

In order to present the research results more intuitively, the above conclusions are further verified by specific arithmetic examples. It is assumed $M=100$, $k=2$, and verified, that the range of the above parameter settings satisfies the condition of the existence of the optimal solution of the model.

Taking α and ρ as independent variables and making $\alpha\in(0,1]$ and $\rho\in(0,1)$ respectively, the variation of each node and overall profit of the supply chain under the supplier, e-commerce platform dominated resale and wholesale channels with α and ρ are plotted as follows.

As can be seen from Figure 2, in both dominant cases, the supplier wholesale profit is smaller than the resale profit when the commission ratio of the e-commerce platform is low, and the supplier resale profit decreases as the commission ratio increases, and the resale profit is lower than

the wholesale profit when a certain threshold value is exceeded. The critical value under the supplier-led model is smaller than the critical value under the e-commerce platform-led model. Therefore, the supplier should choose resale mode when the commission of the partner e-commerce platform is set low; therefore, the supplier should choose wholesale mode when the commission of the partner e-commerce platform is set high. In both dominant cases, suppliers' profit in wholesale mode and profit in resale mode are positively correlated with the social responsibility elasticity coefficient of e-commerce platforms, so suppliers should choose to cooperate with e-commerce platforms that are willing to take social responsibility.

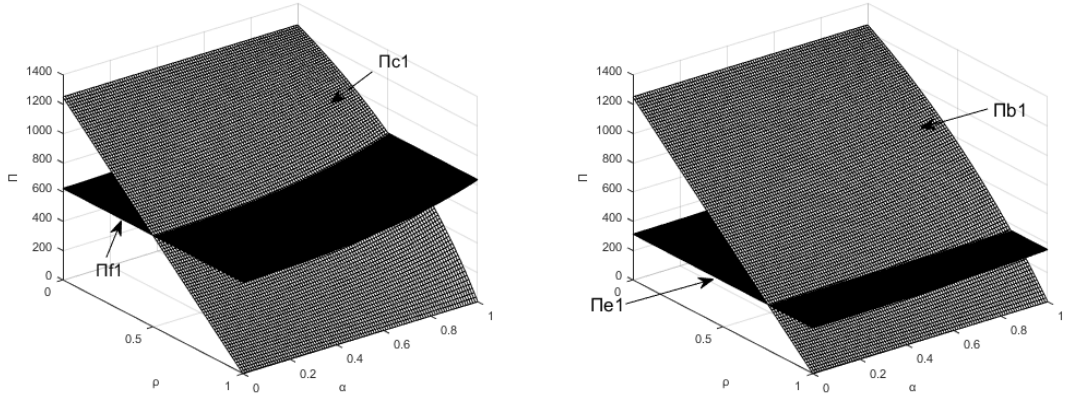

Figure 2: Comparison of profit of supplier resale model and wholesale model under e-commerce platform and supplier dominance

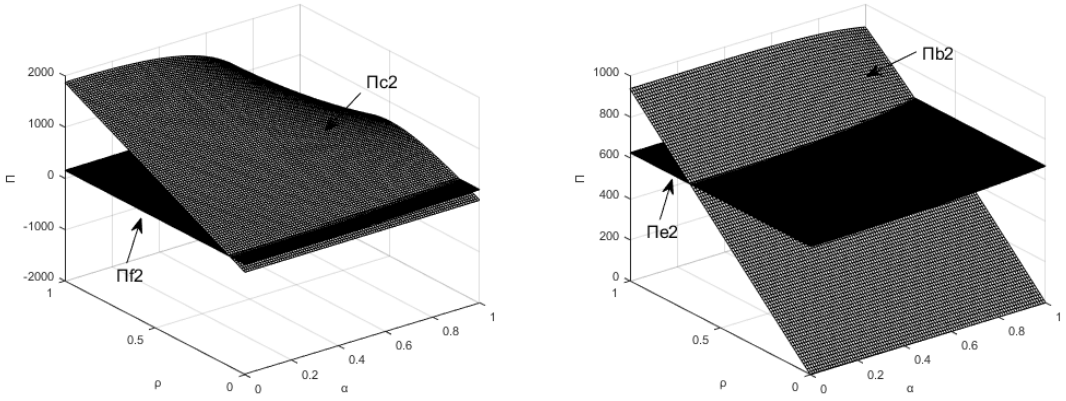

Figure 3: Comparison of profit between resale model and wholesale model under e-commerce platform dominance and supplier dominance

It can be seen from Figure 3 that in both dominant cases, the wholesale mode of e-commerce platform is larger than the resale mode profit when the commission ratio of e-commerce platform is lower, and the profit of resale channel of suppliers increases with the increase of commission ratio, and the profit of resale mode will exceed the profit of wholesale mode when a certain threshold value is exceeded. The critical value under supplier-led is less than the critical value under e-commerce platform-led. Therefore, the e-commerce platform should reasonably set the commission range to make its own profit, and avoid the commission ratio is too high, the supplier choose the wholesale mode and give up the resale mode, especially when it is dominated by itself, it is better to choose the resale mode. In the supplier-led, e-commerce platform social responsibility elasticity coefficient is large, too high commission ratio will instead bring the e-commerce platform

profit reduction. The profit of wholesale mode of e-commerce platform under two dominant cases is positively correlated with the elasticity coefficient of social responsibility of e-commerce platform, and the profit of resale mode of e-commerce platform under two dominant cases is negatively correlated with the elasticity coefficient of social responsibility of e-commerce platform, so the supplier should increase the social responsibility investment when choosing wholesale mode.

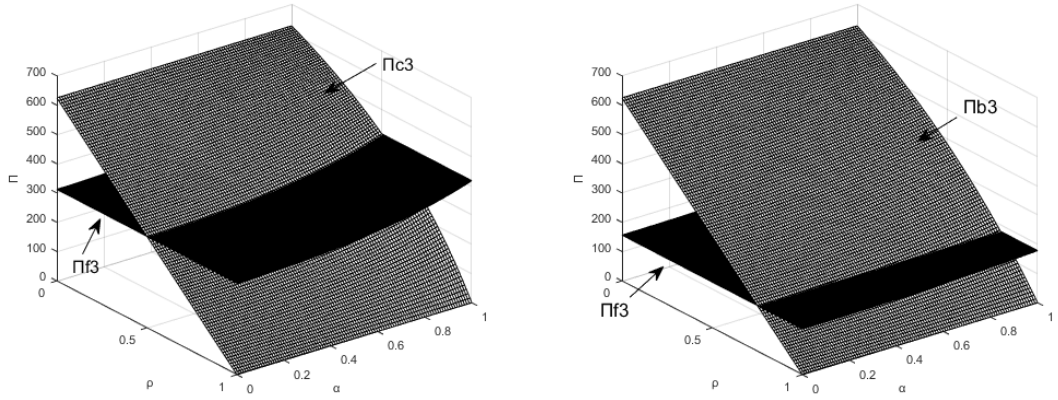

Figure 4: Comparison between resale profit and wholesale profit of logistics enterprises under e-commerce platform and supplier dominance

As can be seen from Figure 4, in both dominant cases, the wholesale mode profit of logistics enterprises is smaller than the resale mode profit when the commission ratio of e-commerce platform is lower, and the resale mode profit of suppliers decreases with the increase of commission ratio, and the resale mode profit will be lower than the wholesale mode profit when a certain threshold value is exceeded. The critical value under supplier-led is smaller than the critical value under e-commerce platform-led. In both cases, the supplier's wholesale profit and resale profit are positively correlated with the social responsibility elasticity coefficient of the e-commerce platform, and logistics enterprises should pursue the supplier-led cooperation model.

5. Summary

This paper constructs a three-tier e-commerce supply chain consisting of e-commerce platform, suppliers and logistics enterprises, establishes four decentralized and centralized decision models under two online sales channels, resale and wholesale, in which suppliers and retailers are dominant respectively, compares and analyzes the impact of socially responsible inputs of e-commerce platform on the optimal pricing decision and profit of supply chain members under different situations, and gives suggestions on the choice of The model also gives suggestions on the choice of sales model under different dominance.

The main conclusions obtained are as follows: (1) Under the situation where suppliers and e-commerce platforms are dominant respectively, the e-commerce platform's assumption of social responsibility will bring about an increase in the wholesale and retail prices of goods, which will help to increase the profits of the e-commerce supply chain as a whole and each member in the wholesale mode, and will help suppliers and logistics enterprises to make profits in the resale mode, but will lead to the damage of the e-commerce platform's own profits. (2) Under the resale channel, the increase of the commission ratio of the e-commerce platform will reduce the profits of suppliers and logistics enterprises and enhance the profits of the e-commerce platform. However, when the social responsibility elasticity coefficient of e-commerce platform is large under the supplier-led model, the excessive commission ratio will bring down the profit of e-commerce platform instead.

(3) In the case where the supplier and the e-commerce platform are dominant, the supplier's wholesale mode profit is smaller than the resale mode profit when the commission ratio of the e-commerce platform is low, and the supplier's resale mode profit decreases with the increase of the commission ratio, and the resale mode profit is lower than the wholesale mode profit when a critical value is exceeded. (4) In the case where the supplier and the e-commerce platform are dominant, the wholesale mode of the e-commerce platform is larger than the resale mode profit when the commission ratio of the e-commerce platform is low, and the resale mode profit of the supplier increases with the increase of the commission ratio, and the resale mode profit exceeds the wholesale mode profit when a certain threshold value is exceeded. (5) In the case where the supplier and the e-commerce platform are dominant, the profit of the logistics enterprise in wholesale mode is smaller than the profit of the resale mode when the commission ratio of the e-commerce platform is lower, and the profit of the supplier in resale mode decreases with the increase of the commission ratio, and the profit of the resale mode will be lower than the profit of the wholesale mode when a critical value is exceeded.

The management implications of this study are: (1) From the perspective of the e-commerce supply chain, e-commerce platforms should take more social responsibility to help increase the overall profit of the supply chain. (2) From the supplier's perspective, suppliers should pursue supply chain dominance and seek cooperation with e-commerce platforms that are willing to take social responsibility. Since the e-commerce platform will not set too high commission ratio, suppliers should choose resale mode when the e-commerce platform is dominant. Choose the resale model when the commission ratio of the supplier-led cooperation platform is low, and choose the wholesale model when the commission ratio is high. (3) From the perspective of the e-commerce platform, the e-commerce platform in resale mode should take more social responsibility in favor of its own profit. The commission should not be set too low to damage their own profits, and should not be too high to make suppliers give up the resale mode, especially when the elasticity coefficient of social responsibility of e-commerce platform is large under the supplier-led, the commission ratio may be set too high to damage their own interests. (4) From the perspective of logistics enterprises, logistics enterprises should pursue cooperation with supplier-led supply chain.

In the study of this paper, only the case of e-commerce platform alone taking social responsibility in the three-tier e-commerce supply chain is considered; in the future, the case of other members in the supply chain sharing social responsibility can be studied and the impact of designing contractual contracts on supply chain coordination can be considered.

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Appendix

Proposition 1:

$$\frac{\partial p_1^a}{\partial \alpha} = \frac{2kM\alpha}{\left(-2k+\alpha^2\right)^2} > 0$$

$$\frac{\partial p_1^b}{\partial \alpha} = \frac{36kM\alpha\rho}{\left(8k-3\alpha^2\rho\right)^2} > 0$$

$$\frac{\partial p_1^c}{\partial \alpha} = \frac{3kM\alpha\rho}{2\left(k-\alpha^2\rho\right)^2} > 0$$

$$\frac{\partial s^a}{\partial \alpha} = \frac{M\left(2k+\alpha^2\right)}{\left(-2k+\alpha^2\right)^2} > 0$$

$$\frac{\partial s^b}{\partial \alpha}=\frac{3M\rho\left(8k+3\alpha^2\rho\right)}{\left(8k-3\alpha^2\rho\right)^2}>0$$

$$\frac{\partial s^c}{\partial \alpha}=\frac{3M\rho\left(k+\alpha^2\rho\right)}{4\left(k-\alpha^2\rho\right)^2}>0$$

$$\frac{\partial w_2^b}{\partial \alpha}=\frac{12kM\alpha\left(1-\rho\right)\rho}{\left(8k-3\alpha^2\rho\right)^2}>0$$

$$\frac{\partial w_2^c}{\partial \alpha}=\frac{kM\alpha\left(1-\rho\right)\rho}{2\left(k-\alpha^2\rho\right)^2}>0$$

Proposition 2:

$$\frac{\partial \pi_1^b}{\partial \alpha}=-\frac{96k^2M^2\alpha\left(-1+\rho\right)\rho}{\left(8k-3\alpha^2\rho\right)^3}>0$$

$$\frac{\partial \pi_1^c}{\partial \alpha}=-\frac{kM^2\alpha\left(-1+\rho\right)\rho}{4\left(k-\alpha^2\rho\right)^2}>0$$

$$\frac{\partial \pi_2^b}{\partial \alpha}=-\frac{27kM^2\alpha^3\rho^3}{\left(8k-3\alpha^2\rho\right)^3}<0$$

$$\frac{\partial \pi_2^c}{\partial \alpha}=-\frac{15kM^2\alpha\rho^2}{16\left(k-\rho\alpha^2\right)^2}<0$$

$$\frac{\partial \pi_3^b}{\partial \alpha}=\frac{48k^2M^2\alpha\left(1-\rho\right)\rho}{\left(8k-3\alpha^2\rho\right)^3}>0$$

$$\frac{\partial \pi_3^c}{\partial \alpha}=\frac{kM^2\alpha\left(1-\rho\right)\rho}{8\left(k-\alpha^2\rho\right)^2}>0$$

Proposition 3:

$$\frac{\partial p_1^b}{\partial \rho}=\frac{18kM\alpha^2}{\left(8k-3\alpha^2\rho\right)^2}>0$$

$$\frac{\partial p_1^c}{\partial \rho}=\frac{3kM\alpha^2}{4\left(k-\alpha^2\rho\right)^2}>0$$

$$\frac{\partial s^b}{\partial \rho}=\frac{24kM\alpha}{\left(8k-3\alpha^2\rho\right)^2}>0$$

$$\frac{\partial s^c}{\partial \rho}=\frac{3kM\alpha}{4\left(k-\alpha^2\rho\right)^2}>0$$

$$\frac{\partial w_2^b}{\partial \rho} = -\frac{2kM\left(8k-3\alpha^2\right)}{\left(8k-3\alpha^2\rho\right)^2} < 0$$

$$\frac{\partial w_2^c}{\partial \rho} = -\frac{kM\left(k-\alpha^2\right)}{4\left(k-\alpha^2\rho\right)^2} < 0$$

Proposition 4:

$$\frac{\partial \pi_1^b}{\partial \rho} = -\frac{8k^2M^2\left[8k+3\alpha^2\left(-2+\rho\right)\right]}{\left(8k-3\alpha^2\rho\right)^3} < 0$$

$$\frac{\partial \pi_1^c}{\partial \rho} = -\frac{kM^2\left(k-\alpha^2\right)}{8\left(k-\alpha^2\rho\right)^2} < 0$$

$$\frac{\partial \pi_2^b}{\partial \rho} = \frac{6k^2M^2\left(8k-9\alpha^2\rho\right)}{\left(8k-3\alpha^2\rho\right)^3} > 0$$

$$\frac{\partial \pi_2^c}{\partial \rho} = \frac{3k^2M^2\left(k-4\alpha^2\rho\right)}{16\left(k-\alpha^2\rho\right)^3}$$

When $\rho < \frac{k}{4\alpha^2}$, $\frac{\partial \pi_2^c}{\partial \rho} > 0$ When $\rho > \frac{k}{4\alpha^2}$, $\frac{\partial \pi_2^c}{\partial \rho} < 0$.

$$\frac{\partial \pi_3^b}{\partial \rho} = -\frac{4k^2M^2\left[8k+3\alpha^2\left(-2+\rho\right)\right]}{\left(8k-3\alpha^2\rho\right)^3} < 0$$

$$\frac{\partial \pi_3^c}{\partial \rho} = -\frac{kM^2\left(k-\alpha^2\right)}{16\left(k-\alpha^2\rho\right)^2} < 0$$

Proposition 5:

$$\pi_1^b - \pi_1^c = \frac{kM^2\alpha^2\left(-1+\rho\right)\rho\left(16k+9\alpha^2\rho\right)}{8\left(8k-3\alpha^2\rho\right)^2\left(k-\alpha^2\rho\right)} < 0$$

$$\pi_2^b - \pi_2^c = -\frac{3kM^2\rho\left(64k^3-240k^2\alpha^2\rho+98k\alpha^4\rho^2+3\alpha^6\rho^3\right)}{32\left(8k-3\alpha^2\rho\right)^2\left(k-\alpha^2\rho\right)^2} > 0$$

$$\pi_3^b - \pi_3^c = \frac{kM^2\alpha^2\left(-1+\rho\right)\rho\left(16k+9\alpha^2\rho\right)}{16\left(8k-3\alpha^2\rho\right)^2\left(k-\alpha^2\rho\right)} < 0$$

Proposition 6:

$$\frac{\partial p_2^d}{\partial \alpha} = \frac{2kM\alpha}{\left(-2k+\alpha^2\right)^2} > 0$$

$$\frac{\partial p_2^e}{\partial \alpha} = \frac{20kM\alpha}{\left(8k-\alpha^2\right)^2} > 0$$

$$\frac{\partial p_2^f}{\partial \alpha} = \frac{kM\alpha}{2\left(-2k+\alpha^2\right)^2} > 0$$

$$\frac{\partial s^d}{\partial \alpha} = \frac{M\left(2k+\alpha^2\right)}{\left(-2k+\alpha^2\right)^2} > 0$$

$$\frac{\partial s^e}{\partial \alpha} = \frac{M\left(8k+\alpha^2\right)}{\left(-8k+\alpha^2\right)^2} > 0$$

$$\frac{\partial s^f}{\partial \alpha} = \frac{M\left(2k+\alpha^2\right)}{4\left(-2k+\alpha^2\right)^2} > 0$$

$$\frac{\partial w_2^e}{\partial \alpha} = \frac{2kM\alpha}{\left(8k-\alpha^2\right)^2} > 0$$

$$\frac{\partial w_2^f}{\partial \alpha} = 0$$

$$\frac{\partial w_1^e}{\partial \alpha} = \frac{4kM\alpha}{\left(8k-\alpha^2\right)^2} > 0$$

$$\frac{\partial w_1^f}{\partial \alpha} = 0$$

Proposition 7:

$$\frac{\partial \pi_1^e}{\partial \alpha} = \frac{8k^2M^2\alpha}{\left(8k-\alpha^2\right)^3} > 0$$

$$\frac{\partial \pi_1^f}{\partial \alpha} = \frac{kM^2\alpha}{4\left(-2k+\alpha^2\right)^2} > 0$$

$$\frac{\partial \pi_2^e}{\partial \alpha} = \frac{kM^2\alpha}{\left(-8k+\alpha^2\right)^2} > 0$$

$$\frac{\partial \pi_2^f}{\partial \alpha} = \frac{kM^2\alpha}{16\left(-2k+\alpha^2\right)^2} > 0$$

$$\frac{\partial \pi_3^e}{\partial \alpha} = \frac{4k^2M^2\alpha}{\left(8k-\alpha^2\right)^3} > 0$$

$$\frac{\partial \pi_3^f}{\partial \alpha} = \frac{kM^2\alpha}{8\left(-2k+\alpha^2\right)^2} > 0$$

Proposition 8:

$$\pi_1^e - \pi_1^f = kM^2\left[\frac{2k}{\left(-8k+\alpha^2\right)^2} + \frac{1}{8\left(-2k+\alpha^2\right)}\right]$$

$16k-8\alpha^2-\left(-8k+\alpha^2\right)\left(-4+\frac{\alpha^2}{2k}\right)=-16k-\frac{\alpha^4}{2k}<0$, therefore $\frac{2k}{\left(-8k+\alpha^2\right)^2}+\frac{1}{8\left(-2k+\alpha^2\right)}<0$, $\pi_1^e-\pi_1^f<0$.

$$\pi_2^e-\pi_2^f=\frac{3kM^2\left(8k-5\alpha^2\right)}{32\left(16k^2-10k\alpha^2+\alpha^4\right)}>0$$

$$\pi_3^e-\pi_3^f=kM^2\left[-\frac{1}{32k-16\alpha^2}+\frac{k}{\left(-8k+\alpha^2\right)^2}\right]$$

$32k-16\alpha^2-\left(-8k+\alpha^2\right)\left(-8+\frac{\alpha^2}{k}\right)=-32k-\frac{\alpha^4}{k}<0$, therefore $-\frac{1}{32k-16\alpha^2}+\frac{k}{\left(-8k+\alpha^2\right)^2}<0$, $\pi_3^e-\pi_3^f<0$.

Conclusion 1:

$$\pi_1^e-\pi_1^b=2k^2M^2\left[\frac{1}{\left(-8k+\alpha^2\right)^2}+\frac{4\left(-1+\rho\right)}{\left(8k-3\alpha^2\rho\right)^2}\right]$$

When $\rho<\frac{2\left(-64k^2+28k\alpha^2-\alpha^4+\sqrt{2}\sqrt{2048k^4-1792k^3\alpha^2+672k^2\alpha^4-100k\alpha^6+5\alpha^8}\right)}{9\alpha^4}$, $\pi_1^e-\pi_1^b<0$, When $\frac{2\left(-64k^2+28k\alpha^2-\alpha^4+\sqrt{2}\sqrt{2048k^4-1792k^3\alpha^2+672k^2\alpha^4-100k\alpha^6+5\alpha^8}\right)}{9\alpha^4}<\rho<\frac{2k}{5\alpha^2}$, $\pi_1^e-\pi_1^b>0$.

$$\pi_1^f-\pi_1^c=\frac{1}{8}kM^2\left(\frac{1}{2k-\alpha^2}+\frac{-1+\rho}{k-\alpha^2\rho}\right)$$

When $\alpha>\frac{2\sqrt{k}}{\sqrt{5}}$, $\pi_1^f-\pi_1^c<0$; When $\alpha<\frac{2\sqrt{k}}{\sqrt{5}}$, When $\rho<\frac{1}{2}$, $\pi_1^f-\pi_1^c<0$, When $\frac{1}{2}<\rho<\frac{2k}{5\alpha^2}$, $\pi_1^f-\pi_1^c>0$.

Conclusion 2:

$$\pi_2^e-\pi_2^b=\frac{2k^2M^2\left[9\alpha^2\rho\left(-1+2\rho\right)-8k\left(-2+3\rho\right)\right]}{\left(8k-\alpha^2\right)\left(8k-3\alpha^2\rho\right)^2}$$

When $\rho<\frac{8k+3\alpha^2-\sqrt{64k^2-80k\alpha^2+9\alpha^4}}{12\alpha^2}$, $\pi_2^e-\pi_2^b>0$, When $\frac{8k+3\alpha^2-\sqrt{64k^2-80k\alpha^2+9\alpha^4}}{12\alpha^2}<\rho<\frac{2k}{5\alpha^2}$, $\pi_2^e-\pi_2^b<0$.

$$\pi_2^f-\pi_2^c=\frac{kM^2\left[k^2\left(1-12\rho\right)-14\alpha^4\rho^2+2k\alpha^2\rho\left(2+15\rho\right)\right]}{32\left(2k-\alpha^2\right)\left(k-\alpha^2\rho\right)^2}$$

Conclusion 3:

$$\pi_3^e - \pi_3^b = k^2 M^2 \left[\frac{1}{\left(-8k + \alpha^2\right)^2} + \frac{4\left(-1+\rho\right)}{\left(8k - 3\alpha^2\rho\right)^2} \right]$$

Because of $\rho < \frac{2k}{5\alpha^2}$, therefore $\pi_3^e - \pi_3^b < 0$.

$$\pi_3^f - \pi_3^c = \frac{kM^2\left(k-\alpha^2\right)\left(-1+2\rho\right)}{16\left(2k-\alpha^2\right)\left(k-\alpha^2\rho\right)}$$

When $\alpha > \frac{2\sqrt{k}}{\sqrt{5}}$, $\pi_3^f - \pi_3^c < 0$; When $\alpha < \frac{2\sqrt{k}}{\sqrt{5}}$, When $\rho < \frac{1}{2}$, $\pi_3^f - \pi_3^c < 0$, When $\frac{1}{2} < \rho < \frac{2k}{5\alpha^2}$, $\pi_3^f - \pi_3^c > 0$.