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A Many-Objective Evolutionary Strategy Based on Angle Dominance

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Abstract: Evolutionary multi-objective optimization (EMO) algorithm is a new multi-objective optimization algorithm developed in recent years, which has a broad application prospect in dealing with multi-objective optimization problems. This paper recognizes some recent efforts and discusses some feasible directions to develop potential EMO algorithms for solving high-dimensional objective optimization problems. A many-objective evolutionary strategy based on angle dominance (MaOES-AD) is proposed. The proposed MaOES -AD is applied to a many-objective test problem with multiple objectives and compared with recently proposed algorithms. It is proved that the proposed MaOES-AD algorithm achieves satisfactory results on all the considered problems.

1. Introduction

Many-objective optimization problems (MaOPs) are the key problems in engineering and scientific research that must be solved. Since MaOP has multiple conflicting objectives, an improvement in the performance of one objective may induce a degradation in the performance of another or more objectives. It is often impossible to achieve the optimum for all objectives simultaneously. With the increase of dimension, all kinds of the dynamic, nonlinear, and non-differentiable characteristics of MaOP can also lead to the complexity of the multi-objective optimization and searching space increase sharply.And it’s hard to find a suitable one for different MaOP general methods to solve all of these. Solving the evolutionary computation field at home and abroad with MaOP has become a hot issue which is difficult to address.

At present, Researchers have proposed many relatively mature multi-objective intelligent optimization improvement methods. Multi-objective optimization algorithms based on the Pareto dominance criterion have proved their effectiveness in dealing with multi-objective optimization problems with 2 or 3 objectives (MOPs), such as SPEA2 0, NSGA-II 0, etc. However, the multi-objective optimization algorithms based on Pareto dominance do not perform well when dealing with super-multi-objective optimization problems with 4 or more objectives (MaOPs). As the number of objectives of the problem to be optimized increases, more and more individuals become non-dominated by each other under the traditional Pareto dominance criterion, resulting in some promising individuals cannot be selected for the next generation, which is also called the "curse of dimensionality".The fundamental reason that the traditional Pareto dominance criterion fails in the high-dimensional objective space is that the traditional Pareto dominance criterion is too strict. In the past period, some studies have relaxed the traditional Pareto rule, which makes the improved Pareto dominance relation adapt to deal with super multi-objective optimization problems. For example, the ϵ-dominance method 0 employs a relaxation factor ϵ to compare the dominant relationships between individuals. Pierro et al. proposed the preference ranking method to replace the traditional non-dominated ranking 0. Based on this, fuzzy dominance methods 0 are proposed to study the fuzziness of Pareto dominance relations and to design ranking schemes to select promising solutions. L-optimality paradigm was proposed in 0, which selects solutions with the same importance as the objective by considering the improvement of the objective value. In addition, Yang et al. proposed grid-based methods 0 to select solutions with higher dominance priority and control the proportion of Pareto optimal solutions by adjusting the grid size.

The density estimation method based on Euclidean distance will bring a very large computational burden when dealing with super multi-objective optimization problems. Therefore, the algorithm proposed a new way of selecting individuals. The basic idea is that the Angle of selecting the critical layer of Lk individual set a minimum of two individual xp and xq, because the Angle between the two individuals to a minimum, shows that the search direction is almost the same, so you just need to delete the poor performance of an individual, and then continue to repeat this step, to choose the appropriate number of individuals. When measuring individual performance, a measurement method based on Shift-based Density Estimation (SDE) and Sum Of Objective (SO) dynamics is used, which pays more attention to convergence in the early stage. And the later period is more focused on diversity.

In this paper, a method based on Angle dominance is used as the first selection criterion. This method only allows a few parameters and is not sensitive to parameters. Finally, the Angle dominance is transplanted to NSGA-II and SPEA2, and compared with the original NSGA-II and SPEA2. The experimental data show that the NSGA-II and SPEA2 based on Angle dominance are better than the original NSGA-II and SPEA2 in dealing with super-multi-objective optimization problems. The experimental results show that MaOES-AD has more advantages than the other algorithms in dealing with high-dimensional problems.

2. Background

2.1. Problems of Traditional Parent Dominance Criterion

Although the multi-objective optimization algorithm based on the traditional Pareto dominance criterion has made remarkable achievements in many studies in the past, there are still some problems with the traditional Pareto dominance criterion. These problems can be roughly divided into two categories: The problem of Dominance Resistant Solutions (DRSs) cannot be solved and the comparison criterion based on the Pareto dominance criterion fails in high-dimensional space, resulting in the problem that most individuals in the population do not dominate each other and the selection pressure is insufficient.

Firstly, anti-dominated individuals are those who have a slight advantage in only one or a few target values but are very poor in most target values. Figure 1 shows the 2 goals under the minimization problem of two typical forms of resistance to dominate the existence of the individual, the individual x1 in the F1 dimensions has a minimum target, but in the F2 dimension the target is very poor, and the individual x1 true Pareto frontier far distance, if the individual x1 into the next generation will damage the convergence of population. However, under the standard of Pareto dominance, individual x1 cannot be eliminated. An effective method is, to sum up, all the objective values of the individual, through which individual x1 will obtain the maximum objective sum and eliminate individual x1, which is reflected in the environmental selection in MaOES-AD. Individual x4 is also a typical anti-domination individual. When compared with individual x3, individual x4 has better F2 and worse F1. When comparing individual x4 with individual x5, F1 is better and F2 is worse. But in fact, individual x4 is farther away from the true Pareto front than individual x3 and individual x5, so it needs to be eliminated.

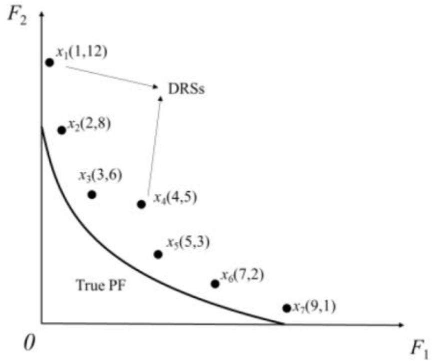

Figure 1: Example diagram.

Secondly, Pareto dominance is widely used in 2-or 3-objective optimization problems and effectively distinguishes individuals, but it fails in many-objective optimization problems. To solve the problem that the Pareto dominance criterion fails in dealing with super-multi-objective optimization problems, the paper adopts an angle-based dominance strategy, which relaxes the definition of Pareto dominance and expands the area of individual dominance, so that the convergence pressure of the algorithm is increased when dealing with super-multi-objective optimization problems.

2.2. The Angle Control

First, find the lowest point in the population zw={z1w, ..., zmw}. The detailed definitions and schemata of the minimum points have been given in Section 2.2. Then, the vector zw is enlarged according to a preset parameter k to obtain z'={k*z1w,..., k*zmw}. It is very difficult for us to do this. Next, z' is projected onto each axis, according to Equation (1):

$$\alpha_i = \arccos \frac{P_i o \cdot P_i p}{|P_i o| \cdot |P_i p|} \quad (1)$$

Calculate the angle Pi between the individual p and the projection point of each axis, and form a vector anglep=(a1,..., aM). Finally, the angle vector anglep of individual p is used to replace the objective value of individual p to do Pareto dominance.

To illustrate Angle dominance, Figure 2 shows a case in point. The shaded part in the figure is the area of the region dominated by individual p after using Angle dominance, which is larger than the area dominated by individual p under traditional Pareto dominance, which is a relaxation of the strict Pareto dominance.

The first benefit of using angular dominance is that it addresses the problem of dominance-resistant individuals (DRSs) under traditional Pareto dominance entering the next generation and compromising the convergence of the population. The second benefit of angular dominance is that it relaxes the definition of traditional Pareto dominance, expands the area of individual dominance, and increases selection pressure.

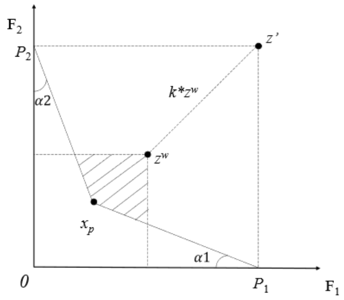

Figure 2: Diagram of Angle domination.

2.3. NSGA-II and SPEA2

The most important concept in NSGA-II 0 is crowding distance, a parameter used to estimate the density of a particular solution. Taking the case of two objectives as an example, the average distance between the solution and the two adjacent solutions under each objective is calculated respectively. This value can be regarded as the perimeter of a rectangle formed by the two adjacent solutions on both sides of a particular solution as the two diagonal vertices. As shown in Figure 3, the black dots represent solutions in the same layer, and the crowding distance of the ith solution is equal to the perimeter of the rectangle formed by its two adjacent solutions as the two diagonal endpoints, as shown in the dashed line. When the distance is larger, the solution and its neighbors are more scattered, and the difference between the solution and its neighbors is larger. The smaller this distance is, the denser the solution and the two adjacent solutions, and the higher the similarity between this solution and the surrounding solutions. Through the computation of this flow, a crowding distance is obtained for each solution in the same layer. For the solution of the same layer, the larger the crowding distance is, the greater the difference between the objective function value of the solution and the adjacent solution is, that is, the better the diversity of the solution is, so it should be preferentially selected in the case of the same order value.

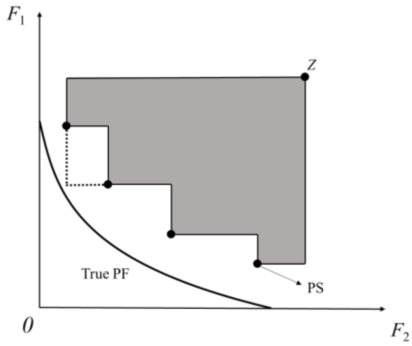

Figure 3: Diagram of strength domination.

Strength Pareto Evolutionary Algorithm (SPEA) 0 is a multi-objective evolutionary algorithm proposed by Zitzler and Thiele et al. Its main feature is to establish an external archive set, which is used to store the Pareto optimal solution generated by each evolution, to realize the elite retention strategy. The size of the external archive set affects the efficiency of the SPEA algorithm. The larger the external archive set is, the lower the efficiency of the algorithm. To maintain the scale of the external archive set, SPEA adopts a clustering and pruning strategy to keep the original properties of the Pareto-optimal solution set. However, SPEA still has the following shortcomings:

(1) Individuals dominated by the same group of non-dominated individuals have the same fitness;

(2) The fitness of individuals is only determined by the dominant relationship between

individuals, without considering the influence of individual distribution;

(3) Using the clustering method to prune the archive set may cause the loss of non-dominated solutions;

In 2001, Zitzler and Thiele proposed SPEA2 0. The improvement of SPEA2 is mainly reflected in the following aspects:

(1) Fine-grained fitness Settings

When calculating fitness, both the number of individuals dominated by an individual in the population and the number of individuals dominated by it are considered.

(2) New environment selection strategy

The environment selection strategy of SPEA2 is to set the external archive set size to a fixed size M, select excellent individuals from the current population P and external archive set E, and copy them into the next generation external archive set Et+1.

3. Design

3.1. Framework

In this section, the many-objective optimization strategy based on Angle dominance (MaOES-AD) will be introduced in detail.

Algorithm 3-1 MaOES-AD
Input:
Number: N; Maximum number of iterations: Gen_{Max}
Output: population P
Initialization:
Randomly generated initial populations P
While the *termination criterion is not fulfilled* Gen_{Max}
P using genetic factors to produce the next populations P'
Combining P and P' to produce a bound population Q
Normalizing Q produces a normalized population Q'
Angle-based non-dominated sorting for Q'
Calculation for Q' based on transfer density estimation and dynamics of the target sum
Environmental selection was performed on Q' and N promising individuals were selected to form the next generation population P
End

Algorithm 3-1 shows the overall framework and flow of the MaOES-AD. MaOES-AD starts with a randomly generated initialized population. In the iterative process, two genetic factors, binary crossover 0 and polynomial mutation 0, will act on the parent population P of the previous generation to produce the next generation P'. Subsequently, the parent population P and offspring population P'are combined to generate a bound population Q of size 2N, and Q is normalized to generate the normalized bound population Q'. Next, the algorithm performs angle-based non-dominated sorting on the normalized combined population Q'and obtains the dominated layer {L1,... , Lk,... , Lmax} which Lk is a critical layer. In addition to the angle-based non-dominated sorting on Q', it is also necessary to calculate the dynamic fitness of Q'based on transition density estimation and objective sum. Finally, the algorithm will perform environment selection to select the N most promising individuals to form the parent population P of the next generation. The evolution iteration will continue until the end condition of the algorithm is satisfied.

3.2. Normalization

After initialization, MaOES-AD uses the simulated binary crossover (SBX) 0 and polynomial mutation 0, to generate the offspring population. By using crossover, mutation genetic agents on the parent population P, an offspring population P' will be generated, and MaOES-AD will combine P and P' to form a combined population Q with population size 2N. Subsequently, MaOES-AD will normalize the bound population Q to produce the normalized bound population Q'. Algorithm 3-2 shows the pseudo-code for the normalization. First, an ideal point z*={z1*,..., zm*}is constructed according to all individuals in the combined population Q. and the least point zw={z1w,... , zmw}, including zi* = minfi(x), ziw = maxfi(x). After that, the target value of xq combined with each individual of Q in the population is transformed.

Algorithm 3-2 Normalization
Input: Combining populations Q
Output: Bound populations after normalization Q'
For i = 1 to M
Calculate $z_i^* = \min f_i(x_q)$
Calculate $z_i^w = \min f_i(x_q)$
End
For i = 1 : 2N
Normalization by $f_i^{'}\left(x_q\right) = \dfrac{f_i\left(x_q\right) - z_i^*}{z_i^w - z_i^*}$ and saved in Q'
End

3.3. Sort

In this step, the Angle vector of all individuals in the normalized combined population Q 'is first calculated according to Equation (1), and then the Angle vector is used to replace the target value vector to obtain the non-dominated layer {L1,..., Lk,..., Lmax} which Lk is a critical layer. Subsequently, each individual will be assigned a non-dominated layer number Li, which will be used as the first selection criterion in the construction of the crossover pool and environmental selection. The smaller the layer number, the closer the individual is to the true Pareto front, and the better the convergence.

3.4. Fitness

It is not enough to take Angle dominance as the first selection criterion, because a non-dominated solution set with good convergence without good diversity is still not what we want. Therefore, measures to ensure diversity need to be introduced, and the pseudo-code for calculating the fitness is shown in Algorithms 3-3. MaOES-AD uses two indicators to comprehensively consider the fitness of an individual, which are the sum of the objective value and the transfer-based Density Estimation (SDE) 0.

In general, solutions in the evolution process have different priorities at different stages, and individual convergence can be achieved in the early stage so that individuals can quickly approach the true Pareto front. However, more priority should be given to diversity and distribution in the later stages of evolution, where the algorithm mainly focuses on obtaining uniformly distributed solution sets with good diversity. According to the above analysis, a calculation method that emphasizes convergence in the early stage and diversity in the later stage is given in Formula (2).

Algorithm 3-3 Fitness
Input:
Q' Current number of iterations: Gen_{cur} Maximum number of iterations: Gen_{Max}
Output: *Fit*
For i = 1 to 2*N*
Calculate SO(x_i)
Calculate SDE(x_i, *Q'*)
Calculate *Fit*(x_i)
End

$$fit\left(x_p\right)=\mu*\frac{1}{SDE\left(x_p,Q'\right)}+(1-\mu)*SO\left(x_p\right) \quad (2)$$

$$\mu=\frac{Gen_{cur}}{Gen_{\max}} \quad (3)$$

According to the above analysis, the fitness calculation function of MaOES-AD is finally defined as Eq. (2). Moreover, μ is defined as equation (3), where Gencur represents the current algebra and Genmax refers to the total algebra. According to equation (2), in the early stage of the algorithm, the fitness Fit (xp) of individual xp mainly depends on the objective of individual xp and SO (xp). The individual objective sum provides an efficient way to measure the convergence of the solution. However, in the later stage of the algorithm, the fitness Fit (xp) of individual xp mainly depends on the transfer-based density estimate SDE (xp, P) of individual xp. Therefore, by a reasonable combination of SDE and SO, we can obtain the individuals we want at different stages of the algorithm, and finally achieve to obtain a set of non-dominated solution sets with good convergence and uniformly distributed on the true Pareto front.

3.5. Selection

Algorithm 3-4 Selection
input: *N*
Output: P_{next}
Add individuals in the $\{L_1,\ldots,L_{k-1}\}$ layer to P_{next}
Calculate the vector angle between every two bodies in L_k
While $\lvert P_{next}\rvert + \lvert L_k\rvert > N$
Find the two individuals x_p and x_q with the smallest vector angles in L_k
If $Fit(x_p) > Fit(x_q)$
Remove x_p from L_k
Else
Remove x_q from L_k
End
Add all individuals in L_k to P_{next}
End

The specific details of the environment selection are given in Algorithms 3-4.

The objective of environmental selection is to select N most promising individuals from the normalized combined population Q' of size 2N to form the parent population Pnext of the next generation. In MaOES-AD's environment selection, the individuals in the first k−1 non-dominated layer are first added to the parent population Pnext in the next generation, and the vector Angle between every pair of individuals in the critical layer Lk is calculated. Then, individuals started to be removed from Lk cyclically until the sum of the number of individuals in Pnext and Lk is equal to N, as in lines 4 to 11 in Algorithm 33-4. In the process of cyclic deletion, the two individuals xq and xq with the smallest vector Angle in Q' are first found. Since individuals xq and xq have the smallest vector Angle between them at Lk, it can be considered that individuals xq and xq search in basically the same direction. Next, the one with the worse fitness value is selected and deleted.

4. Comparison

4.1. Test the Problem and Parameter Settings

The test set used in the experiments in this subsection is the widely used DTLZ1-DTLZ7 0. The parameters for the test set are shown in Table 1.

Table 1: Description of parameter Settings and properties for test problems DTLZ1-DTLZ7.

Problem	Objective	Decision
DTLZ1	8,10,15,20	M-1+k (k=5)
DTLZ2	8,10,15,20	M-1+k (k=10)
DTLZ3	8,10,15,20	M-1+k (k=10)
DTLZ4	8,10,15,20	M-1+k (k=10)
DTLZ5	8,10,15,20	M-1+k (k=10)
DTLZ6	8,10,15,20	M-1+k (k=10)
DTLZ7	8,10,15,20	M-1+k (k=20)

4.2. Experimental Result

In this experiment, SPEA2 and NSGA-II algorithms based on Angle dominance (AD-SPEA2 and AD-NSGA-II) are compared with the original SPEA2 and NSGA-II algorithms, respectively. The performance index used to evaluate the advantages and disadvantages of the algorithm is IGD 0, which is a performance index that can comprehensively evaluate convergence and diversity. Tables 2 show the results of the IGD of SPEA2 and AD-SPEA2 on the DTLZ test problem, and Tables 3 show the results of the IGD of NSGA-II and AD-NSGA-II on the DTLZ test problem. Among them, the black bold part indicates that the results are optimal in this experiment.

From Table 2, we can observe that AD-SPEA2 has significantly better IGD results than SPEA2 on 28 different test problems, and IGD has similar results to SPEA2 on 3 test problems. From Table 3, we can observe that AD-NSGA-II has significantly better IGD results than NSGA-II on 28 test problems, while IGD and NSGA-II have similar results on 4 test problems. According to the experimental results, we can conclude that when dealing with many-objective optimization problems, the IGD index of the strategy based on Angle dominance and Angle density estimation is 40% higher than that of the traditional dominating method SPEA2. This indicates that the strategy based on Angle dominance and Angle density estimation can obtain better distribution results on the Pareto front when dealing with the optimization problem with heavy objectives compared with the traditional dominance method for continuous optimization.

From Table 3, it can be found that the strategy based on Angle dominance and Angle density estimation has a great improvement in IGD index compared with the traditional algorithm based on dominance relationship NSGA-II when dealing with multi-objective optimization problems with heavy objectives, which indicates that compared with the traditional multi-objective optimization problem based on dominance relationship for continuous optimization, the strategy based on Angle dominance and Angle density estimation has a great improvement in IGD index. The strategy based on Angle dominance and Angle density estimation can obtain better approximation results of the Pareto front when dealing with optimization problems with heavy objectives.

Table 2: Results of IGD for SPEA2 and AD-SPEA2 on the DTLZ test problem.

Problem	Objective	Decision	SPEA2	AD-SPEA2
DTLZ1	8	12	1.0340e+2 (4.55e+1) -	**1.2335e+1 (5.69e+0)**
	10	14	2.1643e+2 (2.88e+1) -	**5.5742e+1 (4.37e+1)**
	15	19	2.4332e+2 (1.04e+2) -	**1.5372e+2 (9.70e+1)**
	20	24	2.6330e+2 (3.61e+1) -	**1.9036e+2 (7.06e+1)**
DTLZ2	8	17	2.4353e+0 (2.40e-2) -	**1.4315e+0 (3.41e-1)**
	10	19	2.5386e+0 (1.44e-2) =	**1.9159e+0 (5.08e-1)**
	15	24	2.6434e+0 (3.03e-2) =	**2.1497e+0 (4.83e-1)**
	20	29	2.7379e+0 (1.57e-2) -	**2.3848e+0 (5.02e-1)**
DTLZ3	8	17	1.2537e+3 (2.46e+2) -	**8.9905e+1 (3.27e+1)**
	10	19	1.5772e+3 (1.51e+2) -	**5.9560e+2 (3.40e+2)**
	15	24	1.7527e+3 (6.03e+1) -	**1.0691e+3 (6.24e+2)**
	20	29	1.7517e+3 (8.59e+1) -	**1.3077e+3 (5.16e+2)**
DTLZ4	8	17	2.4565e+0 (2.84e-2) -	**5.2622e-1 (2.46e-2)**
	10	19	2.5350e+0 (3.81e-2) -	**7.7536e-1 (7.06e-2)**
	15	24	2.6594e+0 (2.21e-2) -	**1.0752e+0 (9.79e-2)**
	20	29	2.7371e+0 (2.14e-2) -	**1.3022e+0 (1.33e-1)**
DTLZ5	8	17	2.0886e+0 (5.65e-1) -	**7.3340e-1 (2.16e-1)**
	10	19	1.7392e+0 (7.41e-1) -	**6.9998e-1 (1.88e-1)**
	15	24	2.0490e+0 (7.14e-1) -	**7.8134e-1 (3.70e-1)**
	20	29	2.1446e+0 (6.46e-1) -	**1.0033e+0 (5.14e-1)**
DTLZ6	8	17	9.9511e+0 (4.53e-2) -	**7.8586e+0 (7.79e-1)**
	10	19	9.9667e+0 (3.57e-2) -	**7.7322e+0 (1.06e+0)**
	15	24	9.9983e+0 (5.21e-2) -	**8.9953e+0 (8.35e-1)**
	20	29	1.0057e+1 (6.03e-2) =	**9.4050e+0 (7.23e-1)**
DTLZ7	8	27	1.6090e+0 (2.24e-1) -	**9.2089e-1 (3.52e-2)**
	10	29	2.2867e+0 (3.28e-1) -	**1.2812e+0 (2.61e-2)**
	15	34	6.3068e+0 (2.39e+0) -	**1.8137e+0 (2.62e-2)**
	20	39	7.0284e+0 (2.70e+0) -	**2.2557e+0 (6.58e-2)**
+/-/=			0/25/3	

Table 3: Results of IGD of NSGA-II and AD-NSGA-II on the DTLZ test problem.

Problem	Objective	Decision	NSGA2	AD-NSGA2
DTLZ1	8	12	**1.2117e+1 (5.96e+0) -**	1.3965e+1 (5.11e+0)
	10	14	1.7620e+1 (5.11e+0) =	**1.3710e+1 (7.41e+0)**
	15	19	**1.9835e+1 (5.99e+0) =**	2.0846e+1 (9.76e+0)
	20	24	**1.9364e+1 (7.71e+0) =**	2.1580e+1 (9.78e+0)
DTLZ2	8	17	2.1863e+0 (1.17e-1) -	**1.1165e+0 (1.19e-1)**
	10	19	2.0057e+0 (4.40e-1) -	**1.1871e+0 (1.28e-1)**
	15	24	1.5881e+0 (1.96e-1) -	**1.2532e+0 (3.53e-2)**
	20	29	1.5994e+0 (1.26e-1) -	**1.3543e+0 (7.88e-2)**
DTLZ3	8	17	1.1276e+3 (3.22e+2) -	**1.3439e+2 (3.87e+1)**
	10	19	1.1890e+3 (2.33e+2) -	**2.0193e+2 (4.93e+1)**

	15	24	1.2609e+3 (2.27e+2) -	**1.8619e+2 (5.31e+1)**
	20	29	8.5908e+2 (1.54e+2) -	**1.9231e+2 (3.68e+1)**
DTLZ4	8	17	1.9669e+0 (3.54e-1) -	**4.7496e-1 (8.84e-3)**
	10	19	1.7789e+0 (1.95e-1) -	**5.9171e-1 (1.25e-2)**
	15	24	1.6248e+0 (1.08e-1) -	**7.2934e-1 (1.87e-2)**
	20	29	1.6323e+0 (1.33e-1) -	**7.8506e-1 (6.92e-3)**
DTLZ5	8	17	2.9448e-1 (1.43e-1) -	**2.5233e-1 (9.23e-2)**
	10	19	4.4091e-1 (1.71e-1) -	**2.7247e-1 (1.89e-1)**
	15	24	6.5979e-1 (2.24e-1) -	**3.9717e-1 (1.92e-1)**
	20	29	8.0712e-1 (2.62e-1) -	**4.2745e-1 (2.00e-1)**
DTLZ6	8	17	7.0100e+0 (6.87e-1) -	**5.7686e+0 (7.84e-1)**
	10	19	7.3826e+0 (5.29e-1) -	**6.6279e+0 (6.18e-1)**
	15	24	7.4175e+0 (6.49e-1) -	**6.1911e+0 (5.60e-1)**
	20	29	7.5113e+0 (6.21e-1) -	**6.8704e+0 (7.98e-1)**
DTLZ7	8	27	2.1679e+0 (6.08e-1) -	**1.7590e+0 (2.39e-1)**
	10	29	5.6668e+0 (1.91e+0) -	**5.2267e+0 (1.27e+0)**
	15	34	1.6619e+1 (3.98e+0) -	**1.5496e+1 (5.39e+0)**
	20	39	2.5085e+1 (3.53e+0) =	**2.2913e+1 (5.04e+0)**
+/-/=			0/24/4	

5. Conclusions

In this paper, an alternative strategy for dealing with MaOPs, named MaOES-AD, has been proposed. MaOES-AD not only has a simple structure, but also is free from the use of weight vectors, and indicators. The main characteristic of MaOES-AD is that it makes use of two strategies (i.e., angle-based selection and shift-based density estimation) to delete poor individuals one by one during the environmental selection. The angle-based selection strategy aims to maintain the diversity of search directions. It identifies a pair of individuals with the minimum vector angle, which means that these two individuals search in the most similar directions. Subsequently, shift-based density estimation is conducted to differentiate them by considering both diversity and convergence and to remove the inferior one. We validated that these two strategies play very important roles and are indispensable in MaOES-AD. In addition, we compared MaOES-AD with two state-of-the-art MaOEAs for solving MaOPs with up to 20 objectives in the DTLZ test suites. The results indicate that, overall, MaOES-AD achieves the best performance in terms of both IGD.

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