

Stochastic Model for Integrated Preventive Maintenance Planning

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Abstract: Preventive maintenance planning management is modelled with stochastic approach. It is aimed to prevent stoppages and quality disorders due to disturbances, carriage and maintenance processes in production. Different maintenance policy alternatives were considered in order to develop and sustain more effective maintenance policies. The preventive maintenance process includes the cost of inspection and maintenance status, the cost of repair and other losses in the accidents with operator injuries and damage to the possible value of the situation. The stochastic model approach is applied for determine the possible period intervals of the machinery and equipment and the maintenance process analyzed and discussed in detail. The preventive maintenance approach in the maintenance planning process is aimed to develop a sustainable maintenance policy without any disturbance in the quality of production from any disturbance and disruption of the system in the long term. However, it is aimed to take preventive maintenance measures as well as to analyze the current system condition and predict future situations.

1. Introduction

The uncertainty situation in machine failures and stops are very important during the production activities for this reason we prefer to use stochastic maintenance planning model. The modelling, uncertainty situation and system performance were analyzed by taking into consideration possible situations in machine and breakdowns and faults. In the maintenance process, the costs of the control and maintenance status diagnosis, repair costs and other losses in case of accidents analyzed by taking into consideration the possible values of the injuries and damage of the operators. It is aimed to perform the periodical maintenance process with the machinery and equipment where the maintenance process performed with the stochastic model approach applied.

In this study, production plan activities are evaluated together with the system risk situation and maintenance plan activities are aimed to be formed considering the current situation and conditions with integrated process and maintenance functions. Maintenance costs involve a significant part of

the total operating costs of manufacturing and production facilities. Although, it varies by industry type, maintenance costs are known to account for between 15% and 60% of the total operating costs. According to the studies, a large part of the operation and maintenance costs are wasted due to incorrect, systematic and unplanned maintenance methods. Ineffective maintenance methods can also have a major impact on the quality of the product produced therefore it directly effects the increasing of the costs.

The remainder of the paper is organized as follows. Previous studies on preventive maintenance systems of stochastic structure are reviewed in section 2. The system and integrated preventive maintenance planning stochastic model is presented in section 3. Then proposed stochastic maintenance model with illustrative example and analysis are discussed in section 4. Also, the proposed model and numerical results are given with case study in next section. Section 5 includes the discussion and some concluding remarks which are provided in conclusion.

2. Literature Survey

The integrated preventive maintenance planning model is aimed to predict the disruption situations that the system may encounter. The model has been developed in order to prevent possible unexpected deterioration situations and prevent any such problems and to avoid any negative problems during the production process with the preventive maintenance operations carried out considering the machine wear conditions. In conventional maintenance models, the system and its components are considered to be operating perfectly or fail in two possible situations [1, 2]. The system and its components are taken into consideration in case of defective production of parts or defects that may occur in parts during assembly and distortion situations that may occur in machines during the production of parts. In practice, however, many systems and components can fail and operate in an intermediate operating state. When some of the components that make up the component deteriorates, or when the machine forming the component starts to deteriorate, it may be necessary to either repair the defective machine or replace it with a new one to complete the performance [3, 4, 5]. Also, the system has been evaluated by considering both the part level and the machine level. The structure where multi-state components are taken into consideration and the deterioration conditions of the machinery and equipment used in this structure are taken into consideration [6-12]. Along with the proposed model, the maintenance model in terms of both material and machinery produced was examined [13-17].

3. Stochastic Models in Discrete Events

Markov chains are a special type of discrete time stochastic processes. In addition, Markov chains have the ability to predict the long-term state (equilibrium state) of the system in addition to its ability to predict the situation at a certain moment. In this study, by considering a multi-state system consisting of N , multi-state components i ($i = 1, 2, \dots, N$) in series have $K + 1$ different states, each of which is different. The system performance ratio is shown in g_k ($k = 0, 1, \dots, K$). K indicates perfect working condition and 0 indicates full failure condition. Transition times between component states, Markov process approach, and time between transitions are expressed by the exponential distribution. γ_i, k is the structural distortion or transition rate from the component i [18].

$$S_s(t) = \min \{s_1(t), s_2(t), \dots, s_N(t)\} \quad (1)$$

In particular, one component falls into a lower state, other neighbouring and / or related components that affect functionally deterioration rates. Initially, all components are in excellent working condition and no interaction between components, all degrading at internal degradation rates [19].

The state of 1 to k as $i_k(t)$ in the state transition rate (t) of the component i ($i = 1, 2, \dots, N$) can be expressed as:

$$\gamma_{i,k}^m(t) = \gamma_{i,k} \cdot f(G_s(t), n_t(t), \delta), \quad k = K, K-1, \dots, 1 \quad (2)$$

The modified failure rate $\gamma_{i,k}^m$ consists of two elements: the internal failure rate $\lambda_{i,k}$, and the interaction effect on decay is the ratio of $f(\bullet)$ caused by other failure components. System performance rate represents the $f(\bullet)$ at $G_s(t)$ time $t_s(t)$ [20].

($G_s(t) = \min \{g_1(t), g_2(t), \dots, g_N(t)\}$; $g_i(t)$ = system performance}) is the number of components that affect the transition of $n_I(t)$ to a lower level. It expresses the state of t in it. Also, the interaction effect is stochastic in nature, and the system consists of operational conditions, including the state of environmental / non-critical components. Therefore, we add the δ parameter to capture the uncertainty. It is caused by random variations in these factors. We assume that the process takes place with the normal distribution of δ mean zero and standard, deviation of σ [21]. We update the number of components and affected components $f(\bullet)$ interaction effect and the transition rates of all components in the system.

Then, we use this updated transition in accordance with equations. (2) and (3). Accordingly the term $f(\bullet)$ in the equation (2) can be expressed

$$f(G_s(t), n_I(t), \delta) = \left(\frac{G_K}{G_s(t)} + |\delta| \right)^{N_I(t)/N}, \quad G_s(t) \neq 0 \text{ and } f(\bullet) \geq 1 \quad (3)$$

When all components are in perfect condition, i.e. $n_I(t) = 0$ and $G_s(t) = G_K$, $f(\bullet)$ is equal to 1 and $\gamma_{i,k}^m$. In the case of the system ($G_s(t) < G_K$), $f(\bullet)$ value will be less than 1 and $\gamma_{i,k}^m(t)$: 's $f(\bullet)$ value will be greater than 1. The uncertainty parameter (δ) and the remaining parameters are obtained using historical data and subjective input from experts. After modelling the rates of disruption of multi-state components, the components that can be evaluated considering the reliability rate is:

$$R_s(t, D) = \Pr \{G_s(t) \geq D\} \quad (4)$$

System reliability can then be evaluated as the sum of the following and the possibilities for all acceptable states of the system:

$$R_s(t, D) = \sum_{k=0}^K [P_k(t) \cdot I(G_s(t) \geq D)] \quad (5)$$

$P_k(t)$ is the probability of the state of k at time t and I is an indicator function with a value of 1, a performance ratio higher than the system demand level ($G_s(t) \geq D$) and 0 in any other situation. Zero value is considered a system error in the display function. The possibility of the system state at time T is given as:

$$P_k(t) = \Pr \{S_s(t) = k\} = \sum_{\min\{s_i\}=k} P_{s_1, s_2, \dots, s_N}(t) \quad (6)$$

where $P_{s_1, s_2, \dots, s_N}(t)$, is the s_i ($= 1, 2, \dots, N$) s_i of the state of each component at time t . At t time, the state of the system, k , and if only the state of at least one component is k , and the state of the other is higher than the components k . The state probabilities of each component are calculated by solving the $P_{s_1, s_2, \dots, s_N}(t)$ Chapman - Kolmogorov differential system.

3.1. Mathematical Model

In this section covers the stochastic preventive maintenance mathematical model.

Indices:

i	product type
j	machine type used in production
t	production period

Parameters:

D	demand level
$g_i(t)$	performance rate of component i at time t
$g_{i,k}$	performance rate of component i state k
G_k	system performance rate in its perfect state
$G_s(t)$	system performance rate at time t
$n_i(t)$	number of influencing components up to time t
R_t	the period of time necessary for the production of the product i during the period t
O_t	the period of overtime in the production of the product i ot t period
$R_{\max t}$	normal working time available during the period t
$O_{\max t}$	available overtime available during period t
M_{jt}	preventive maintenance variable for machine j in period t. if the variable value is 1, maintenance is performed, if 0, no maintenance is performed.
W_{jt}	weekend preventive maintenance variable for machine j in period t. if 1 is weekend maintenance, 0 is not a weekend maintenance.
$Z_{jt\theta}$	if 1 is the variable that tells the machine j in period t whether the last preventive maintenance was performed before the period θ . If 1, maintenance operation is performed, if 0, maintenance operation is not performed.
D_{it}	demand for i product during the period t
M_t	maximum duration of possible preventive maintenance activity performed during the normal working time during the t period
WM_t	maximum duration of preventive maintenance activity possible at the weekend in the t period
$\lambda_{j\theta t}$	probability of machine j deterioration in period
$\gamma_{j\theta t-1}$	actual deterioration value of machine j in period t-1
C_T	total cost
C_o	beginning cost
C_m	maintenance cost
V_k	material volume for k th unit
C_{ok}	beginning unit cost
C_m	total maintenance cost
C_q	q th unit cost for improvement
t_q	q th unit production period
v	volume
C_{kq}	q th unit cost of for kth fixed cost
α_q	C_o weight percentage of initial cost
δ_{kq}	index of damage
T	expected life of the machine
C_q	q th unit's annual costs;

$$C_{\alpha q} = \alpha_q C_o \quad (7)$$

$$C_o = \sum_k c_{ok} V_k \quad (8)$$

$$C_q = C_{\alpha q} + \sum_k \delta_{kq} (\lambda^{C_{kq}}) V_k \quad (9)$$

$$C_m = \sum_q \frac{C_q}{(1+v)^{\tau_q}} \quad (10)$$

$$C_T = C_o + C_m \quad (11)$$

$$C = C_T \frac{v(1+v)^T}{(1+v)^T - 1} \quad (12)$$

This study aims to minimize system costs by minimizing cumulative maintenance costs and minimizing disruption and quality deterioration in production due to machine downtime and downtime. We call the opportunistic preventive maintenance model cause of the model considering reliability threshold status. Also, in case of machine deterioration and stoppages, revision of other machines in standing system considered and periodic maintenance after certain production levels are applied to regardless of reliability threshold.

3.2. Determination of Stochastic Preventive Maintenance Reliability Threshold Level

Determining of the reliability threshold level for component j is considering the reliability threshold value R_j

$$R_j = \exp \left[-\int_0^{T_j} h_j(t) dt \right] \quad (13)$$

The value h_j indicates the hazard ratio function. T_j represents the optimum preventive maintenance time interval for component i. The equation $\int_0^{T_j} h_j(t) dt$ gives the stochastic deterioration probabilities of component j in each preventive maintenance cycle.

The H_j value represents the hazard ratio function. T_j represents the optimum preventive maintenance interval for the component.

$$C_j = \frac{c_j^m \tau_j^m \int_0^{T_j} h_j(t) dt + c_j^p \tau_j^p + c^d \tau_{j0}^d \left[\int_0^{T_j} h_j(t) dt + 1 \right] + c^s \tau_j^s}{T_j + \tau_j^m \int_0^{T_j} h_j(t) dt + \tau_j^p + \tau_{j0}^d + \tau_{j0}^d \left[\int_0^{T_j} h_j(t) dt + 1 \right]} \quad (14)$$

All these assumptions, c_j^m, c_j^p represents the minimum repair cost and preventive maintenance cost per component time for component j, while representing the cost of dismantling equipment per unit time. c^s represents the cost of downtime per unit of the system for one system component, together with downtime due to repair and maintenance. $\int_0^{T_j} h_j(t) dt + 1$ indicates not only the deterioration state of component j, but also the default number of disassemblies in the preventive maintenance period. τ_j^m represents the total minimum stop of the component τ_{j0}^d j; in the removal state, while τ_j^p indicates a single minimum repair state

$$\tau_{j0}^d = \tau_j^d + \tau_j^{dp} \quad (15).$$

τ_j^d represents the downtime of the work piece while τ_j^{dp} represents the cumulative downtime. τ_j^s represents the total downtime of the system at a standstill. Preventive maintenance interval;

$$\tau_j^d = \tau_j^m \int_0^{T_j} h_j(t) dt + \tau_j^p + \tau_{j0}^d [\int_0^{T_j} h_j(t) dt + 1] \quad (16).$$

The objective of this study is to determine the optimal preventive maintenance interval by minimizing C_j and determining the R_j reliability threshold.

Preventive maintenance reliability threshold level k reaches the preventive maintenance threshold; the system is stopped to perform the maintenance process. Preventive maintenance time intervals are given in a TW time window that are indicated by $t = t_k$ and $t = t_k + TW$. When the number of components is indicated by r , k , the total maintenance cost refers to the combination of component and G components which is;

$$C^G = C_k \cup (U_{\eta-1}^r C_\eta) \quad (17)$$

C_k is the maintenance cost for component k and C_η is the maintenance cost for component η in the G composition. Component k can be repaired or prevented with a minimum maintenance cost C_k .

$$C_k = C_k^{mop} + C^d \tau_{k0}^d + C^s \tau_k^s = \begin{cases} C_k^m r_k^m + C^d r_{k0}^d + C^s r_k^s & \text{if minimum maintenance for } k \text{ component} \\ C_k^p r_k^p + C^d r_{k0}^d + C^s r_k^s & \text{if preventive maintenance for } k \text{ component} \end{cases} \quad (18)$$

Minimum repair presented for Preventive Maintenance in Eq. (18). C_k^{mop} denotes the minimum cost of the minimum repair condition or preventive maintenance condition for component k , while the preventive maintenance together with the combination of C with other components.

4. Case Study

In this section, possible necessary maintenance situations were examined, taking into account the different forms of the machine's operating states. Four different situations of machine operation, namely, machine operation and non-operation, slow operation of the machine and noise from the machine were taken into account. At the same time, a possible situation analysis was made by comparing the machine's malfunction state structure with the applied process states. The feedback data regarding the disorder status received for a total of 3 months are given in Figure 1. These data were arranged with state parameters considering Eq. (1) and Eq. (2). Below the alternative cases are discussed. The operation performed with the parameter D is expressed.

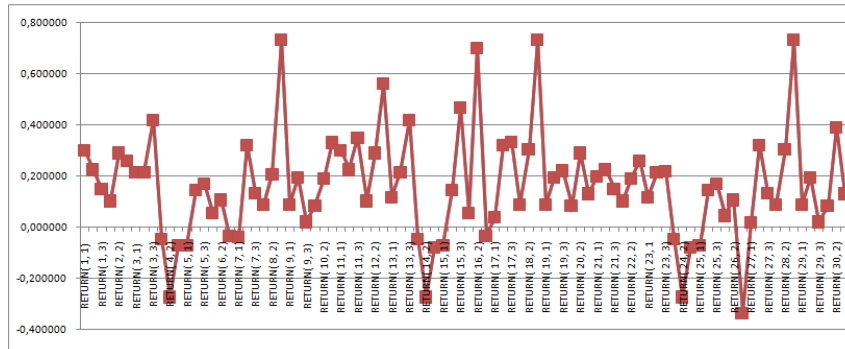


Figure 1: Case dataset

Definitions of states and decisions

S0- The machine works perfectly

S1- The strange sound when the machine is running

S2- The machine sometimes slows down while running

S3- The machine is at a standstill from time to time

D1-Do nothing

D2- Perform periodic maintenance

D3- Overhaul the machine, change the spare part or change itself.

As a result of the current data analysis, the reduced cost values were included in the model according to the D1, D2 and D3 state according to the transaction status. Especially by changing, revising and continuing production of machine parts, took place as the factor reducing cost in the system the most. Figure 2 shows the correlation status chart. Figure 3 gives the probability distributions of the situations encountered

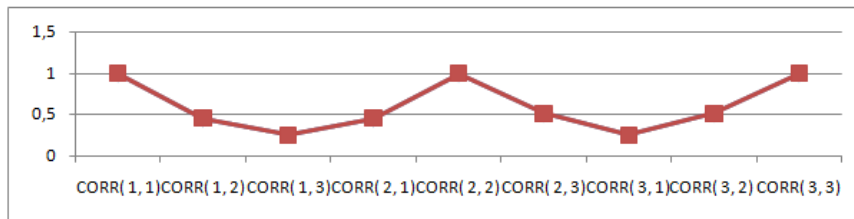


Figure 2: Dataset correlation result diagram

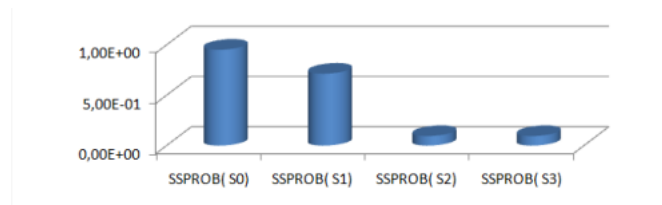


Figure 3: Data set state probability distribution

Figure 4 shows the decision results of the situations. As a result of the analysis, S2-D3, in case of slow operation of the machine, D3 option is selected, the option of replacing the defective part of the machine or replacing the machine is selected. With the change made in this context, any disruption or stopping situation that may occur in production will be prevented. The transition state between states and possible situations and decision chains are given in Figure 5.

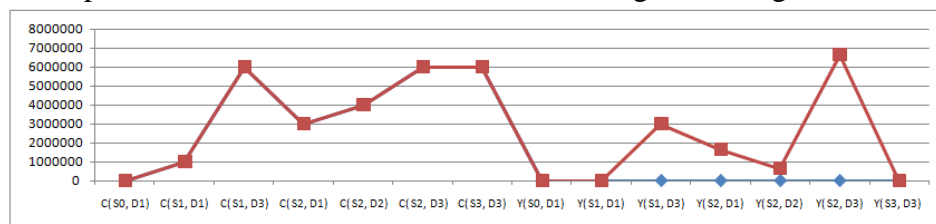


Figure 4: Case decision results (C-Case; Y-Decision)

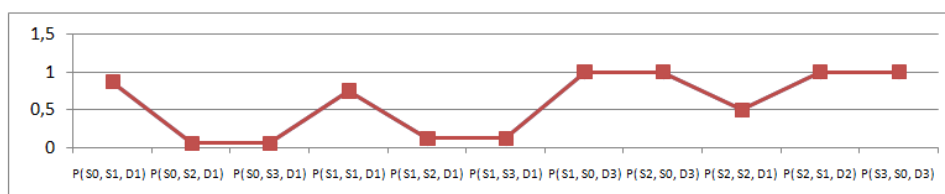


Figure 5: Situations and possibilities of applied maintenance policies

5. Conclusion

In this study, the structure of the integrated preventive maintenance planning problem in

different situations, different probability and possible situations that may be encountered in different scenarios are examined. They were examined and aimed to prevent negative situations such as production stops and faulty product production situations. With the recommended preventive maintenance, the behaviour of the system was examined; possible downtimes and repair maintenance operations were analyzed comprehensively in the system. In this way, cost increases caused by possible deterioration in production were tried to be prevented, and poor quality production caused by machine defects was tried to be prevented. By the way, the importance of carrying out preventive maintenance studies and risk management studies together with maintenance management in later studies has emerged. Thanks to the proposed model, it has been tried to estimate the state value that the system can take in the next state, taking into account the possible situations that may be encountered in the system.

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