

# ***Research on an extended composite proportional navigation law based on error adaptation and error anti saturation***

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**Abstract:** According to the two-dimensional spatial relationship model of relative motion between missile and target, a constant value proportional navigation law based on error anti saturation is proposed on the basis of conventional proportional navigation and extended proportional navigation; Thirdly, the adaptive estimation design of guidance parameters is carried out from the angle error of longitudinal plane and lateral plane, angle coupling error, monocular distance, reciprocal of monocular distance and other aspects, and finally a composite three-dimensional proportional guidance method with error adaptation and error anti saturation is formed. The case digital simulation results also show the correctness and effectiveness of the proposed method.

## **1. Introduction**

The coefficient of conventional proportional navigation is generally selected between 2-6, but when it is designed as a constant value, it is difficult to obtain the automatic adaptability of different targets under different situations, so the miss distance is unstable<sup>[1-4]</sup>.

With the rapid development of missile technology in modern war, many missiles not only hope to obtain the minimum miss distance when hitting the target, but also often hope that the missile attitude is the best when hitting the target, so as to maximize the effectiveness of the warhead and achieve the best damage effect. Therefore, many experts use different modern control methods to improve the landing angle of the guided terminal attack target, such as for the situation that anti-ship and anti Tan tactical missiles need to attack the target at a specific angle, songt 50. Shin S.J. and Cho h. studied an optimal guidance law in two-dimensional plane. The key point of the guidance law is to consider the influence of target maneuver and missile speed time-varying on guidance, combine the optimal angle of fall constraint control problem with target estimation and filtering, and deduce the optimal guidance law with angle of fall constraint for anti-ship missiles in Cartesian coordinate system based on the energy optimization criterion and Schwartz inequality. However, the falling angle of conventional proportional navigation can not be controlled, so the research of improved proportional navigation is expected to make effective improvements in the falling angle. Therefore, this paper attempts to use adaptive method to improve the proportional

navigation<sup>[5-8]</sup>.

The method of automatically adjusting the guidance coefficient according to the error is a good attempt in this regard. Therefore, the adaptive adjustment design of the proportional guidance coefficient according to different monocular distances and different angular errors has achieved good results in the improvement of proportional guidance<sup>[9-13]</sup>. At the same time, if only the error is used to adjust, it is easy for the proportional guidance coefficient to diverge with the increase of error when the coefficient is too large. Therefore, if the proportional guidance coefficient is too large at this time, the miss distance will also diverge. At this time, the error anti saturation design can limit the divergence of the proportional coefficient through the saturation function. Based on the above reasons, a method of error adaptation and error anti saturation is proposed in this paper, and has achieved good results in simulation cases, so it has high engineering application and promotion value.

## 2. Two dimensional proportional navigation model

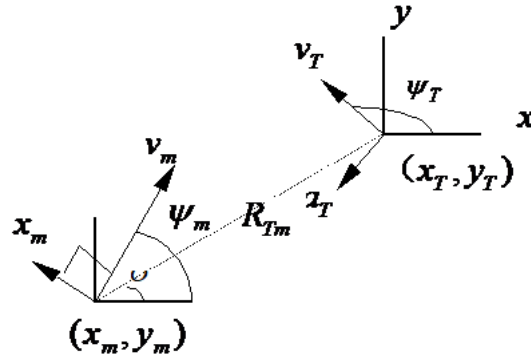


Figure 1: The location relationship between target and missile

According to reference[2], the model of conventional proportional guidance law can be built as follows. The motion of target and missile can be shown as figure 1.  $(x_m, y_m)$  are the location of missile and  $(x_T, y_T)$  are the location of target.  $(\psi_m, v_m)$  are the azimuth and speed of missile and  $(\psi_T, v_T)$  are the azimuth and speed of target.  $(a_m, a_T)$  are the acceleration of missile and target, and  $\sigma$  is the sight angle. And the model can be written as:

$$\ddot{x}_m = -a_m \sin \psi_m, \dot{x}_m(t_0) = v_{mx0}, x_m(t_0) = x_{m0} \quad (1)$$

$$\ddot{y}_m = a_m \cos \psi_m, \dot{y}_m(t_0) = v_{my0}, y_m(t_0) = y_{m0} \quad (2)$$

$$\ddot{x}_T = -a_T \sin \psi_T, \dot{x}_T(t_0) = v_{Tx0}, x_T(t_0) = x_{T0} \quad (3)$$

$$\ddot{y}_T = a_T \cos \psi_T, \dot{y}_T(t_0) = v_{Ty0}, y_T(t_0) = y_{T0} \quad (4)$$

$$\psi_m = \arctan(\dot{y}_m / \dot{x}_m), \psi_T = \arctan(\dot{y}_T / \dot{x}_T) \quad (5)$$

$$\Delta y = y_T - y_m, \Delta x = x_T - x_m, \sigma = \arctan(\Delta y / \Delta x) \quad (6)$$

According to the proportional law, it has

$$\dot{\psi}_m = k\dot{\sigma} \quad (7)$$

We assume that the velocity of the missile is approximately constant during its movement, and the control force is perpendicular to the velocity, so only the direction of the missile velocity is changed. Then there are:

$$a'_m = v_m \dot{\psi}_m, \quad v_m = \sqrt{v_{mx0}^2 + v_{my0}^2} \quad (8)$$

$$T\dot{a}'_m + a'_m = a_m \quad (9)$$

The above is a complete proportional navigation motion relationship model, where T represents the time constant of the control loop.

### 3. Design of extended proportional navigation law in three-dimensional space

According to the model of proportional navigation in the previous section, we extend it to three-dimensional space, where  $\Delta_x, \Delta_y, \Delta_z$  are the distance between missile and target on x、y、z axis direction. Then the azimuth of zox plane and yox plane can be solved as

$$\begin{aligned} q &= \arctan(\Delta_z / \Delta_x) \\ w &= \arctan(\Delta_y / \Delta_x) \end{aligned} \quad (10)$$

And the initial value of  $q_0$  and  $w_0$  are set as 0, then its derivative can be solved as

$$\begin{aligned} dq &= (q - q_0) / dt \\ dw &= (w - w_0) / dt \end{aligned} \quad (11)$$

And the speed angle of missile of track angle and flight path angle  $\theta_1$  and  $\theta_2$  are used to solved error angle between speed angle and azimuth angle as

$$\begin{aligned} e_1 &= q - \theta_1 \\ e_2 &= w - \theta_2 \end{aligned} \quad (12)$$

The self-adaptive regulation law of guidance coefficient is designed as follows:

$$d_1 = \frac{1}{(\sin e_1)^2} + \hat{c}_1 + \hat{c}_2 e_1 + \hat{c}_3 e_2 + \hat{c}_4 (r - r_0) + \hat{c}_5 \frac{1}{r} \quad (13)$$

$$d_2 = \frac{1}{(\sin e_2)^2} + \hat{c}_6 + \hat{c}_7 e_1 + \hat{c}_8 e_2 + \hat{c}_9 (r - r_0) + \hat{c}_{10} \frac{1}{r} \quad (14)$$

The adaptive estimation law of unknown parameters is as follows:

$$\dot{\hat{c}}_1 = j_1 e_1 \quad (15)$$

$$\dot{\hat{c}}_2 = j_2 e_1^2 \quad (16)$$

$$\dot{\hat{c}}_3 = j_3 e_1 e_2 \quad (17)$$

$$\dot{\hat{c}}_4 = j_4 (r - r_0) e_1 \quad (18)$$

$$\dot{\hat{c}}_5 = j_5 \frac{1}{r} e_1 \quad (19)$$

$$\dot{\hat{c}}_6 = j_6 e_2 \quad (20)$$

$$\dot{\hat{c}}_7 = j_7 e_1 e_2 \quad (21)$$

$$\dot{\hat{c}}_8 = j_8 e_2^2 \quad (22)$$

$$\dot{\hat{c}}_9 = j_9 (r - r_0) e_2 \quad (23)$$

$$\dot{\hat{c}}_{10} = j_{10} \frac{1}{r} e_2 \quad (24)$$

Where  $j_1, j_2, \dots, j_{10}$  are constant parameters which are used to adjust the convergence speed of adaptive law.

Here, the extended proportional guidance coefficient is set as 2. According to the extended proportional guidance formula, the acceleration of the missile in the horizontal and vertical directions can be obtained:

$$\begin{aligned} a_1 &= 2 \cdot dq \cdot v_m \cdot d_1 + a_{10} \\ a_2 &= 2 \cdot dw \cdot v_m \cdot d_2 + a_{20} \end{aligned} \quad (25)$$

The error self-adjusting proportional navigation term of anti saturation is designed as follows

$$\begin{aligned} a_{10} &= dq \cdot v_m \frac{l_1}{|e_1| + \varepsilon_1} \\ a_{20} &= dw \cdot v_m \frac{l_2}{|e_2| + \varepsilon_2} \end{aligned} \quad (26)$$

#### 4. Simulation Example

We set the initial location of missile as  $(x_m, y_m, z_m) = (0, 500, 0)$ , and set the initial coordinate of target as  $(x_t, y_t, z_t) = (12000, 40, -300)$ . Set the maximum value of missile movement time to  $t_f = 50$ . In addition to the introduction of extended proportional navigation, the overload constraint is still 100. Therefore, only the simulation program of extended proportional navigation is given here:

```
ag=2*dq*vm;
ag2=2*dw*vm;
amax=1000;
a=ag;
a2=ag2;
if abs(ag)>amax
a=sign(ag)*amax;
end
if abs(ag2)>amax
a2=sign(ag2)*amax;
end
sita2=atan(vym/vxm);
sita1=atan(vzm/vxm);
```

Figures 2 to 9 below show the three-dimensional motion curve of the missile and the target, the lateral motion curve of the missile and the target, and the vertical motion curve of the missile and the target respectively; Monocular range change curve, lateral velocity angle change curve of missile and target, vertical velocity angle change curve of missile and target, lateral acceleration change curve of missile and target, vertical acceleration change curve of missile and target.

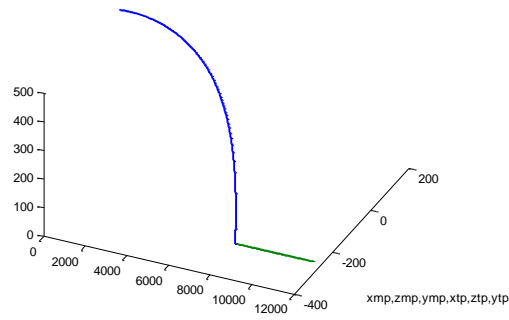


Figure 2: Three dimensional image of missile and target trajectory

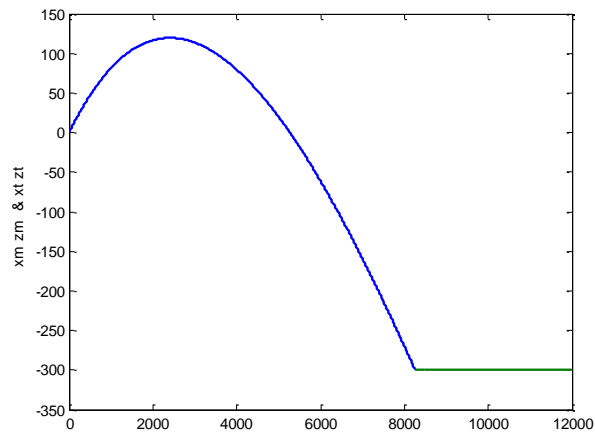


Figure 3: Lateral motion diagram of missile target

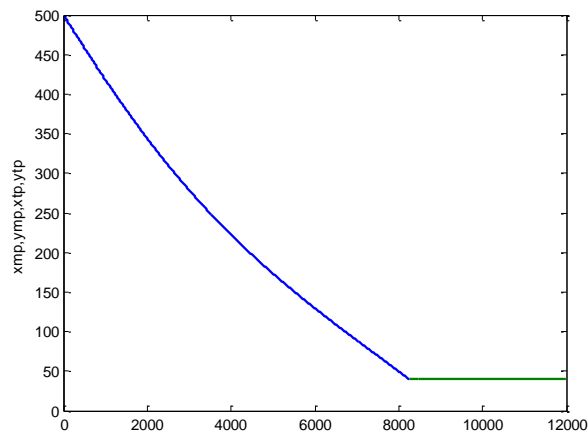


Figure 4: Vertical motion diagram of missile target

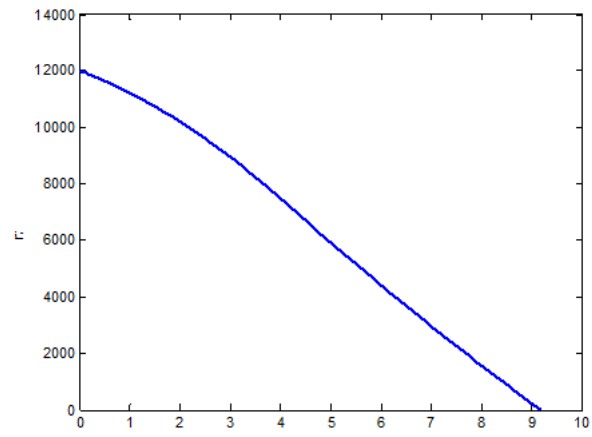


Figure 5: Missile target distance map and miss distance

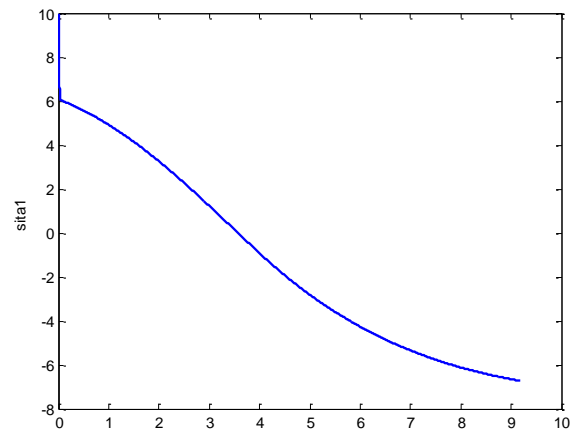


Figure 6: Lateral speed angle

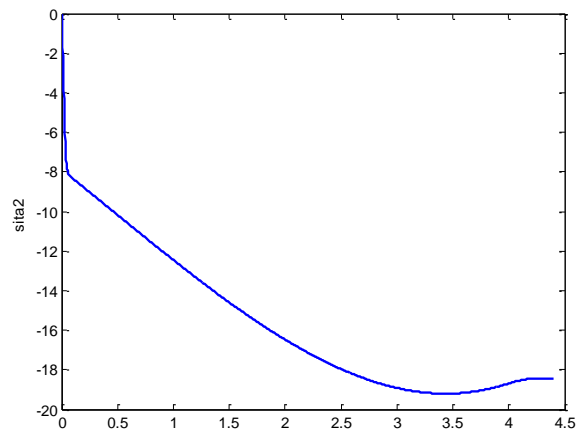


Figure 7: Vertical speed angle

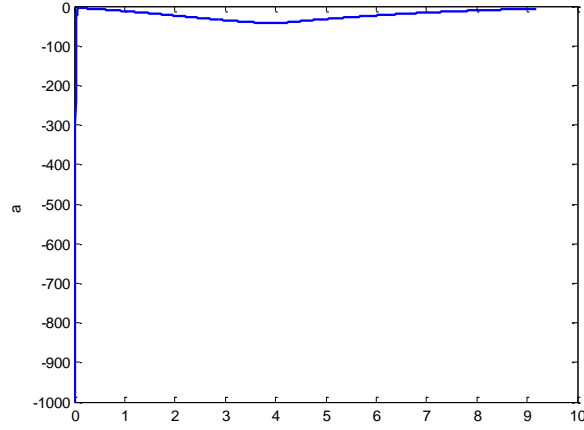


Figure 8: Lateral acceleration

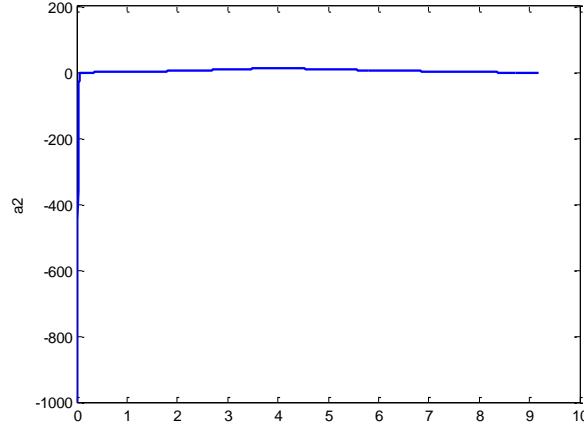


Figure 9: Vertical acceleration

It can be seen from the simulation curve that the final miss distance between the missile and the target according to the guidance law is less than 0.2m. In the process of tail chasing and attacking low altitude targets, overload constraints play an obvious role in reducing the missile miss distance, but the final attitude of the missile hitting the target is close to the level. The acceleration changes of the two channels are within the limited range, and the end segment is small, and only the initial stage is large at an instant. Because the missile and the target are in relative motion, the final falling angle reaches about 20 degrees. From the whole simulation results, the miss distance is very small, and the parameters have good robustness, which can be suitable for different missile target relative situations. Therefore, the error adaptive guidance law and the error anti saturation guidance law proposed in this paper are effective, and have high engineering promotion value.

## 5. Conclusions

In this paper, a composite guidance method combining error anti saturation proportional guidance and error adaptive extended proportional guidance is proposed. The design of error anti saturation makes the coefficient of proportional navigation not divergent with the increase of error; At the same time, the error adaptive extended proportional guidance considers many factors, such as

the proportional coefficient with error, with monocular distance and the reciprocal of monocular distance. Finally, the combination of the two methods achieves a good guidance effect. The final digital simulation analysis shows the effectiveness and correctness of the method proposed in this paper.

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