The Ball on the Rubber Band

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Abstract: Connect the two metal balls with a rubber band, then twist the rubber band and put the metal ball on the table. The ball will start to rotate in one direction and then in the other. This strange phenomenon is similar to the so-called "pendulum" movement. This paper makes a theoretical and experimental study of this phenomenon, and explores the relevant parameters that affect the motion, such as the type of rubber band, the material of the contact surface, and the size of the metal ball.

1. Introduction

Rubber band is a short ring made of rubber and latex, which is generally used to bind things together. If you connect the two metal balls with a rubber band, then twist the rubber band and put the metal ball on the table, you can find that the ball will start to rotate in one direction, and then rotate in the other direction, making a cyclic "pendulum" movement.

In order to explain this phenomenon, this paper has carried out theoretical and experimental research, and the main research object is the relevant parameters that affect the motion. Through analysis, it is found that in this system, the size of the metal ball, the type of rubber band, and the material of the contact surface will have a certain impact on the movement. Therefore, based on the principle of control variables, this paper studies each single variable step by step.

2. Theoretical model analysis

2.1 Model establishment

The theoretical model is shown in Figure 1. Considering the limitations of this study, the following two reasonable assumptions can be made to simplify the model: (1) During the study, the velocity of the ball is not very fast, so the non slip rolling can be ignored. (2) The air resistance during the study is negligible.
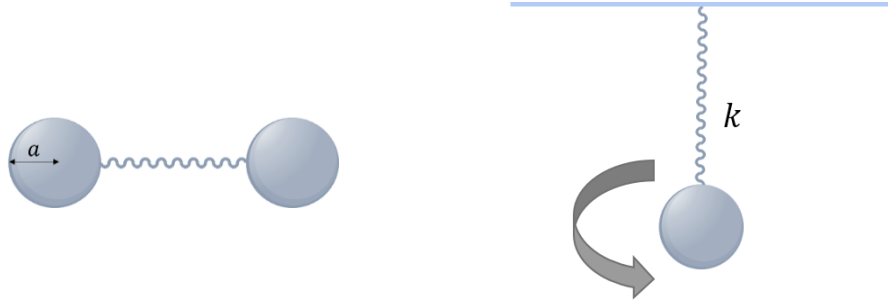
Figure 1: Construction of theoretical model

Therefore, considering only rolling friction, the metal ball is regarded as a rigid body, and the rotation angle of the metal ball around its own axis is θ analysis. At this time, there are two systems, one is a small system when each metal ball is rolling itself, and the other is a large system composed of two metal balls and rubber bands, in which two metal balls are in circular motion. In this paper, the self rolling small system of each metal ball is studied firstly.

Since there is only rolling friction and no sliding friction, each minute time element can be regarded as the moment equilibrium state of the metal ball. The metal ball is subjected to sliding friction torque, rubber band torque and self rotation torque through torque analysis[1].

The above three moments are analyzed respectively. I is the moment of inertia of a single ball, α is the angular acceleration of a single ball, which can be obtained from the formula of rigid body motion. The formula for calculating the moment of rotation M_b of the metal ball itself is:

$$M_b = I\alpha \tag{1}$$

Rolling friction is essentially static friction. Assume that the rolling friction torque is M_f, the positive pressure is N, and the rolling friction coefficient is δ.

$$\backslash M_f = N\delta \tag{2}$$

Through literature research, the relationship between pressure and speed here is approximately linear, and the size of rolling friction torque is proportional to the positive pressure[2]. In this case, suppose a coefficient b related to rolling friction coefficient, then:

$$v = a\dot{\theta} \tag{3}$$

$$M_f = ab\dot{\theta} \tag{4}$$

Where: a is the radius of the metal ball, v is the tangential velocity when the metal ball moves around its own axis, θ is the angle at which the metal ball rotates around its own axis. It can be known that the proportional relationship between b and rolling friction coefficient δ, and the subsequent discussion of rolling friction coefficient δ can be transformed into the discussion of b.

In this system, the rubber band will provide torque M_r, where k is the torsion constant of the rubber band. The calculation formula of rubber band torque is:

$$M_r = k\theta \tag{5}$$

Since the ball rolls without sliding in the tangential direction, we can draw a conclusion:

$$I\alpha + ab\dot{\theta} + k\theta = 0 \tag{6}$$

At the same time, we can get the following initial conditions:

$$\begin{cases} \theta \mid_{t=0} = \theta_0 \\ \dfrac{d\theta}{dt} \mid_{t=0} = 0 \end{cases} \tag{7}$$

The expression of θ is obtained by combining (6) and (7):

$$\theta = \sqrt{\theta_0^2 + \frac{b^2\theta_0^2}{4I^2\omega^2}}\, e^{-\frac{bt}{2I}} \cos\left[\omega t + \arctan\left(-\frac{b}{2I\omega}\right)\right] \tag{8}$$

At this time, the relationship between the rolling angle of the metal ball around its own axis and time has been obtained. It is not difficult to find that the rotation angle of the metal ball itself changes periodically. It is known from the physical formula that since the radius of the metal ball is certain, the velocity of the metal ball and the angular velocity of the metal ball will change periodically.

Since the ball rolls without sliding, the tangential velocity v of the ball around its own axis is equal to the velocity of the ball moving in a circle around the rubber band.

$$v = \dot{\theta} r \tag{9}$$

At this time, the movement of the metal ball itself will be transformed into the movement of the whole system. At this time, the movement of the large system formed by two metal balls can be studied. Because the expression θ contains a cosine function, which itself or after finding the n-order derivative contains a periodic property, for large systems, the velocity change period of the metal ball moving in a large circle is the same as the velocity change period of the metal ball rolling itself.

The value of ω is:

$$\omega = \sqrt{\frac{k}{I} - \frac{b^2}{4I^2}} \tag{10}$$

$$T = \frac{2\pi}{\omega} \tag{11}$$

Therefore, the theoretical curve between ω and different variables can be obtained by analyzing the formula, as shown in Figure 2, Figure 3 and Figure 4.
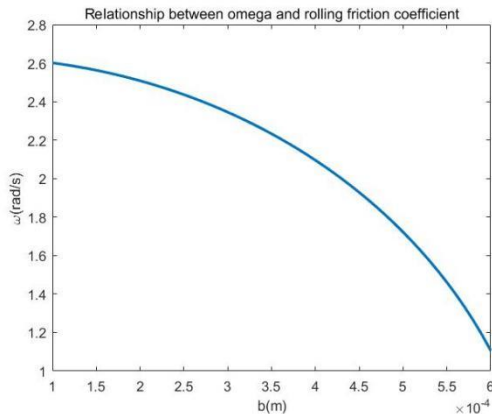

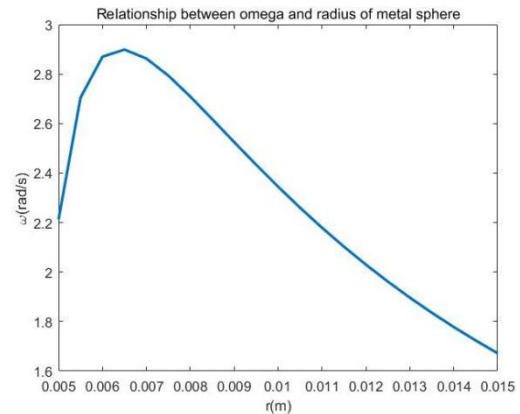Figure 2: ω and rolling friction coefficient   Figure 3: ω and the radius of the ball
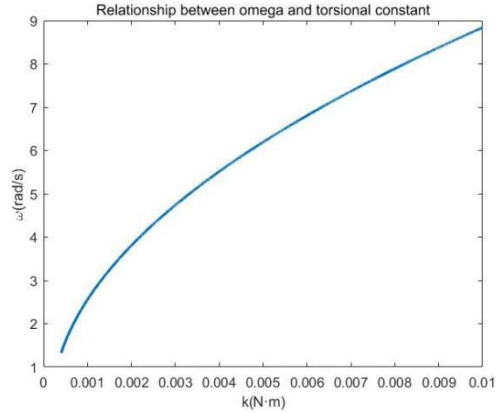
Figure 4: ω and torsion constant

It can be seen from the figure that the larger the rolling friction coefficient is, the smaller ω is; The larger the torsion constant is, the larger ω is; The larger the radius (when the diameter is greater than 13mm), the smaller the ω.

2.2 Calculation of model parameters

The schematic diagram of the experimental device is shown in Figure 5. In order to obtain the torsion constant of the rubber band, it is necessary to twist the rubber band and measure the rotation period of the ball when the ball is suspended, and bring it into the following formula to calculate k:

$$\frac{2}{5}ma^2\ddot{\theta}+k\theta=0 \tag{12}$$

At the same time, place the ball on different contact surfaces, push the ball to make it roll, and track the trajectory of the ball. Then b can be calculated by the following formula:

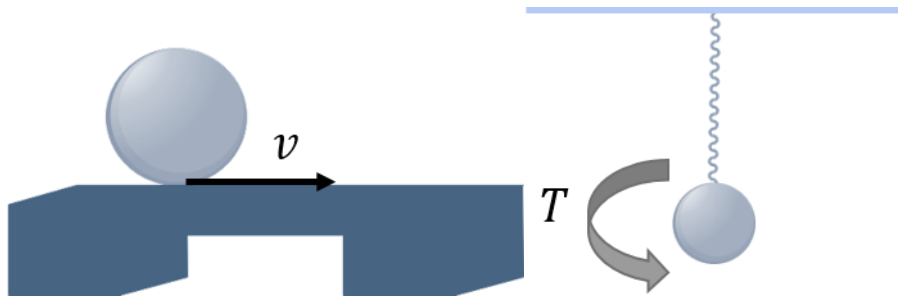$$\frac{2}{5}ma^2\ddot{\theta}+ab\dot{\theta}=0 \tag{13}$$

Figure 5: Schematic diagram of experimental device

Through Tracker, it can be simply measured that the torsion constant of 75 mm rubber band is about 10^{-5}. For the parameter coefficient b, it can be known that it is roughly between $10^{-4}-10^{-5}$.

2.3 Theoretical calculation results

Combined with theoretical analysis, in order to explore the factors that affect the experimental results, this paper uses the control variable method to explore the effects of four ball diameters, two rubber band materials and three contact surface materials on the experiment. Through the above

expression, the theoretical value of the maximum velocity of the ball under different ball diameters, different rubber band materials and different contact surfaces can be calculated, which is 1/4 of the period, as shown in Table 1.

Table 1: Results of theoretical calculation

Serial number	Diameter (mm)	Rubber band material	Material of contact surface	$\frac{1}{4}T(s)$
1	16	First	Rubber	1.4101
2	18	First	Rubber	2.5860
3	20	First	Rubber	3.7620
4	22	First	Rubber	4.9380
5	20	First	Paper	3.7751
6	20	First	Board	3.2169
7	20	Second	Rubber	3.3620

2.4 Simulation experiment

In order to have a deeper understanding of the "pendulum" movement, this paper uses MATLAB to simulate the movement of the ball on the rubber band, and simulates the movement of two balls in space through the numerical solution of the differential equation[3]. The simulation results are shown in Figure 6. From the simulation experiment, it can be seen that the distance between the two metal balls changes from small to large and then from large to small. The speed also changes periodically, and the amplitude gradually decreases, making a circular motion[4].
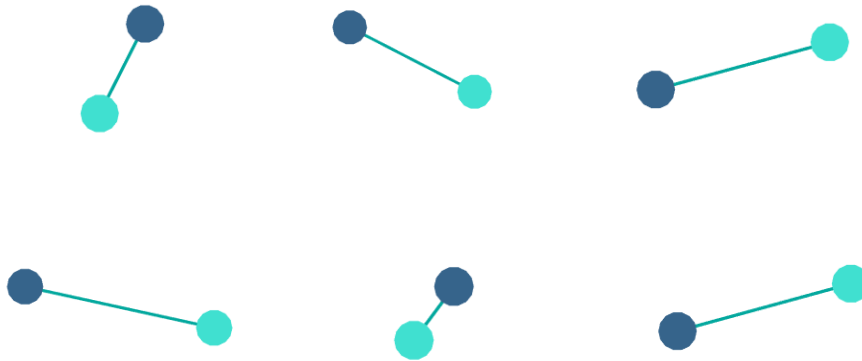

Figure 6: Simulation result

3. Physical experiment

3.1 Experimental operation

The main idea of this experiment is to control the variable method. The variables of this experiment are: the radius of the ball, the rubber band material, and the contact surface material. The invariants of the experiment are: the number of turns of the rubber band and the length of the rubber band.

For the metal ball used in the experiment, mark the center of the metal ball with a fluorescent pen to facilitate follow-up tracking. Then the values of k and b can be calculated through the theoretical calculation of the experimental parameters mentioned in the theory. Finally, tie the two balls to both

ends of the rubber band, twist and release them, track the trajectory of the balls, and then analyze the results through Tracker. The results are shown in Figure 7.
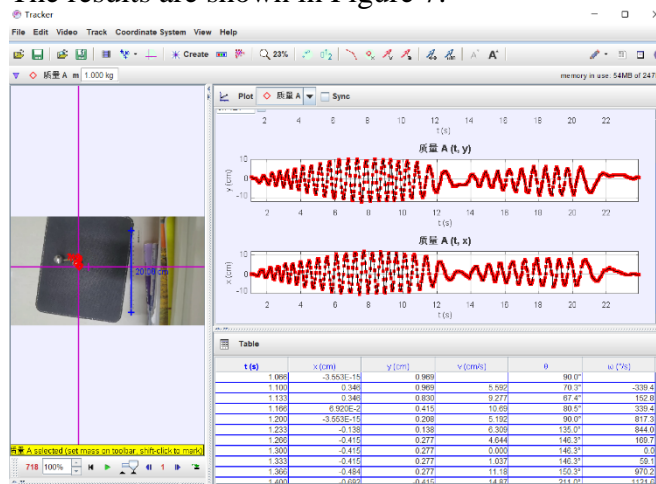

Figure 7: Tracking the motion track of metal ball with Tracker

3.2 Comparison between experimental results and theoretical values

Through the Tracker tracking results, it can be found that the metal ball makes obvious periodic movement. Therefore, in order to compare with the theoretical value, this paper chooses to study the time when the metal ball reaches the maximum speed, and compares the time when the actual maximum speed is reached with the time when the theoretical maximum speed is reached.

3.2.1 Study on the coefficients of different contact surfaces of the same ball and different rubber Band materials

In the first and second group of experiments, the diameter of the metal ball and the material of the rubber band were first controlled unchanged, the material of the contact surface was changed, and the time for the metal ball to reach the maximum speed under different conditions was explored; Then control the diameter of the metal ball and the material of the contact surface unchanged, and explore the time when the metal ball reaches the maximum speed under different conditions[5]. The results are shown in Table 2 and Figure 8.

Table 2: Study on the coefficients of different contact surfaces of the same ball and different rubber Band materials

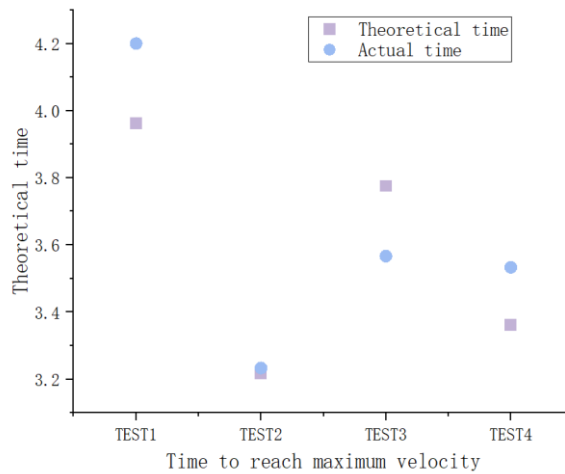| Serial number	Diameter (mm)	Rubber band material	Material of contact surface	Theoretical value	Actual value
1	20	First	Rubber	3.9620	4.2000
2	20	First	Board	3.2169	3.2333
3	20	First	Paper	3.7751	3.5666
4	20	Second	Rubber	3.3620	3.5333

Figure 8: Comparison between theoretical and actual maximum speed time

It can be seen from the experimental data that the difference between theory and practice is small, and the error is within an acceptable range.

3.2.2 Study on balls with different diameters

In the third group of experiments, the material of the contact surface and the rubber band were controlled unchanged, the diameter of the metal ball was changed, and the time for the metal ball to reach the maximum speed under different conditions was explored. The results are shown in Figure 9.
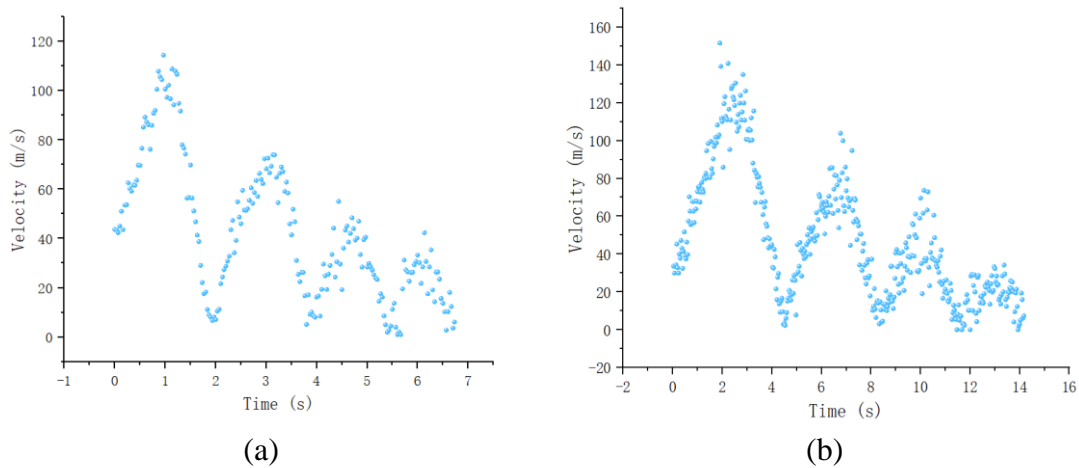

(a)   (b)

Figure 9: Variation of movement velocity of metal balls with different diameters with Time

Table 3: Study on balls with different diameters

Serial number	Diameter (mm)	Rubber band material	Material of contact surface	Theoretical value	Actual value
1	16	First	Rubber	1.4101	1.2000
2	18	First	Rubber	2.5860	2.8000
3	20	First	Rubber	3.7620	4.2000
4	22	First	Rubber	4.9380	5.4000

Using Tracker software to track the trajectory of the ball, you can get an image of the speed changing with time. By studying the physical quantity of vibration speed, we can find that it has the

characteristics of periodic vibration, which can well describe the characteristics of pendulum movement described in the title. The results are shown in Table 3.

4. Conclusions

Combining the theoretical and experimental results, the following conclusions can be drawn:

(1) The torsion constant of rubber band is negatively correlated with the period of motion; The rolling friction coefficient between the contact surface and the ball is positively related to the motion period; The radius of the ball is positively correlated with the period of motion.

(2) The ball with larger torsion constant and the ball with larger diameter can maintain longer motion.

(3) Increasing the length of the rubber band is conducive to increasing the movement time, but the excessive length of the rope will cause the winding part to hang, making the movement unable to continue.

For the overall analysis of the experiment, the errors are as follows: the factors such as air resistance and ball sliding friction are ignored in the experimental hypothesis, which may lead to deviation in the period of theoretical calculation; During the experiment, due to factors such as uneven winding caused by manual winding of rubber band, the center of mass of the whole movement will change, which will affect Tracker tracking and the final experimental data; Even though the selected length of the rubber band has been studied, it is still difficult to avoid the knotting problem of the rubber band, which may make the measured movement cycle sometimes increase and make the movement end early.

References

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