

Analysis of Elbow Stress Intensification Factors for Piping System

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Abstract: In order to study the stress intensification effect of pipe elbow in the secondary stress check, the U-shaped pipeline commonly used in engineering was taken as the research object. The experiment platform for analysing pipe elbow stress was established, and the maximum stress with the displacement load was measured and compared with the results of the finite element analysis results and ASME B31.3 Code. On this basis, a correction formula calculating the Stress Intensification Factor (SIF) of elbows was proposed, and the influence of the wall thickness and the bend radius on the elbow stress distribution was studied. The results showed that pipeline displacement significantly affects stress for pipe bends, the stress of the elbow increased with in-plane displacement load. To ensure structural integrity for reliable working conditions for piping components, pipe displacement needs to be considered when designing bends. On this basis, a modified formula for calculating the SIF of in-plane elbow is proposed. Compared with the ASME code formula, this formula is closer to the actual stress value of the elbow due to considers the influence of pipe displacement on elbow stress. The stress value of the elbow obtained by finite element analysis essentially in agreement with the experimental value, and the average error is less than 5.16%. With simultaneous increase in bend radius and wall thickness there is a reduction in SIF. When either of the above parameters is increased on, and keeping others constant the SIF decreases. The influence of pipeline displacement on SIF is more for short bend radius and its effect decreases with increased bend radius.

1. Introduction

The piping systems as an important channel for fluid transmission and energy conversion are widely used in petrochemical, energy power, aerospace and other engineering fields^[1]. Elbow is one of the important components in the piping system, and its Stress Intensification Factor (SIF) is an important parameter to ensure the safety of the elbow. During the service process of the pipeline, elbows are prone to local deformation and stress concentration under the combined effect of structure and fluid load. The cross-section of the elbow tended to be ovalization deformation when an external load is applied, which leads to a further increase of the elbow stress. Therefore, a series of piping codes such as ASME B31.1^[2] and ASME B31.3^[3] proposed by the American Society of Mechanical Engineers stipulate that the elbow stress intensification factor should be considered

when calculating the stress of the piping system. However, the existing research results are mainly used to analyse the single elbow structure, and the little research on the practical application of continuous elbow structure in pipeline. Nordham and Kaldor^[4] proposed a formula for calculating the SIF of elbows with different lengths of attached pipes based on the numerical analysis of the piping structure. Nikola^[5] proposed a new calculation equation for the SIF of the elbow based on the numerical model. The analysis results show that the equation is more suitable for the numerical data and the experimental data used to establish the original SIF formula. Nikola et al.^[6] presents new equations for calculation of elbow's SIF for large diameter-to-thickness ratio elbows, and proposed equations prove to be a better fit to the database than the existing ASME B31.3 code equations. Gao et al.^[7] used the FE Tools software to simulate and calculate the value of the SIF of the elbow, and compared it with the results of the ASME B31.3 code.

The main objective of this paper is to study the dependence of elbow's SIF on the effect of the cross section ovalization caused by displacement of pipeline subjected to in-plane opening mode of bending. The U-shaped continuous elbow structure commonly used in engineering was taken as the research object. By comparing the experimental results with the finite element analysis, the variation law of the elbow stress with the displacement of the pipeline is studied, and on this basis, a calculation formula of the modified elbow's SIF suitable for engineering application is proposed.

2. Finite Element Analysis

Calculation of SIF using finite element analysis is straightforward and extensively used in the engineering practice^[8]. This paper has been performed using the ANSYS software for parametric modelling and analysis. Input program files are created using Parametric Design Language which performed modelling and simulation of pipe bends of differing geometry.

2.1. Numerical Analysis Model

Numerical analysis model presented here is the U-shaped piping with four 90 degrees pipe elbows, as shown in Figure 1. In the process, shell element and pipe element are respectively used for modelling to obtain actual peak stress and nominal stress. The shell element is a three-dimensional finite element, which is suitable for simulating thin-walled to medium-thick-walled pipeline structures. It can perform deformation analysis such as tensile and bending of the structure. The pipe element is evolved from the beam element, which is a simplified pipe structure element.

The shell model adopts Shell181 element overall modelling and divided into 12480 elements in total. The pipe model adopts Pipe16 element and Pipe18 element to simulate the straight pipe section and the elbow section respectively, and the model has 83 elements in total. The elbow's geometrical parameters considered for analysis is presented in Table 1. Different bend radius and thicknesses are considered for analysis to obtain the wide range of the h factor.

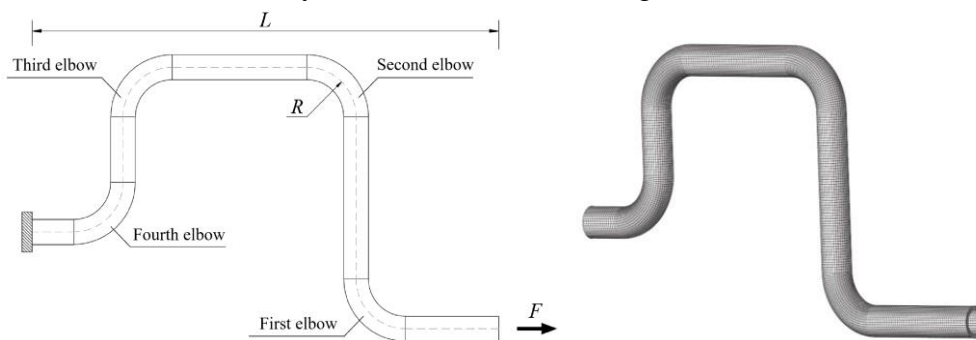


Figure 1: Numerical analysis model of U shape piping

Table 1: Details of the geometrical parameters of the pipe bends

Outside diameter D_o (mm)	Nominal pipe thickness t (mm)	Mean radius r (mm)	r/t	Bend characteristic $h = Rt/r^2$			
				$R=28.5$	$R=38.0$	$R=47.5$	$R=57.0$
				mm	mm	mm	mm
19	0.75	9.125	12.167	0.257	0.342	0.428	0.513
	1.25	8.875	7.100	0.452	0.603	0.754	0.905
	1.75	8.625	4.929	0.670	0.894	1.117	1.341
	2.25	8.375	3.722	0.914	1.219	1.524	1.828

2.2. Material and Boundary Conditions

The material considered for the present study is Type 304 Stainless Steel, non-hardening J2 flow theory was used assuming the material type as elastic-perfectly plastic. The material parameters are shown in Table 2.

In order to study the influence of the displacement of the pipeline on the elbow's SIF, considering that in the pipeline system, the elbow is generally not directly constrained, but indirectly constrained by the connected straight pipes when load is impacted, so boundary conditions are that one end of all fixed, and the axial displacement is applied to the other end. All cases were simulated without taking into account internal pressures, and therefore pressure hardening effects were not taken into account.

Table 2: SUS304 material parameters of stainless steel

Young's modulus GPa	Poisson's ratio	Density 10^{-3} kg/m ³	Elongation %	Yield stress MPa
195	0.31	7.85	35	270

3. Elbow Stress Intensification Factors

In the 1950s, Markl^[9] completed a series of pipeline fatigue tests and analysis, and based on the elastic stress theory of elbows used dimensional extrapolations led to

$$i_i = \frac{0.9}{h^{2/3}} \geq 1.0 \quad (1)$$

$$h = \frac{\text{Bend ratio}}{\text{Pipe ratio}} = \frac{R/r}{r/t} = \frac{Rt}{r^2} \quad (2)$$

In which, i_i is in-plane stress intensification factor, the flexibility characteristic h is given in Eq. (2). where R = bend radius, t = the nominal thickness of pipe, and r = mean radius of the pipe.

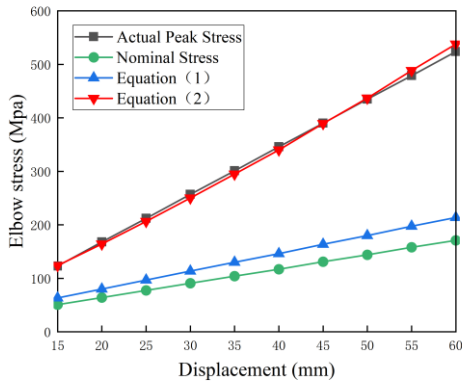
The study shows that the presence of ovality at the elbow has a momentous effect on B2 stress indices associated with stress intensification factor^[10]. But the Markl's equation hardly take into account the influence of the ovality at the elbow caused by the pipeline displacement load. When the displacement load produces a large bending moment at the elbow, the stress concentration phenomenon of the elbow will also be intensified due to ovalization deformation, which will

increase the peak stress of the elbow, and the real stress at the elbow will be higher than the stress value calculated by using Markl's formula. Considering the influence of ovalization at the elbow caused by displacement load, combined with numerical simulation, this paper proposes the following modified calculation formula of elbow stress intensification factors:

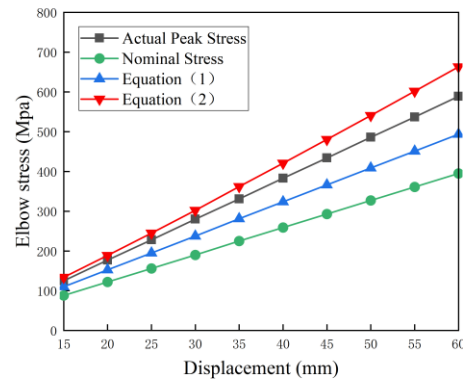
$$i = \frac{0.9}{h^{2/3}} + 1.093 \left(\frac{\Delta \cdot r^7}{R^2 T^4 L^2} \right)^{\frac{1}{3}} \quad (3)$$

In which: Δ = Pipeline end displacement; L = Effective length, $L = a L' / (b L' + c)$, where L' is the end displacement vector to the moment arm of each elbow, a , b and c are constants. The Errors about in-plane stress intensification factor in Eq.(1) was replaced with a Displacement related terms which could be chosen based on the value of bend characteristics to obtain Eq.(3).

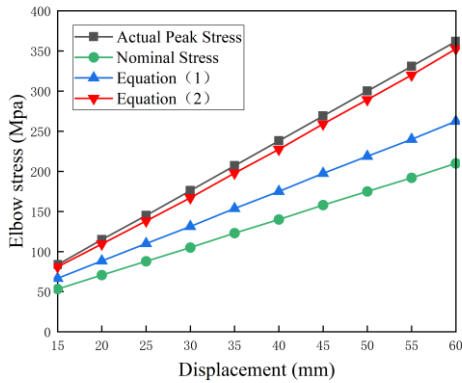
The study of the errors associated with actual peak stresses and nominal stresses about the proposed Eq. (3) and the original Eq. (1) is presented below. The comparison is given in Figure 2. It can be seen from the figure that the in-plane stress intensification factor Eq. (3) of the elbow is in good agreement with the finite element analysis results, and produces relatively accurate results within the inspection range. The maximum relative error at the first elbow is 2.69%, the second elbow is 12.62%, the third elbow is 2.58%, and the fourth elbow is 1.49%. Compared with Eq. (1) proposed by Markl in his paper, the correction Eq. (3) proposed has better correlation, higher calculation accuracy and smaller error.



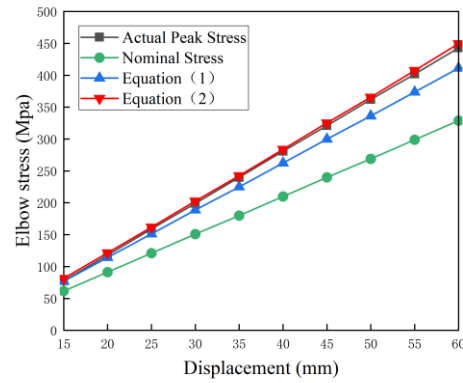
(a) First elbow



(b) Second elbow



(c) Third elbow



(d) Fourth elbow

Figure 2: Plot of elbow stress analysis data and equation (1) and equation (3)

It can be observed that in some cases, the percentage difference was higher. This may be due to the mismatch in the value of bend radius. It is quite difficult to fix a suitable bend radius using bend characteristic ranges. Hence the validation is confined only to the observed range of bend characteristics.

4. Validation of Finite Element Analysis

In order to measure the stress value of the pipe elbow under the action of pipeline displacement load, the U-shaped pipeline specimen was fabricated, and the tensile test was carried out on the specimen by an electronic universal testing machine.

4.1. Test Specimen

As shown in Figure 3, the U-shaped pipeline specimen is welded by a stamped elbow with a pipe diameter of 19 mm, a wall thickness of 1.5 mm, and a curvature radius of 38 mm, and a straight pipe with the same pipe diameter and wall thickness. The total length of the specimen is $L=384$ mm, and the material used is SUS304 stainless steel. The center point of the inner side, outer side, and neutral layer of each elbow is used as the stress measuring point, and the strain gauge model is BX120-3AA. The quasi-static tensile test was carried out on the WDW-50E program controlled electronic testing machine^[11]. During the experiment, the external force F was applied on the end of the elbow and the displacement change of the end of the specimen under the external force is recorded.



Figure 3: Test specimen

4.2. Analysis of Experimental and Numerical Results

The finite element results are now compared with the experimental values. Figure 4 shows the curve of the maximum stress at the center of the intrados of the four elbows versus the external load. It can be seen that with the increase of external load, the maximum stress of the four elbows gradually increases linearly. Compared with the calculation results of Eq.(1), the maximum value of elbow stress calculated by modified Eq.(3) is closer to the experimental value, and its average error is less than 5.16%. The maximum value of elbow stress calculated by the original Eq.(1) is generally smaller than the experimental value, and the error is large with an average error of 16.34%.

It is worth mentioning that due to the lack of experimental reference, this paper failed to carry out the experiment according to Markl's experimental method in 1952^[12], but adopted the method of structural static load analysis, and tried to propose a comprehensive consideration of the stress of the pipeline structure in actual work. It can be seen that the finite element simulation based on shell element and pipe element is true and effective in a certain extent from the experimental results.

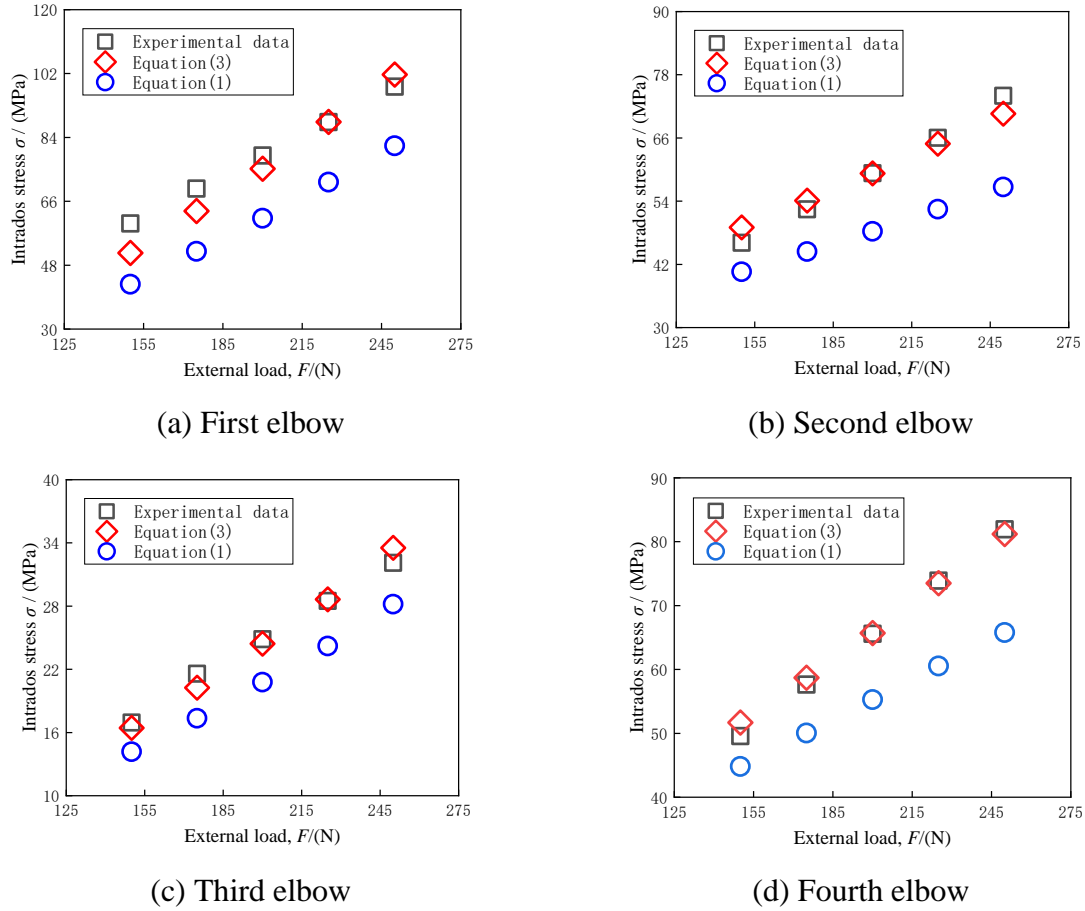
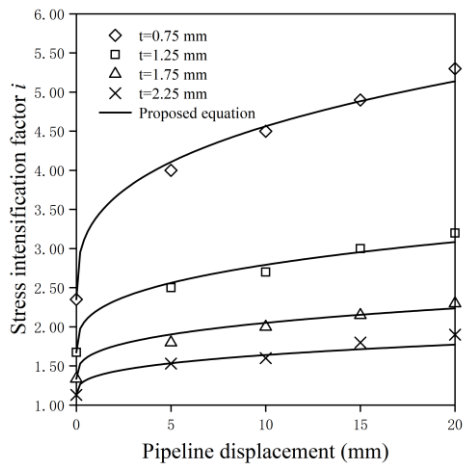


Figure 4: Curve of elbow stress with external load

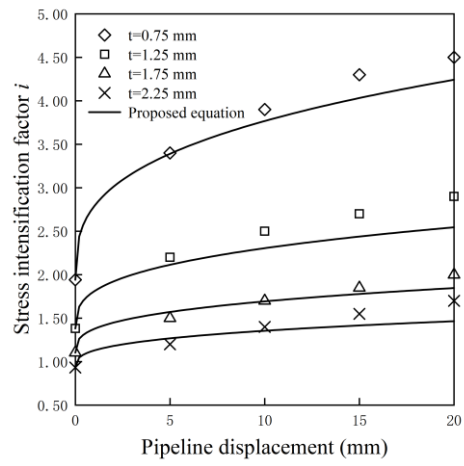
5. Results and Discussion

The basic approach of ASME code implies that the SIF value for straight pipes is always one and it exceeds one for bends and elbows. Hence this approach states that the increase in the SIF value reduces the load carrying capacity of the structure. Figure 5 depicts the variation of SIF value with pipeline displacement for pipe bends of different thickness combinations and bend radius. It can be seen from the figure that the proposed Eq.(3) has a good correlation with the finite element analysis results. For all models, increase of pipeline displacement will increase SIF value. The percentage difference in the magnitude of the SIF value increases with pipeline displacement for pipe bends with different geometric parameters due to the ovalization of the elbow section. From the analysis the presence of the ovalization in the cross section of the bend model, results in an overall minimum and maximum percentage difference of SIF value ranging from 1.18% to 9.88% for bend models with 28.5 mm bend radius, these values vary from 2.57% to 13.03% for a bend radius of 38.0 mm. The percentage difference varies from 4.18% to 15.47% for bends with 47.5 mm bend radius. When bend radius is increased to 57.0 mm it ranged from 0.97% to 8.73%.

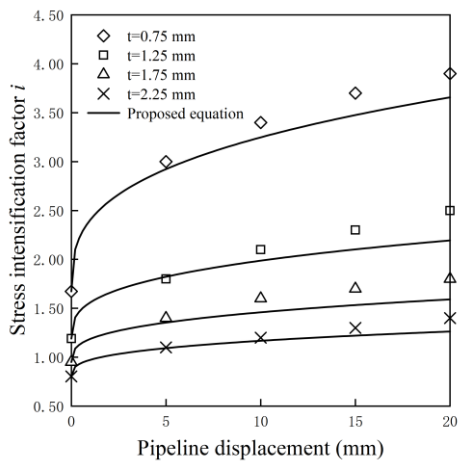
It is clear that, ovalization weakens the pipe geometry by reducing the stiffness effect caused by the opening mode of bending. When pipe bends under in-plane opening moment load, area at mid cross section increases due to the ovalisation when considering geometric nonlinearity in the analysis. This is due to the increase in distance between intrados and extrados, and decrease in the distance between crowns^[13]. This increases the second moment of area adding stiffness to the geometry due to which additional stiffness increases the load bearing capacity.



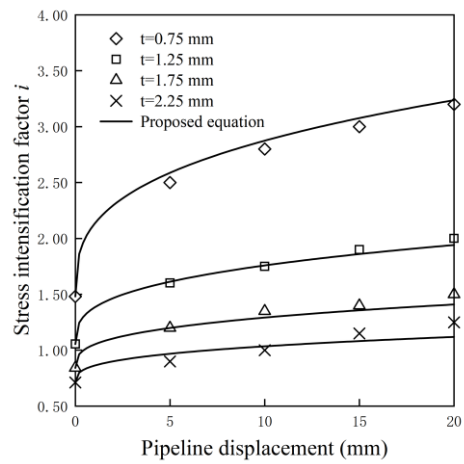
(a) R=28.5mm



(b) R=38.0mm



(c) R=47.5mm



(d) R=57.0mm

Figure 5: Comparison of stress intensification factor between the FE results and equation (3)

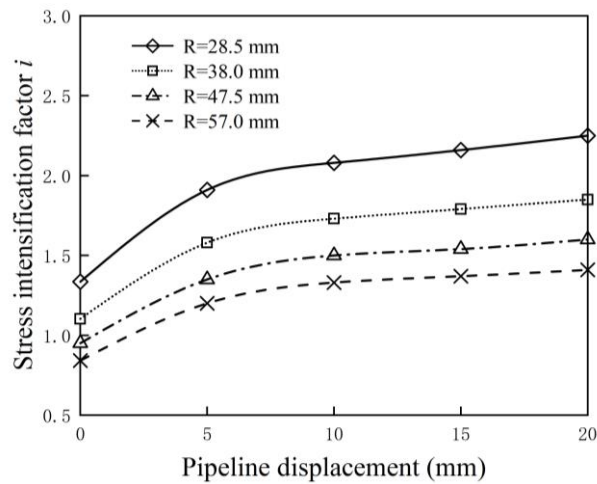


Figure 6: Effect of bend radius on SIF value for models with $t = 1.75$ mm

The combined effect of bend radius and elbow section ovalization on SIF value for models with 1.75 mm thickness and 19 mm average pipe diameter is shown in Figure 6. For any specific ovality the percentage difference in SIF value is higher for models with 28.5 mm bend radius and lower for 57 mm bend radius models. Similar behavior is noticed for all other thicknesses considered in the analysis. The above details clarify that the weakening effect (increase in SIF value) produced by ovality is reduced for pipe bends with long bend radius. For pipe bend models, increased wall thickness increases stiffness and resistance against bending load and reduces SIF value. Therefore, in pipe bends with specific bend radius and ovality, SIF value is reduced with increased thickness.

6. Conclusion

1) Pipeline displacement significantly affects stress for pipe bends, the stress of the elbow increases with increased in-plane displacement load. To ensure structural integrity for reliable working conditions for piping components, consideration of pipeline displacement is mandatory when designing pipe elbows.

2) A modified formula for calculating the SIF of in-plane elbow is proposed. Compared with the ASME code formula, this formula is closer to the actual stress value of the elbow due to considers the influence of pipe displacement on elbow stress. The stress value of the elbow obtained by FE analysis is in good agreement with the experimental value, and the average error is less than 5.16%.

3) With simultaneous increase in bend radius and wall thickness there is a reduction in SIF. When either of the above parameters is increased on, and keeping others constant the SIF decreases. The influence of pipeline displacement on SIF is more for short bend radius and its effect decreases with increased bend radius.

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