

Exploring the Pathway of Developing a Sense of Quantity in Primary School Mathematics

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Abstract: Sense of quantity is a very important element in the development of primary school students' core literacy in mathematics. It is a new core keyword in mathematics added to the Mathematics Curriculum Standards (2021 Edition) to prompt teachers to focus on the development of students' sense of quantity. This article is a study of quantitative awareness, which will help teachers move from a vague to a clear understanding of quantitative awareness, thereby improving the effectiveness of teaching and learning and helping students to acquire quantitative awareness and improve their core literacy. In teaching practice, there is a real dilemma that the content of teaching lacks a connection with life, the teaching process lacks activity experience, and the homework assignment lacks optimal design. Teachers need to develop students' sense of quantity by linking them to real-life situations, developing their sense of quantity by enriching their perceptual experiences, and consolidating their sense of quantity by optimizing their homework design.

1. Introduction

“Sense of quantity is one of the core mathematical literacies that primary school students should have ^[1].” It is a very important element in the development of primary school students' core mathematical literacy today, and has been one of the very hot topics of discussion in mathematics teaching in recent years. Sense of quantity begins with the perception of quantity, which is both related to and distinct from ‘sense of number’. The development of a sense of quantity in mathematics teaching can help students to master quantitative methods, examine problems and solve them with a quantitative perspective, and help students to develop a sense of basic mathematical ideas. As scholars point out, “Metrics is a fundamental idea of mathematics, it is a quantitative and qualitative portrayal of the real world borrowed from the precision of quantity and the shape of space. ^[2]” Therefore, the Mathematics Curriculum Standards (2021 Edition) have explicitly added the term ‘sense of quantity’ as a core keyword in mathematics to prompt teachers to focus on the development of students' sense of quantity.

2. Theoretical overview of quantitative sense in primary mathematics

There is no clear definition of “quantity sense” in mathematics education. The expression “quantity sense” exists abroad, but it is not yet as clear a concept as “number sense” ^[3]. Some

Chinese scholars believe that “the sense of quantity is the visual or tactile sense of the scale, degree, speed and other aspects of various objects. It is the perceptual understanding of the size, quantity, length, thickness, roundness, thickness, weight, speed and tightness of an object. And it expresses the feeling of lightness, thickness, size and quantity of an object with the help of light and darkness, color, line and other modelling elements. [4]” In a nutshell, the sense of quantity, or the perception of measurement, is an intuitive perception of quantities such as length, weight and area, which students acquire through personal experience without the use of professional tools of measurement. In the content of mathematics at primary level, learning about Yuan, jiao, fen, seconds, minutes and hours, meters, centimeters and kilometers are all part of the learning of the sense of quantity.

The development of students’ sense of quantity at primary school level has important implications: firstly, it helps to implement students’ core literacy in mathematics. The Mathematics Curriculum Standards (2011 Edition) clearly state that emphasis should be placed on developing students’ core literacies. The so-called core literacy in mathematics refers to the comprehensive embodiment of the quality of thinking, key competencies, and emotions, attitudes and values that are fundamental characteristics of mathematics, it is the ultimate goal of mathematics education related to human behavior, and it is gradually formed and developed in the teaching activities in which students themselves participate [5]. It is primarily concerned with the training specifications for education, i.e. the standards that students should achieve after mathematics education, which can be expressed as students being able to see the world mathematically, to think about the world mathematically and to express the world in mathematical language. Teachers pay attention to the development of students’ sense of quantity, which helps students develop quantitative concepts, master quantitative methods and develop quantitative thinking, thus laying a solid foundation for the implementation of students’ core literacy. Secondly, it helps to improve students’ estimation skills. In mathematics, number and quantity go hand in hand. Number is an expression of the result of a measure and a symbolic, and the essence of mathematics lies in measurement, and the essence of measurement lies in the expression of number. However, there is no strict distinction between number and quantity in quantities, and teachers teach the content of quantities with more emphasis on calculation and conversion, neglecting to develop students’ estimation and perception of quantity. Teachers should pay attention to the development of students’ sense of quantity by using objects familiar to students to form references and helping them to experience the sense of quantity through activities. Students have an internal thinking process of perceiving-comparing-estimating-correcting quantities in their learning, thus improving their sense of quantity, accumulating experience in estimation, mastering estimation methods in real-life experiences, and enhancing their awareness and ability to estimate.

3. The dilemma of developing a sense of quantity in primary school mathematics

3.1. Lack of life connection in teaching content

As an important part of the core literacy of mathematics, it is the teacher’s responsibility to help students develop a sense of quantity. However, in the past, teachers neglected students’ existing life experiences and failed to link the teaching content with students’ lives, focusing on direct indoctrination of students, especially on the results of students’ learning and receiving knowledge, but not on the process of developing students’ sense of quantity and emotional penetration. For example, in the lesson “hectares and square kilometers”, the textbook uses the example of the “bird’s nest” to introduce the hectare, by showing that the area of a square with a side of 100 meters is 1 hectare. As students are unfamiliar with the bird’s nests, it is difficult to create effective cognitive conflict. If teachers do not make connections between hectares and the familiar living environment of students, such as schools, parks and city buildings, and cannot find references to

hectares, it is difficult for students to build a clear representation of hectares.

The reason for this dilemma is related to teachers' outdated concepts of teaching quantity sense. As the organizers and guides of learning activities, Teachers need to fully explore the teaching content related to quantity sense to develop students' quantity sense. However, in practice, some teachers believe that teaching quantity sense is a form of teaching conceptual understanding of mathematics, so in order to enable students to better grasp mathematical concepts, they use a fill-in-the-blank approach to instil mathematical knowledge into students, who can quickly grasp mathematical concepts but do not develop a sense of quantity. In addition, the teaching of quantity has its own stage of presentation, which requires teachers to grasp the characteristics of the grade level when teaching. For example, in the lower grades, "yuan, jiao and fen" and "centimeters and meters" are quantities that students can visualize in the classroom, whereas in the middle grades, "knowledge of kilometers" "tons" and "hectares" are more abstract in content because the units of quantity are larger and more difficult to measure.

3.2. Lack of activity experience in the teaching process

The development of a sense of quantity is linked to students' experiences. If students do not have relevant activity experiences, there is no way to build a sense of quantity. Therefore, the prerequisite for the development of students' sense of quantity is that they first establish a clear cognitive representation of quantity, and then estimate - revise - reconstruct it from their activity experiences. However, in the actual teaching process, teachers spend their time on the teaching process to help students understand mathematical concepts and exercise training in order to better achieve their knowledge objectives, and tend to neglect students' sense of quantity and lack the experience of activities to verify the estimation of quantity. For example, in the lesson "Understanding the square centimeter", some teachers spend a lot of time teaching the concept of "square centimeter" in the teaching process, but there are no specific activities for students to experience how big "1 square centimeter" really is.

Teachers' lack of activity experience design in the teaching process is related to their lack of ability to teach quantity sense. To develop students' sense of quantity, teachers must first know how to set up diverse activity experiences according to the characteristics of students at different grade levels. In the past, teachers have not been aware of the value of quantity sense and have lacked in-depth research, which has led to a lack of experience in teaching quantity sense and a lack of competence in teaching quantity sense. For example, when some teachers teach quantity, the activities focus on how to measure, lacking the link for students to explore and perceive quantity, and lacking the process of thinking about and imagining quantity. Therefore, in teaching activities, if teachers neglect to guide students to estimate and perceive quantity, students do not experience quantity deeply in the activities, the representation of measurement units is not well established, and it is naturally difficult for students to form a sense of quantity.

3.3. Lack of optimal design of homework assignments

Teachers set homework according to the teaching situation and students' characteristics, which can effectively consolidate classroom knowledge and thus achieve teaching objectives. However, some teachers give more consideration to the need for examinations when assigning homework, and tend to neglect the effectiveness of homework. For example, after teaching a lesson, teachers often assign students to complete the exercises in the relevant exercise book or textbook. Such homework is set in a single format and has a high repetition rate, which is not conducive to the improvement of students' abilities and the development of their thinking. In addition, teachers assign paper-based assignments, which are usually on mathematical concepts or number operations, and lack

personalization and hierarchy for each student's specific situation in order to facilitate marking and to standardize the criteria. In this way, the development of students' sense of quantity and their mathematical thinking is undoubtedly hindered.

The reason for this dilemma is that teachers have a misunderstanding of the function of quantitative work in mathematics. As an important part of teaching and learning, homework has its own unique role to play, and an effective homework assignment can facilitate the development of students' quantitative sense. However, in the past, teachers believed that homework was for examinations and that students could master knowledge as long as they practiced in repetitive homework. They pursued the quantity rather than the quality of homework and assigned a large amount of knowledge-based homework to students, which led to a high repetition rate and a single form of homework. If teachers do not understand the function of homework, then it will not play the role it should, and the special nature of quantitative work means that it should not be limited to the acquisition of knowledge, but should be integrated with practical work and life practice. Only in this way can students' knowledge, skills and literacy be developed simultaneously.

4. Pathways for developing a sense of quantity in primary school mathematics

4.1. Connecting to life situations to promote the perception of quantity

The abstract nature of mathematical knowledge requires teachers to express the teaching content in a visual and life-like language so that students can better understand mathematical knowledge. Teachers make use of students' familiar life situations to teach quantity sense, guide students to perceive quantity, develop their sense of quantity and promote the development of students' core mathematical literacy. In fact, "the development of a sense of quantity requires teachers to relate the content of the lesson not only to what they have already learnt, but also to real life, and they need to guide students continuously in the classroom to achieve a refinement of their sense of quantity ^[6]."

On the one hand, teachers need to study and analyze the textbooks and strengthen the links between their content and students' lives. Mathematics textbooks are universal, and the material in them may not be suitable for all students in all regions. For example, in the lesson 'hectares and square kilometers', the content of the textbook is the "Bird's Nest" and "Tiananmen", however, the realistic nature of students' learning of mathematics determines that there will be differences in students' understanding of the same material content. This requires teachers to add and subtract from the content of the material based on students' life experience, using examples familiar to them to help them understand, and achieving the effect of simplifying the complexity. On the other hand, constructivists assert that the learning context created by the teacher must be real, concrete and vivid, and that the effectiveness of quantitative teaching largely depends on the authenticity of the context created in teaching. Therefore, in teaching, teachers create familiar life situations for students and guide them to compare the quantities in teaching with familiar things in their lives to form familiar and diverse references, thus promoting students' perception of quantities and helping them to build clear representations. For example, in the lesson "hectares and square kilometres", teachers can use sports grounds or school buildings to help students build up an image of one hectare, and then go to familiar parks, museums and other living buildings to strengthen the link between mathematical knowledge and the realities of life, and help students to perceive the quantity.

4.2. Enriching perceptual experience and developing students' sense of quantity

"Experience is an important way of developing a sense of quantity and an important basis for accumulating representations and building concepts ^[7]." Students can only gain a deeper understanding of abstract mathematical knowledge such as relevant concepts, principles and rules if

they have a rich perceptual experience when learning mathematics. On the one hand, teachers need to make use of information technology to assist teaching and learning, allowing students to visualize quantities in visual and intuitive images. Because information technology can effectively visualize abstract content in teaching, using information technology to carry out teaching can visualize and make boring, abstract mathematics interesting, attracting students' attention and enriching their experience, thus helping students to better experience the sense of quantity and improving the effectiveness of teaching. For example, in 'hectares and square kilometers', teachers can show pictures of playgrounds, classrooms and schools to help students understand hectares, and they can also use "panoramic maps" to show the buildings around them and the cities they live in, so that students can move from the point to the surface and from the intuitive to the abstract, to better understand and experience the sense of quantity.

On the other hand, teachers need to create a series of activities to enrich teaching and learning, and allow students to experience the quantities in a rich and varied activity. "Much of the world cannot be passed on, it can only be experienced first hand ^[8]. "Therefore, when designing teaching, teachers should make good use of group investigations, extra-curricular activities, experimental operations, contextual demonstrations and other forms of activities to enrich classroom teaching. Students can only experience the process of discovering mathematical knowledge in a wide variety of activities, and "participate in them personally to 'sense quantity', i.e. use all their senses to realize the real experience of the size, number, weight and length of objects, etc., and gain a unique feeling of 'quantity' in the experience, gradually enhancing their 'sense of quantity' ^[9]." For example, in the lesson "hectares and square kilometers", teachers show pictures of the Yuanmingyuan, the wetland park, the school and so on, after the students have learned about hectares and square kilometers, and carry out group discussions as an inquiry activity. The group discussion activity is about choosing the appropriate unit of area and justifying it. After discussion, students answer that the area of the Yuanmingyuan is about 350 square meters, teachers should make clever use of classroom generation and follow up by asking students why they think so. Teachers discuss with students in a series of 'whys', and allow them to revise their sense of quantity in their experiences, to estimate, to verify, to further clarify their sense of hectares, and help them to revise their sense of quantity, to express their thinking in their answers, to strengthen their sense of quantity in their thinking.

4.3. Optimize the design of homework to consolidate students' sense of quantity

The "double reduction" policy explicitly calls for reducing the burden of homework on students and increasing its effectiveness, which requires teachers to improve the effectiveness of homework when designing it. The effectiveness of homework is reflected in, on the one hand, the integration of mathematical knowledge and life practices, so that students can achieve the application of knowledge and transfer ability in the process of completing homework. Therefore, the abstract nature of quantitative knowledge makes it necessary to set quantitative work not just in writing, but in combination with practical life work, in which quantitative knowledge is consolidated and developed. Different types of practical work can be set for different lesson types. For example, after learning about units of length, teachers can assign the practical work of "making a meter ruler by yourself", so that students can understand the relationship between millimeters, centimeters, decimeters and meters in the process of making a meter ruler and build up their sense of quantity. After learning the units of area, teachers can assign the "I'm a little designer" inquiry-based practical work, in which students choose their own floor tile size according to the size of their living room and explore how many tiles they need for their own living room. The variety of practical assignments is designed to effectively consolidate knowledge of units of measurement and also

enhance problem-solving skills.

On the other hand, effective assignments have to be designed according to the actual situation of students. There are differences in students' learning bases and the same assignments cannot meet the needs of all students, which requires teachers to carefully design graded, quality and personalized assignments so as to meet the needs of students at different levels. When designing assignments, teachers can divide them into "basic", "integrative" and "exploratory" levels. Basic assignments are the most basic understanding of quantitative concepts. Integrative assignments are the application of quantitative concepts with a certain degree of flexibility and difficulty, taking into account the differences in students' levels. Exploratory assignments require students to master knowledge and apply it on the basis of practical operations and life experiences. For example, in the lesson 'Understanding the kilometer', teachers can design "the concept of the kilometer and its conversion to other units" as a basic assignment, "flexible problem solving using knowledge of the kilometer" as a comprehensive assignment, and "experience and express your feelings about 1 kilometer in life" as an open-ended assignment.

5. Conclusion

To sum up, to develop students' sense of mathematical quantity, we should start from students' familiar life experience, focus on perceptual experience, guide students to construct a knowledge network while comparing and projecting, and then consolidate and apply it with the help of practical work, so as to promote the formation of core literacy while learning knowledge and skills. In addition, teachers should not only focus on the development of students' sense of quantity from the perspective of their practice and feelings, but also realize that students' reflective activities also contribute to the development of their sense of quantity. In fact, it is not enough for students to experience and perceive the learning of the sense of quantity, they also need to "enrich the 'sense of quantity' in their reflections continuously^[10]" on the base of their perceptual experience and combining it with thinking and reflective activities, and then develop an effective quantitative sense.

This study uses case studies to analyze the practice of teaching quantitative sense in primary schools at this stage. The findings reveal that teachers' teaching of quantitative sense suffers from the practical dilemma of a lack of life connection in the teaching content, a lack of activity experience in the teaching process, and a lack of optimized design in the assignment, thus resulting in a random, scattered and fragmented state in the development of primary school students' quantitative sense. However, due to the small number of cases in this study and the lack of relevant quantitative research, the results may be slightly less convincing. As the development of quantitative and numerical senses are complementary, further research is needed to clarify their opposing and unifying relationships. In addition, a holistic approach to measurement knowledge at primary level, integrating key elements of measurement such as units, rates, tools and strategies, and a structured design will facilitate the exploration of pathways for developing a sense of quantity and the steady improvement of students' sense of quantity literacy.

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