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3D Bin Packing of Online Constrained Variable Size Sphere Based on Computer Simulation Technology

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Abstract: Three dimensional packing problem is widely used in many fields. At present, the research on off-line 3D packing of rectangular objects is more extensive, while the research on on-line 3D packing of rectangular objects is relatively simple. In this paper, an online constrained variable size sphere three-dimensional packing problem based on computer simulation technology is proposed, and the solution to this problem is given. Due to the different weight of the sphere, the sphere is divided into different levels of cell size, and the sphere is loaded into the appropriate rhombic dodecahedron to form cells, and then loaded; Furthermore, according to the weighted method, the competitive ratio in the bounded environment can be obtained, which solves the packing problem of the same kind of goods with different weights on the assembly line.

1. Introduction

The three-dimensional loading problem widely exists in the logistics industry such as port and airport. The optimization of three-dimensional loading problem is beneficial to reduce the cost of loading and logistics, and has become an important competitive node of enterprises. At present, the research on the rectangular shape of goods to be loaded is quite extensive [1]. If the loading algorithm does not know any subsequent cargo information when processing each cargo to be loaded in order,and immediately gives the packing scheme of the current cargo, it is called online packing.

Kamali et al [2] packed equilateral triangles into squares in two-dimensional space. In the sphere packing, the sphere can be put into the cube, that is, origami technology [3] or cylinder [4] and then put into the box.

In the problem of packing cubes into cell cubes, Han et al. [5] proposed an algorithm with an asymptotic ratio of 2.6161 in the online state. A recent survey conducted by Christense et al. [6] showed that in the two-dimensional or three-dimensional space, some bin packing methods adopted approximate algorithms [10], including offline and online algorithms [9]. Only hokama et al. [7] considered the competitive algorithm of online round packing: on the competitive ratio of any algorithm in bounded space, the lower bound of 2.292, an algorithm with asymptotic competitive ratio of 2.439, and a sphere with the same radius packing into a cube [8] were given.

At present, there are few researches on the three-dimensional packing with the constraint of loading goods as spheres by using computer simulation technology, and the previous researches are usually for the spheres with the same radius, which are relatively familiar with the rectangular packing, but not for the sphere packing under other constraints. This paper presents an online three-dimensional packing problem of variable size spheres with weight constraints. In this problem, the spheres to be loaded have different radii and weights. By using weight condition factor, the spheres are divided into different levels and loaded into the corresponding cell units (the diamond dodecahedron is used in this paper), and then the packing operation is carried out, The computer simulation results show that the method is feasible.

It solves the actual loading problem of different weight and size spheres in real life online packing.

2. Mathematical Model

In this paper, the online packing of goods is about the packing of spherical objects. A sphere with a radius of r_i is filled. A sphere with a radius of i means finding coordinates x_i, y_i, and z_i pointing to its center so that they can be next to the boundary of the container. For sphere j adjacent to r_i, coordinates x_j, y_j, and z_j have $(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 \geq (r_i + r_j)^2$. When packing, the unit box is divided into big box $Lbin$ and small box $Sbin$. When encountering different weights of spheres, they will be put into different boxes respectively. Here $Lbin$ and $Sbin$ are only unit boxes. After that, we will further classify the spheres and containers to be entered, so that the small spheres in i categories can enter $Sbin$, and the large spheres in i categories can enter $Lbin$.

Take a value of ℓ, and let $\ell = 6 / C^p i \sqrt{6}$, because it is cut into rhombic dodecahedron, each rhombic dodecahedron can be called a cell. A $q - bin(i, p)$ is divided into a number of cells with ℓ r sides to package the spheroid (i, p). It should be noted that you can work normally only under the condition of $1/C^{p+1} \geq 2/(iC^p)$. Because of $i < CM$, you must choose $M > 2$, and the other ℓ must be accurate, so as to ensure that the cell can hold (i, p). Figure 1 (left) shows the specific data relationship of cells. The specific process of packing is to place a cell in the lower left corner of the box. The cell has four faces parallel to the two sides of the box, and each face shares two non adjacent vertices with the other two faces. Figure 1 (right) below shows the relationship between cells and boxes.

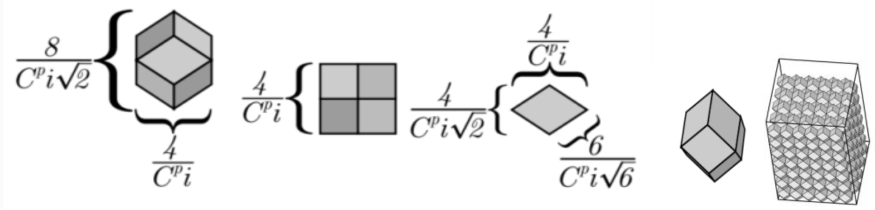

Figure 1: Specific numerical relationship of cell units (left); fill a $q - bin(i, p)$ (right)

Theorem 2.1: for a closed duty cycle of type i and type $Sbin$, for $M \leq i < 3M$, then the duty cycle of type i and type $Sbin$ is at least $\frac{649}{702}(1 - \frac{40.98}{M} + \frac{61.26}{M^2} - \frac{2443.77}{M^3}) \frac{\pi}{\sqrt{18}} \frac{M^3}{(M+1)^3}$

prove: In closed $Sbin$, the volume loss is caused by three factors

(I) : there is a loss of space when the ball fills the diamond

(II): loss when inlaid with rhombic dodecahedron and $q - bin(i, p)$

(III): $q-bin(i,p)$ non full load

For factor (I): packing type (i,p) with volume of at least $\frac{4\pi}{3}(\frac{2}{3^p(i+1)})^3$ into a rhombic dodecahedron with volume of $\frac{16\sqrt{3}}{9}(\frac{6}{3^{p_i}\sqrt{6}})^3$, the occupation ratio is $\frac{\pi}{\sqrt{18}}\frac{i^3}{(i+3)^3}$, that is, $\frac{\pi}{\sqrt{18}}\frac{M^3}{(M+3)^3}$.

For factor (II): it can be observed from Figure 2 that when $q-bin(i,p)$ is filled, three parts are lost in the box. From face $GG'F'F$, the depth of the missing block is at most $\frac{4}{3^p i}$, the height is $\frac{1}{3^{p+1}}$, and the width is $\frac{1}{3^{p+1}}$. From $HEFG$ sides, the depth of the missing block is at most $\frac{8}{3^p i\sqrt{2}}$, the height is $\frac{1}{3^{p+1}}$, and the width is $\frac{1}{3^{p+1}}-\frac{4}{3^p i}$ (the reason for the loss is that it intersects with the last block). As viewed from face $EF'FE$, the missing block has a depth of at most $\frac{4}{3^p i}$, a height of $\frac{1}{3^{p+1}}-\frac{8}{3^p i\sqrt{2}}$ (because it intersects the last block), and a width of $\frac{1}{3^{p+1}}-\frac{4}{3^p i}$ (because it intersects the first block).

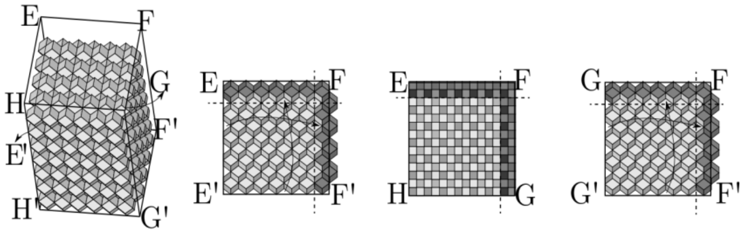

Figure 2: For the online cube sphere

The darker rhombic dodecahedral cells are lost by subdividing a $q-bin(i,p)$ into a hexagon. The boundary of $q-bin(i,p)$ is a thick line. The lines and boundaries of the area between the dashed lines are considered missing.

Therefore, it can be seen that the maximum volume lost in the diamond edge zone of $q-bin(i,p)$ is:

$$(\frac{4}{3^p i})(\frac{1}{3^{p+1}})(\frac{1}{3^{p+1}})+(\frac{8}{3^p i\sqrt{2}})(\frac{1}{3^{p+1}})(\frac{1}{3^{p+1}}-\frac{4}{3^p i})+(\frac{4}{3^p i})(\frac{1}{3^{p+1}}-\frac{8}{3^p i\sqrt{2}})(\frac{1}{3^{p+1}}-\frac{4}{3^p i})=$$

$$\frac{1}{3^{3p+2}i}(4+\frac{8}{\sqrt{2}}+4)-\frac{1}{3^{3p+1}i^2}(\frac{32}{\sqrt{2}}+16+\frac{32}{\sqrt{2}})+\frac{1}{3^{3p}i^3}(\frac{128}{\sqrt{2}})<\frac{13.65}{3^{3p+2}M}-\frac{61.25}{3^{3p+3}M^2}+\frac{90.50}{3^{3p}M^3}$$

If $q-bin(i,p)$ is filled in this way, its duty cycle is at least: $\frac{\pi}{\sqrt{18}}\frac{M^3}{(M+1)^3}(\frac{1}{3^{3p+3}}-\frac{13.65}{3^{3p+2}M}+\frac{61.25}{3^{3p+3}M^2}-\frac{90.51}{3^{3p}M^3})\Big/(\frac{1}{3^{3p+3}})=\frac{\pi}{\sqrt{18}}\frac{M^3}{(M+1)^3}(1-\frac{40.99}{M}+\frac{61.26}{M^2}-\frac{2443.77}{M^3})$.

For factor (III): when a $Sbin$ is closed, for $p\geq 0$ and the box is not full, there can only be one $q-bin(i,p)$; Similarly, no $q-bin(i,0)$ can be empty, but for each $q-bin(i,p)$ of $p\geq 1$, it can be up to $3^3-1\ q-bin(i,p)$. Therefore, the maximum volume loss caused by q cases of (i,p) that are not filled is:

$\sum_{p\geq 0}(1-\frac{1}{3^3-1}-\frac{1}{3^3})(1-\frac{40.99}{M}+\frac{61.25}{M^2}-\frac{2443.77}{M^3})\frac{\pi}{\sqrt{18}}\frac{M^3}{(M+1)^3}$. Each enclosure $Lbin$ of type i retains at most i spheres of i types. Because any sphere has a radius of at least ρ_{i+1}, its occupancy ratio is at least $i(4/3)\pi\rho_{i+1}^3$.

With respect to the weight function w, the weight of a large sphere I_i is $w(Ii)=1/i$. If the radius of a small sphere s of type (i,p) is r, its weight is $w(s)=4\pi r^3/(3OR)$, so the algorithm of progressive competition ratio is obtained.

3. Algorithm of progressive competitive ratio

Accordingto hokama [7], we can deduce some properties of a sphere loaded into a lattice. Take three integers σ、δ、ε, and their values or ranges are $0<\sigma<1$, $\delta=\pi 4\sigma\sqrt{18}$ and $\varepsilon<\delta$ respectively, and establish a series of spheres $S_0, S_1\dots$ Until S_k, in which S_k satisfies $V(S_k)\ge(1-\varepsilon)^2$. Starting from $S_0=S$, for each $n\ge1$, build a S_n by adding S_{n-1} with a radius of $\sqrt{6}\ell_n/3$ (of which $\ell_n<\ell_{n-1}$, $\ell_1<\min_{s\in S_0} r(s)$). For $n\ge1$, let $\ell_n\le\varepsilon/(\frac{6}{3\sqrt{3}}A(S_{n-1})+\frac{16\pi}{3}r(S_{n-1})+\frac{32\pi}{9\sqrt{3}}\left|S_{n-1}+\frac{12\sqrt{2}+44}{3\sqrt{3}}\right|)$, suppose S_{n-1} can be packed into a unit box and one of the spheres is fixed. A rhombic dodecahedron is arranged on side ℓ_n and placed in a box containing S_{n-1}.

According to theorem 2.1, the maximum volume of the disjoint rhombic dodecahedron and the element box is $\frac{2\sqrt{2}\ell_n}{\sqrt{3}}+\frac{4\ell_n}{\sqrt{3}}(1-\frac{2\sqrt{2}\ell_n}{\sqrt{3}})+\frac{2\sqrt{2}\ell_n}{\sqrt{3}}(1-\frac{4\ell_n}{\sqrt{3}})(1-\frac{2\sqrt{2}\ell_n}{\sqrt{3}})<\frac{(4\sqrt{2}+4)\ell_n}{\sqrt{3}}+\frac{32\ell_n^3}{3\sqrt{3}}$. If a rhombic dodecahedron is tangent to the interior of a sphere with a radius of r and a center of P, the rhombic dodecahedron with a center of P and a center of $r+2\ell_n/\sqrt{3}$ can be inscribed (see Figure 1 (left) for the model). Then the total volume is at least:

$$1-(\frac{4\sqrt{2}+4}{\sqrt{3}})\ell_n-(\frac{32}{3\sqrt{3}})\ell_n^3-\sum_{s\in S_{n-1}}(\frac{4\pi}{3}(r(s)+\frac{2\ell_n}{\sqrt{3}})^3) =$$

$$1-(\frac{4\sqrt{2}+4}{\sqrt{3}})\ell_n-(\frac{32}{3\sqrt{3}})\ell_n^3-V(S_{n-1})-\frac{6}{3\sqrt{3}}\ell_n A(S_{n-1})-\frac{16\pi}{3}\ell_n^2(S_{n-1})-\frac{32\pi}{9\sqrt{3}}\ell_n^3|S_{n-1}| >$$

$$1-(\frac{4\sqrt{2}+4}{\sqrt{3}})\ell_n-(\frac{32}{3\sqrt{3}})\ell_n^3-V(S_{n-1})-\frac{6}{3\sqrt{3}}\ell_n A(S_{n-1})-\frac{16\pi}{3}\ell_n(S_{n-1})-\frac{32\pi}{9\sqrt{3}}\ell_n|S_{n-1}| \ge 1-V(S_{n-1})-\varepsilon$$

So, if each new sphere occupies the volume of dodecahedron $\frac{\pi}{18}$, it can get $V(S_n)\ge V(S_{n-1})(1-V(S_{n-1})-\varepsilon)\frac{\pi}{18}$, so there is $k\ge0$, so that $V(S_k)\ge(1-\varepsilon)^2$, according to the transitivity angle, S_k can be loaded into box $S_0\in S_k$.

4. Example

Now let's take an example of N disjoint S_k. S_0 has the property of i, q_i sphere, radius between $\rho_{i+1} and \rho_i$. It should be noted that the optimal off-line solution uses N boxes to pack such an instance, and considering the online algorithm, the radius of the sphere does not increase during the packing process. Any on-line algorithm in bounded space B uses at least $Nq_i/i-B$ boxes for each type of ball in S_0. Suppose sphere n_j with volume of v_j is added to S_{J-1} queue to construct S_j. At present, the best density of sphere packing is $\pi/\sqrt{18}$. Any algorithm uses at least $Nn_jv_j\sqrt{18}/\pi-B$ boxes to load n_j balls in bounded space B.

Because $N \geq 2(t+k)B/\sigma$, where t is the number of spheres of different sizes in S sphere, through the above analysis, any bounded space B uses at least:

$$\sum_{i\in S_0}(\frac{Nq_i}{i}-B)+\sum_{j=1}^{k}(Nn_jv_j\frac{\sqrt{18}}{\pi}-B)>N(w(S)+\frac{\sqrt{18}}{\pi}(1-2\varepsilon-v(S))-(t+k)B =$$

$$N(w(S)+\frac{\sqrt{18}}{\pi}(1-V(S)))-N\frac{\sqrt{18}}{\pi}2\varepsilon-(t+k)B >$$

$$N(w(S)+\frac{\sqrt{18}}{\pi}(1-V(S)))-\frac{N\varepsilon}{2}-(t+k)B \geq N(w(S)+\frac{\sqrt{18}}{\pi}(1-V(S)))-N\varepsilon$$

To sum up, if a group of spheres are compressed into a box, the competition ratio of online algorithm in each bounded space is at least $w(S)+\frac{\sqrt{18}}{\pi}(1-V(S))$.

5. Conclusions

In this paper, an online constrained variable size sphere 3D packing scheme is proposed. In the online constrained variable size sphere packing, the size of the sphere to be loaded is divided into the sphere size range by a certain factor, and then the sphere size can be distinguished. Therefore, the corresponding *Sbin* or *Lbin* storage locations can be allocated. The online algorithm competition ratio of *Sbin* and *Lbin* packing is at least $w(S)+\frac{\sqrt{18}}{\pi}(1-V(S))$, Furthermore, it can be concluded that the competition ratio of online approximation algorithm in bounded space is at least 2.8809.

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