

Effects of Network Structure and Traffic Allocation on Traffic Network Efficiency

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Abstract: For decades, research shows that network structure determines statistical properties and dynamical characteristics, even for directed networks. However, as observed in traffic flow networks, traffic flow (edge weight) on roads (edges) affects traffic network state. To evaluate the impact of network structure and flow distribution on network statistics, we introduce a method of network efficiency for weighted traffic flow networks considering weights on edges in calculation. Furthermore, this paper adopts 6 network structures (including the random network, scale-free network, small-world network, grid network, the road network in Beijing, and the road network in Xiamen) and 3 kinds of flow distributions (Normal distribution, Power-law distribution and exponential distribution) to analyze the impact on network efficiency. For analyzing the impact of network structure and flow distribution on traffic network efficiency, two strategies are adopted: 1.) network structure comparison under a certain flow distribution, 2.) flow distribution comparison under a certain network. The work covered in this paper provides an effective tool for comparing network structure and flow distribution, which can analyze the statistical properties of real traffic networks reasonably.

1. Introduction

Nowadays, methods and tools from network science have been applied to many areas including urban traffic research domain [1]. As for traffic networks, the volume on road is described as weight, road as edge. To evaluate the impact of network structure and flow distribution on network statistics, based on LM [2] method, this paper introduces a method of network efficiency for weighted traffic flow networks considering weights on edges in calculation.

For analyzing the impact of network structure and flow distribution on traffic network efficiency, two types of experiments are carried out with MATLAB. One aims at network structure comparison under a certain flow distribution by network efficiency. Another is at the comparison of traffic flow distributions under a certain network. The experimental results show the impact of network structure and flow distribution on the network, respectively. By comparing the results, the impact of six types of network structures (the road network of Beijing, the road network of Xiamen, ER random network, BA scale-free network, the WS small-world network and the grid network) with three kinds of flow

distributions (Normal distribution, Power-law distribution and Exponential distribution) is analyzed.

2. Network Efficiency Evaluation

To evaluate the reliability and vulnerability of i.e. complex network, Latora and Marchiori [3] proposed the network performance issue by measuring the “global efficiency” in a weighted network compared to that of the simple non-weighted network. Although this method has been applied to a variety of networks [4], it only considers geodesic information so that ignores other important factors such as costs, flows and behaviors.

In recent years, the fields of road network vulnerability and transport reliability have received increasing attention; see, e.g., the special issues edited by Lam [5] and Sumalee and Karauchi [6], and the books edited by Bell and Cassir [7], Iida and Bell [8] and Murray and Grubestic [9]. Several authors have noted that there is a need for methods to assess the consequences of severe, albeit seemingly unlikely, disruptions of the transport system [10-13]. Nagurney and Qiang [14, 15] proposed a method called the NQ method to efficiency measure the transportation networks, which captures cost, flow, and travel behavior information based on the topology. The drawback of this method is that O/D demand vector needs to be given. However, in the actual traffic network, O/D demand is difficult to be accurately determined, so it has its limitations.

These attempts provide a new direction for efficiency evaluation of transportation networks. However, the complexity of the behaviors on large-scale urban transportation networks makes the existing methods cannot be used in applications. In various large-scale cities, such as Beijing, the traffic monitor sensors are widely installed, which provides well data support for studying on transportation network efficiency evaluation and vertex importance ranking. In this paper, we will further study the efficiency evaluation and vertex ranking of urban transportation networks by introducing a data-driven pattern to determine the efficiency networks.

The urban transportation systems are characterized by the following aspects: 1, geographical distributed; 2, network-like; 3, flow dynamics. Therefore, urban traffic networks are directed and weighted. The approaches proposed in this paper are constructed on weighted directed network models of urban transportation systems.

The efficiency evaluation approach should consider the effect of weights on edges. The LM method provides a way of efficiency evaluation for abstract network, in which d_{ij} represents the shortest path length between node i and node j . Furthermore, adding the weights of edges into the LM, we introduce the Extended LM (ELM) method as follows:

$$E' = \frac{1}{n(n-1)} \sum_{i \neq j \in G} \frac{1}{\omega_{ij}} \quad (1)$$

Where n is the number of nodes in the network G and ω_{ij} denotes the weight value on the minimum accumulated weight path(s) from i to j . The specific definition of ω relies on the aim of model constructors. In this paper, we define networks with traffic volume (v) as edge weights. The above equation is presented in the following:

$$E'' = \frac{1}{n(n-1)} \sum_{i \neq j \in G} \frac{1}{v_{ij}} \quad (2)$$

Where v_{ij} is the minimum accumulated weight from i to j .

3. Network Structures and Flow Distributions

3.1. Network Structures

In this paper, we generate 4 directed networks: the random network (by ER model), the scale-free network (BA model), the small-world network (WS model), and the grid network (1). As example, each network has 100 nodes and more detail parameters are explained in following.

The ER random network is generated with 100 nodes and $p = 0.04$ (the connect probability). The in-degree and out-degrees are both following the normal distribution, as shown in Figures 1 and 2.

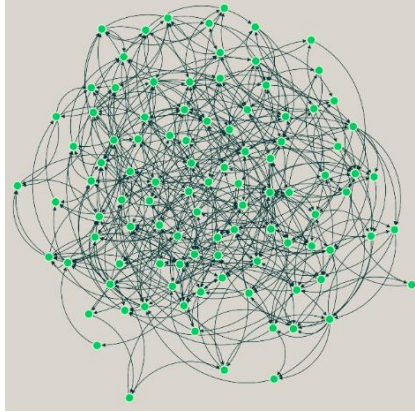


Figure 1: The random network structure (ER Model)

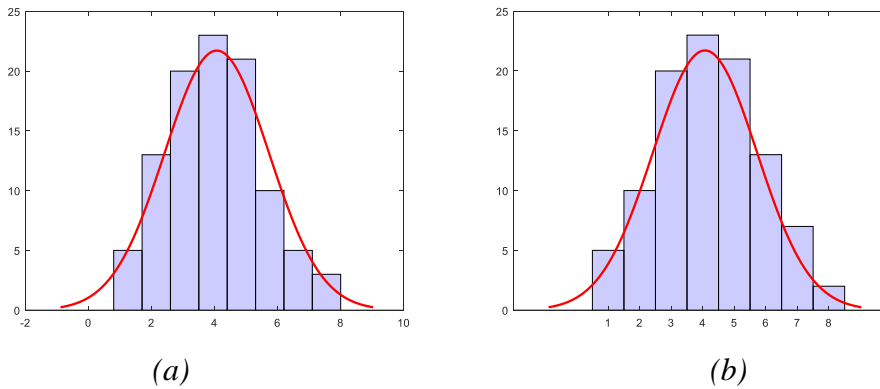


Figure 2: The in-degree (a) and out-degree (b) distribution of the ER random network

The BA scale-free network is generated with 100 nodes and $\alpha = 2.5$ (the scaling exponent). The in-degree and out-degrees are both following the power-law distribution, as shown in Figures 3-6.

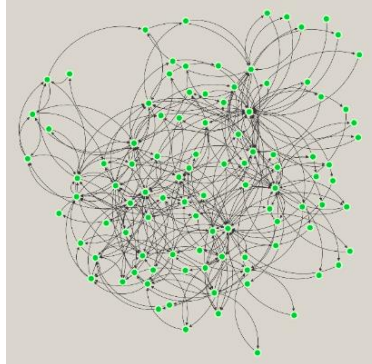


Figure 3: The scale-free network structure (BA Model)

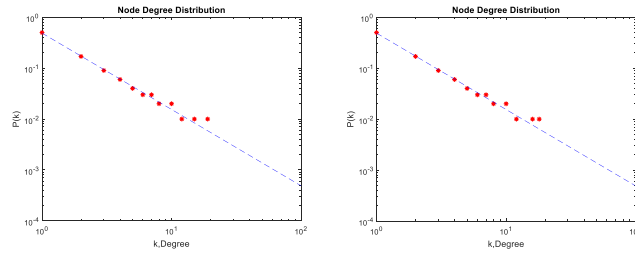


Figure 4: In-degree (left) and Out-degree (right) distribution of the BA scale-free network

The WS small-world is with 100 nodes, $k = 2$ (the mean degree) and $\beta = 0.15$ (the rewiring probability).

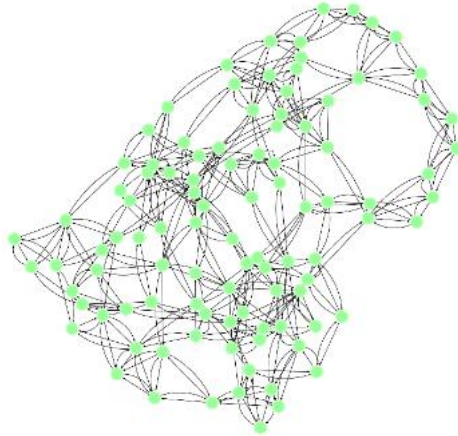


Figure 5: The small-world network structure (WS Model)

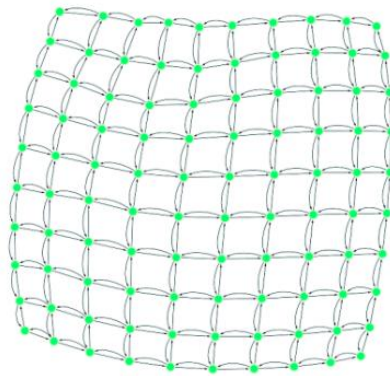


Figure 6: The grid network structure

The grid network has also 100 nodes and 360 edges, whose structure is shown above.

3.2. Flow Distributions

This paper adopts three kinds of flow distributions for analysis: the normal distribution, power law distribution, and the exponential distribution. By using MATLAB, we generate the flow data of one whole network according to the vehicle amount (from 100k to 5000k) and edge amount. Then edges are assigned to the generated volume values by their betweenness: one edge with higher betweenness has a bigger volume value, as shown in Figure 7.

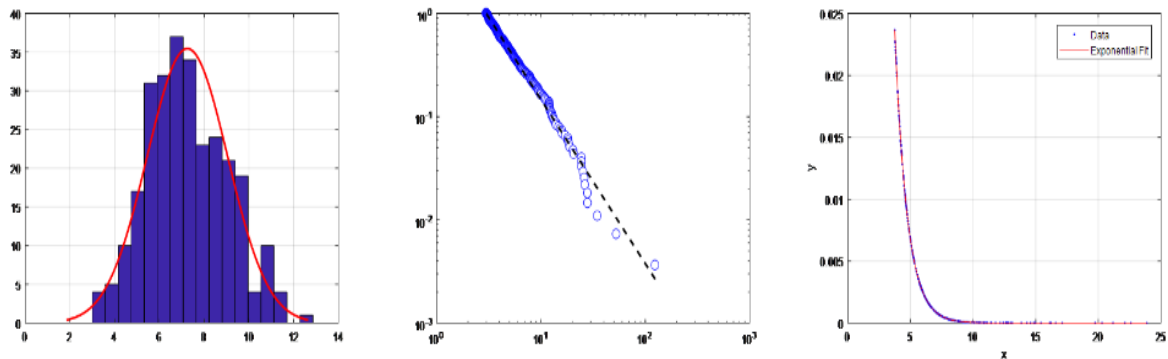


Figure 7: The Normal Distribution (left), the Power Law Distribution (middle), and the Exponential Distribution (right) (100k vehicle number, 400 edges)

Finally, 4 class weighted directed networks following 3 kinds distributions with flow amount from 100k to 5000k are formed.

3.3. Road Traffic Networks of Beijing and Xiamen

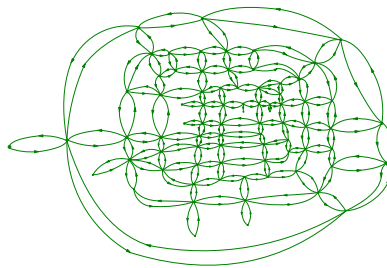


Figure 8: Digraph of the Beijing road network

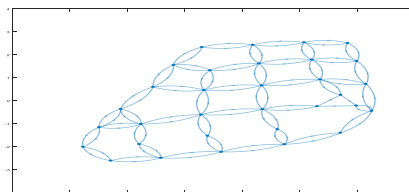


Figure 9: Digraph of the Xiamen arterial road network

As its structure shown above, Figures 8, and 9, Beijing road network is with 167 nodes and 580 edges. Xiamen road network whose structure is shown above has 32 nodes and 93 edges. Based on the Beijing road network structure and the Xiamen road network structure, we generate the flow data following 3 kinds of distributions in the same way.

The 4 generated network structures and 2 real road network structures with the generated flow data with 3 kinds of distributions provide the basis for the further experiments and analysis in this paper.

4. Experimental Results and Analysis

For each weighted directed network, we calculate its network efficiency value. Then the impact analysis from two perspectives is introduced.

4.1. Impact Analysis of Flow Distributions

As shown in Figure 10, the Beijing road network has the highest efficiency under Power-law traffic flow distribution. Under exponential distribution and normal distribution, the network efficiency is very close.

In Figure 11, the Xiamen road network has the highest efficiency under Power-law traffic flow distribution. Under exponential distribution and normal distribution, the network efficiency is very close. Clearly, the efficiency under Power-law distribution flow decreases faster. Consequently, the efficiency is very close when the amount increases beyond 1000 k (veh). It's because the capability of Xiamen road network is smaller than Beijing road network.

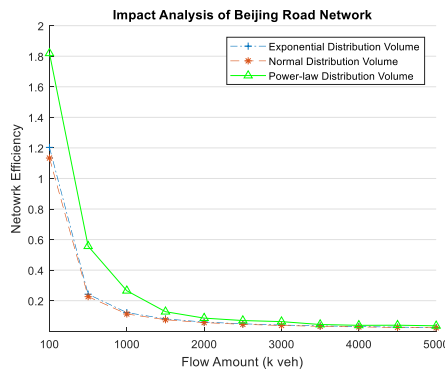


Figure 10: Analysis of Beijing road network under different flow distributions

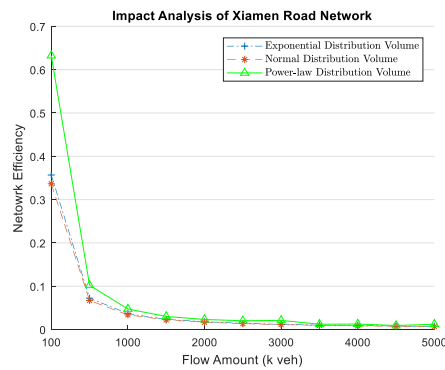


Figure 11: Analysis of Xiamen road network under different flow distributions

The same analysis leads to the conclusion:

The Grid network has the highest efficiency under Power-law traffic flow distribution. Under exponential distribution and normal distribution, the network efficiency is very close. Clearly, the efficiency under normal distribution flow is lowest. The BA scale-free network has the highest efficiency under Power-law traffic flow distribution. In general, the efficiency under three traffic flow distributions has a minor difference with the flow number increasing. The ER random network has the highest efficiency under Power-law traffic flow distribution. In general, the efficiency under three traffic flow distributions has a minor difference with the flow number increasing. The WS Small-world network has the highest efficiency under Power-law traffic flow distribution.

In conclusion, networks with Power-law traffic flow distribution have the highest efficiency. Beijing road network, Xiamen road network, the ER network, and the WS network have minor difference between the efficiency under normal distribution flow and exponential distribution flow. Especially, the BA network does not show significant difference under three kinds of flow distributions, which means the BA network is not susceptible to flow distribution.

4.2. Impact Analysis of Network Structures

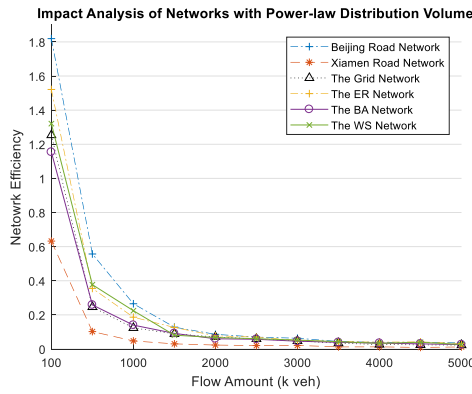


Figure 12: Analysis of network structure with Power-law distribution flow

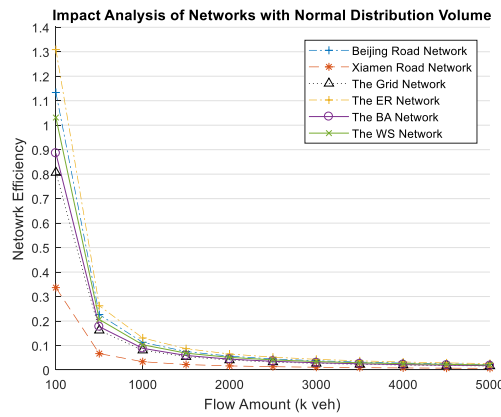


Figure 13: Analysis of network structure with normal distribution flow

As shown in Figure 12 and 13, with Power-law flow distribution, Beijing road network and the ER network have higher efficiency at low flow amount. Beijing road network decreases faster as the flow amount increasing. Xiamen road network has the smallest efficiency. It is worth to note that the grid network has higher efficiency than the BA network with Power-law flow distribution.

In the above figure, under normal flow distribution, the ER network has the highest efficiency. Xiamen road network has the smallest efficiency. Conversely, the BA network has higher efficiency than the grid network. And the efficiency of the WS network is close to Beijing road network's.

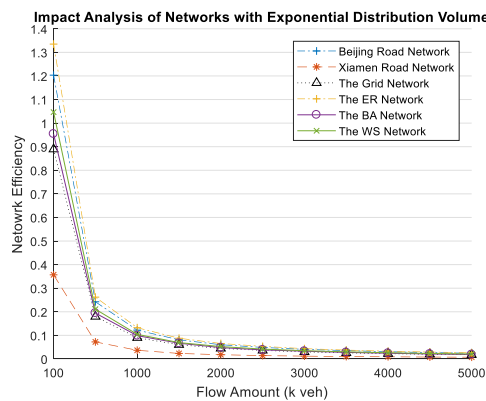


Figure 14: Analysis of network structure with exponential distribution flow

Figure 14 shows that, under exponential distribution flow, the ER network has the highest

efficiency, Xiamen road network is lowest. Similarly, the BA network has higher efficiency than the Grid network. And the efficiency of the WS network is close to Beijing road network's.

In summary, Beijing road network has the highest efficiency in the situation that traffic flow follows a Power-law distribution. The ER network shows the highest efficiency with normal flow distribution and exponential flow distribution. In all situations, Xiamen road network has the smallest efficiency because its smallest network scales causes its little network capacity. Noticeably, the BA network has smaller efficiency than the grid network with normal flow distribution and exponential flow distribution. Oppositely, the BA network is higher with Power-law flow distribution. Clearly, the efficiency of all networks sinks slightly as the flow number increases over 1000k.

5. Conclusions

As observed in traffic flow networks, traffic flow on roads affects traffic network state. That is to say, at least in traffic networks, weights are important in network evaluation. For a certain network, different flow distributions may cause significant different features. To evaluate the impact of network structure and flow distribution on network statistics, based on LM and NQ methods, we introduce a method of network efficiency for weighted traffic flow networks considering weights on edges in calculation.

Furthermore, this paper adopts 6 network structures and 3 kinds of flow distributions. By comparing the network efficiency values, the experimental results are shown in two strategies: the difference among networks under a certain flow distribution and the difference among distributions under a certain network. The results show that networks with Power-law traffic flow distribution have higher efficiency than other distributions. Especially, the BA network does not show significant difference under three kinds of flow distributions, which means the BA network is not susceptible to flow distribution. Beijing road network has the highest efficiency in the situation that traffic flow follows a Power-law distribution. The ER network shows highest efficiency with Normal flow distribution and Exponential flow distribution. Clearly, the efficiency of all networks sinks slightly as the flow amount increasing over 1000k.

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