

Layout Method of Customized Furniture Rectangular Parts Based on Ant Colony Algorithm

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Abstract: In this study, the layout of customized furniture parts was taken as the object. According to the actual situation of the layout of customized furniture parts, the layout method of customized furniture rectangular parts based on ant colony algorithm was proposed. According to the production characteristics of custom furniture parts, this paper describes the construction of rectangular layout path of custom furniture parts and the updating rules of information retrieval, designs the appropriate ant colony algorithm flow, determines the value of ant colony algorithm parameters through the analysis of ant colony algorithm parameters, and completes the solution of custom furniture parts order layout scheme. Experimental results show that the raw material utilization rate of the ant colony algorithm optimized in this study is significantly higher than that of the traditional empirical algorithm under the conditions of different order sample sizes.

1. Introduction

At present, the customized furniture market has gradually become the mainstream mode of manufacturing development with its advantages of high efficiency and low cost. However, due to the diversity of product structure and appearance of customized furniture, it brings certain difficulties to the production of enterprises[1]. How to meet the personalized needs of customers while improving the economic benefits of enterprises, shorten the delivery time of orders, and efficiently and quickly respond to customer needs is an urgent problem for customized furniture enterprises. Customized furniture rectangular parts layout refers to the arrangement of several parts with different sizes and specifications on a certain specification of raw material board village according to certain rules, so as to achieve certain optimization goals on the premise of meeting the requirements of order matching and cutting[2]. Ant colony algorithm is a swarm intelligence search algorithm, which has strong robustness and global search ability. Through the positive feedback mechanism, the search results continue to converge and gradually reach the optimal solution. By simulating the behavior of ant colony looking for food in nature, according to the real information transmission and path selection behavior between ant colonies, some functional behavior is finally realized through the transformation of population behavior. In this paper, ant colony algorithm is applied to the layout of rectangular parts of customized furniture. The research objective is to maximize the use of raw materials and avoid waste of raw materials by combining the production characteristics of customized furniture [3].

2. Layout Method of Customized Furniture Rectangular Parts

2.1 Mathematical Model of Furniture Rectangular Parts Layout

Furniture rectangular parts layout refers to the arrangement of several parts with different sizes and specifications on the raw materials of certain specifications according to certain rules, so as to achieve certain optimization goals on the premise of meeting the requirements of order matching and cutting[4]. The specific description of the problem in this study is as follows. The known raw material sheet length is E , width is R , There are l kinds of parts to be cut in a batch, The width of each part is V and the length is P_l , The quantity of each part is n_i ($i = 1, 2, \dots, k$). Finally, the optimized cutting plan of a batch order is obtained, which is composed of t raw material plates. A total of m parts are discharged in the k raw material board, the rectangle where the part is located is named a_{ij} , and the position of each part on the raw material board is represented by the coordinate (x, y) of the lower left corner of the rectangle corresponding to the part[5]. Parts with texture requirements shall be arranged as required and shall not be rotated; Parts without texture requirements can be rotated and placed horizontally or vertically. Parts can be cut horizontally or vertically. Different cutting methods make the remaining area of raw materials different, which affects the layout results[6,7]. The constraints of furniture rectangular parts layout are expressed by mathematical model as follows:

$$\begin{cases} E_l V + [A_l R_l + l(1-tR_l)]^{n_i} > 1 \\ R_l V + [A_l(1-tR_l) + lP_l]^{n_i} \leq 1 \end{cases} \quad (1)$$

$$\begin{cases} \delta = xK_m + a_{ij}E_l + [A_l R_l + l(1-tR_l)]^{n_i}, 1 \leq m \leq l \\ \sigma = yK_m + a_{ij}R_l + [A_l(1-tR_l) + lP_l]^{n_i}, 1 \leq m \leq k \end{cases} \quad (2)$$

Based on the above algorithm, the objective function is:

$$F = \max \left(\frac{\sum_1^k \sum_{i=1}^{n_i} \delta E_l + [A_l R_l + l(1-tR_l)]^{n_i}}{\sum_1^N \sigma R_l + [A_l(1-tR_l) + lP_l]^{n_i}} \right) \quad (3)$$

The formula is the objective function of the plate layout problem, which means to maximize the utilization of raw materials in batch layout[8]. In order to maximize the utilization of materials and speed up the layout, the following constraints are formulated when studying optimal layout: the shapes of two parts do not overlap each other; Parts must be placed in materials[9]. The arrangement of parts in materials shall be carried out according to the principle of leftmost and lowest; For parts, try to layout according to the principle of large to small area, so as to ensure that there are no small parts in the area where large parts can be placed.

2.2 Ant Colony Optimization Algorithm for Layout

The layout idea of rectangular parts based on ant colony algorithm is as follows: take the

rectangular parts as the nodes through which ants search the path, through the search of ants, through the probability selection between nodes, and the continuous updating of pheromones between nodes, use the algorithm to find out the sequence and mode of rectangular parts discharge, and finally form the layout sequence of rectangular parts with the maximum utilization of raw materials. Record the plate set of the batch as $X = \{x_1, x_2, \dots, x_n\}$, Z_i It refers to the i th plate[10]. There are s plates in this batch, Let the number of ants in the system be m , and each ant selects the plate in turn according to the probability. The serial number of the plate selected by each ant is taken as a solution, and the solution formed by the k th ant can be expressed as $\{A_{k5}, A_{k10}, \dots, A_{kj}, \dots, A_{kn}\}$, $f(k_n)$ means that in the current iteration times, the k -th ant selects the plate numbered n for the first time, and so on.

$$P_{ij}^k(t) = \begin{cases} \frac{F - Z_i \cdot [A_{kj} - A_{kn}]^2}{X \sum f(k_n) - s} \\ 0 \end{cases} \quad (4)$$

Ant α transfers from path i to path j at t time according to the pseudo-random proportional rule $\eta_{ij}(t)$ [11]. Based on this, the ant colony search rule expression is:

$$\varpi = \begin{cases} \arg \max_{j \in lab_{u_k}} \left\{ [P_{ij}^k(t) - \alpha]^\alpha [\eta_{ij}(t) - q]^2 \right\} \\ 0 \end{cases} \quad (5)$$

Among them, q is a random number in $[0,1]$, a_n is the parameter $0 \leq a \leq 1$ set at the beginning of the algorithm, and other symbols are the same as those defined above[12-14]. λ is determined according to the state transition probability rule of formula. It can be seen that whenever the ant wants to choose the next city to be transferred, it is in the interval $[0,1]$ generate a random number in, and then compare the size of this random number with q to determine the ant's transfer criteria. Based on this, the pheromone on the path of all ants is updated, but only the pheromone on the path of the optimal ant in each iteration is updated[15]. The update rule is as follows:

$$\tau_{ij}(t+n) = (1-q) \cdot a_n + \varpi \cdot \lambda \quad (6)$$

This global update strategy enhances the pheromone on the path of the optimal ant, while the pheromone on other paths will gradually weaken due to volatilization, which increases the pheromone difference between the optimal path and other paths, so that the search path of the ant can concentrate near the optimal path as soon as possible[16]. After t_i moments, all ants have completed a traversal, and their memory list A_{ij} will be full this time, at this time, it should be cleared. Put the current ant's city into σ_x for the next traversal. At this time, calculate the path L that each ant has traveled, and save the shortest path L_s . With the passage of time, the pheromone left before gradually decays[17]. After the ant completes a cycle, the amount of information on each path should be obtained according to the formula:

$$\varepsilon_{\max} = \sigma_x \sum_{i=1}^n A_{ij} \tau_{ij}(t+1) + t_i + L - L_s \quad (7)$$

$$\varepsilon_{\min} = \sigma_x \sum_{k=1}^m L \Delta k \tau_{ij}(t+1) - L s \mu_x * V(t, t+1) \quad (8)$$

Where, $V(t, t+1)$ indicates that the k th ant is at the moment $(t, t+1)$ is the amount of pheromone left on the path depends on the performance of ants. The shorter the path, the more pheromones will be released; Δ^k Represents the increment of pheromone of path i in this cycle; μ_x Is the attenuation coefficient of pheromone trajectory[18]. Relying on computer technology, the solution of nesting problem is from theoretical research to practical application. The depth of research on packing problem ranges from one-dimensional linear packing problem to two-dimensional or even three-dimensional packing problem. The scope of research extends from the basic mechanical manufacturing industry to other industries, from basic problems to variant problems that meet various practical production constraints[19]. The cutting and blanking problems are summarized and classified from five perspectives, as shown in figure 1:

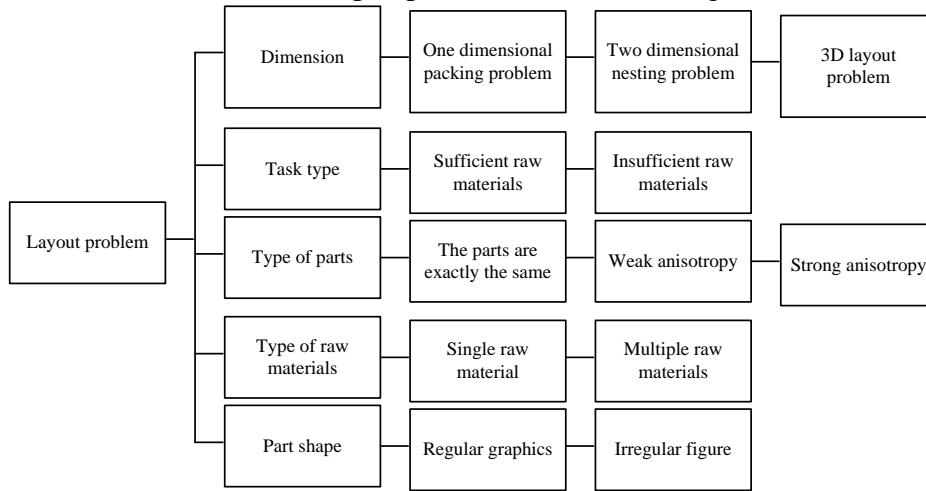


Fig.1 Classification of Layout Problems

In the dimension of raw material type, a single raw material is divided into two categories: all dimensions are fixed and there are variable dimensions. Multi raw materials are divided into three categories: single specification, multi specification weak anisotropy and multi specification strong anisotropy[20]. According to the classification method of watcher2, the layout optimization problem studied in this paper is a two-dimensional rectangular layout problem with sufficient raw materials, and it is also a strongly heterogeneous layout problem with regular part shape. According to the types of raw materials, the two-dimensional rectangular packing problem is divided into two basic problems: rectangular strip packing and packing packing packing. The former belongs to the variable dimension packing problem in a single raw material, and the latter belongs to the multi-dimensional packing problem with immutable dimensions. There are many similarities between the two kinds of problems, starting from the rectangular parts layout optimization problem, and then studying the rectangular packing layout optimization problem. The research on the layout problem is mainly divided into two directions: first, starting from the actual needs of production, analyze the problem, especially explore new layout problems, and enrich the types of cutting and cutting problems: for some basic problems, constantly explore and study new solving algorithms, in order to obtain a layout plan with high utilization. The focus of the research is to explore and study new solving algorithms to improve the utilization rate. According to the classification of the problem solving algorithms, the research on rectangular part nesting problem is mainly divided into

three categories: deterministic algorithm, ant colony algorithm and ant colony algorithm. Ant colony algorithm is to apply a strategy to place parts on the plate and determine the position of parts, which is also called positioning algorithm. Based on ant colony algorithm, meta heuristic algorithm optimizes the sequence of parts to further improve the utilization of materials. The solution framework of rectangular packing problem can be summarized into two modules. The overall solution framework of the algorithm is shown in Figure 2.

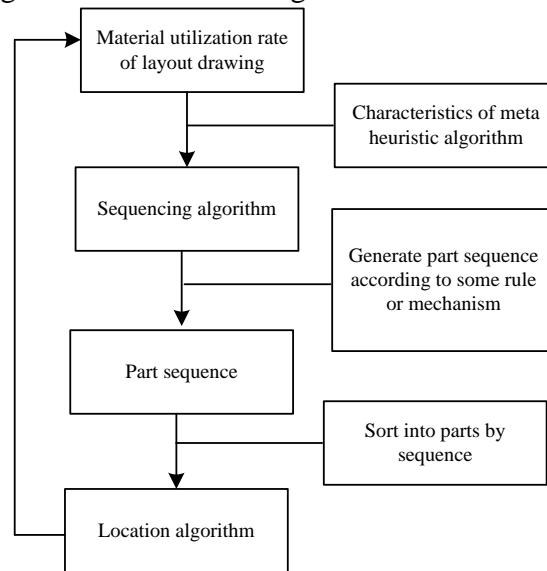


Fig.2 Rectangular Part Layout Problem Solving Framework

The first core module is to arrange a group of rectangular parts into the rectangular plate in turn according to some method or rule, determine the position of each part, so as to form a layout map, and calculate the utilization rate. This module is called rectangular part positioning algorithm or rectangular part layout algorithm, such as BL and BLF algorithm, BF algorithm of the lowest horizontal line, residual rectangle algorithm, etc. The second module is to determine the sequence of this group of rectangular parts according to some method and rule. This module is called rectangular parts layout and sequencing algorithm. For example, the parts are sorted according to the area from large to small, and then they are discharged into the plate according to the positioning algorithm in order.

2.3 Optimal Layout Scheme of Customized Furniture Rectangular Parts

For ant colony algorithm, the early research is a simple sequencing algorithm combined with a simple positioning algorithm. With the deepening of the research, a search mechanism is added to the positioning algorithm to select the appropriate parts to change the order of discharge. At the same time, the sequencing algorithm also defines a variety of rules, such as sorting according to the width, height and perimeter of parts. In the ant colony algorithm, it further enhances the ability to search the type of rectangular parts in the sequence, and generates various parts sequences through some mechanism, rather than interacting with the positioning algorithm when generating sequences according to a single rule, so as to guide the direction with high sequencing utilization to generate sequences. Although the solution algorithm of rectangular packing problem is divided into two modules: sequencing algorithm and positioning algorithm, the framework of the whole algorithm is mainly ant colony algorithm, and the positioning algorithm is only an integral part of the whole algorithm. Set the length of the plate used for layout as l , width is h , The number of plates is enough to arrange all the rectangular pieces to be arranged. All the rectangular pieces to be arranged have a

common kind is d_i , type I rectangular pieces, the number of rectangular pieces is g , then the total number of rectangular pieces to be arranged is:

$$b_n = g \sum_{i=1}^g d_i - lh(\varepsilon_{\max} - \varepsilon_{\min}) \quad (9)$$

Based on the above algorithm, the number of plates used in nesting is minimized to improve the utilization of materials. The basic constraints of nesting are: there can be no overlapping areas between rectangular parts, and rectangular parts can not have parts outside the plate, which meets the process requirements. The layout rule is: each rectangular piece can be discharged horizontally or vertically. The layout method is: start from the bottom left corner of the plate and end at the top right corner of the plate. The coordinates of the bottom left corner of a plate are (0,0). We know that the position of a rectangular piece on the plate can be completely determined by the coordinates of the upper left corner and the lower right corner of the rectangular piece. set up $(x_l, y_l), (x_r, y_r)$ is the coordinates of the upper left corner and the lower right corner on the plate of the i -th rectangular piece. The nesting process is to determine the upper left corner of each rectangular part on the plate according to certain optimization rules l^+ . The coordinates of the lower right corner r^- marked as (x_{r^-}, y_{r^-}) . In general rectangular packing problem, rectangular pieces can be arranged horizontally or vertically. Therefore, there are two relationships between the two coordinates of a rectangular piece:

$$\begin{cases} w_{ij} = x_{l^+} + b_n(x_l - x_r) \\ m_{ij} = y_{r^-} - b_n(y_l - y_r) \end{cases} \quad (10)$$

Rectangular strip layout problem is a basic problem in rectangular part layout problem, and widely exists in actual production. Its main feature is the width of rectangular plate F_L is fixed, But the height F_L Unlimited, that is, the variable dimension layout problem in a single raw material, which is specifically described as: a group of rectangular pieces with a number of N and a known length and width P_m discharge to a width of F_H , on rectangular plates with unlimited height, making the utilization rate of plates the highest, the material utilization rate is defined as:

$$U = P_m \sum_{i=1}^n (w_{ij} \times m_{ij}) / (F_H \times F_L) \quad (11)$$

Rectangular part layout problem refers to the optimal layout of several given rectangular parts on the plate, which requires that all parts must be arranged in the plate without overlap between parts, and meet certain process requirements, so as to maximize the utilization rate of the plate and achieve the purpose of saving raw materials. From the point of view of mathematics, the layout problem of rectangular parts is an integer programming problem. To facilitate the description, the following variables are defined: Given n parts to be arranged, $S = \{1, 2, \dots, N\}$ is a group of numbers of rectangular parts. The so-called sequencing rule is to determine the sequence of rectangular parts' discharge, which will ultimately affect the utilization rate of materials. A better discharge sequence can effectively improve the utilization rate of plates and save raw materials. The sequencing rules can be determined according to the length width ratio, area and other characteristics of rectangular parts. Then the optimal layout of rectangular parts can be obtained as follows:

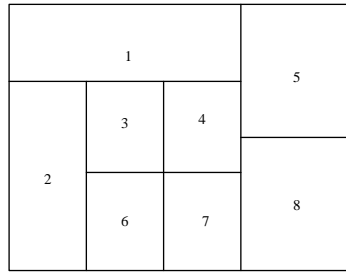


Fig.3 Optimal Layout of Rectangular Parts

Suppose a given length is L , and the width is W target parts, and the plates for splicing the target parts are n , record as $A(1,2,3,\dots,m)$, the final layout plan has a total of P layers, and on each layer there are $y(1,2,3,\dots,n)$ rectangular plates are arranged, and the width of rectangular plates arranged on each layer is the same, and the width of each layer is $w(1,2,3,\dots,i)$. The rectangular plates arranged on the first layer (the bottom layer) are marked as $l(1,2,3,\dots,y)$, By analogy, all rectangular plates arranged on the p -th layer can be represented as $l_p(1,2,3,\dots,y)$. In order to simplify the problem, we do not consider the gap generated during plate splicing, but only consider the splicing cost. According to the above assumptions and definitions, we can get the mathematical model of rectangular layered nesting problem as follows:

$$\min f(A_i) = \left[(p-n)U + \sum_{j=1}^p (L-W) \right] - Ay + wl_p \quad (12)$$

The overall structure of the rectangular optimized nesting system is shown in the figure, including system management module, user management module, data management module and rectangular optimized nesting module.

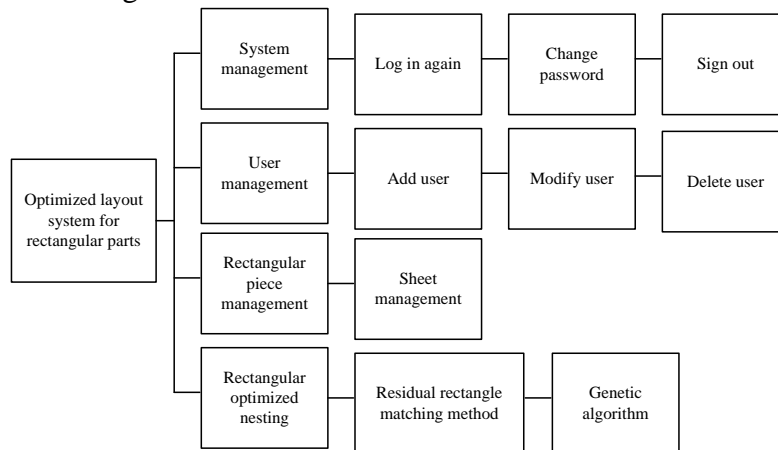


Fig.4 Optimization of Furniture Parts Layout Mode

The management module includes re login, password change and exit functions. User management includes the functions of adding users, modifying users and deleting users. Data management includes rectangular part management and plate management. Rectangular part optimal nesting includes residual rectangle matching method, search algorithm based on the lowest horizontal line and ant colony algorithm. Set rectangular sheet $\tau(i)$ put at the rightmost maximum height of the target part, and then move down and left alternately from this position. First move the

rectangular part down as much as possible, and then move the rectangular part to the left as much as possible until the rectangular part can no longer move down and left (touching the boundary of the target area or other rectangular parts); compare $\tau(i)$ And the width of each layer of rectangular plates arranged on the leftmost side. If the width of a layer t on the leftmost side is the same as that of T , then n discharge at m The right side of the layer; If there is no width of any layer and $\tau(i)$ are similarly, discharge t to the top layer on the left, calculate the sum of the widths of the plates discharged to the left, and record it as μ , compare μ and the width of the target part. The pheromones left on the rectangular pieces are updated globally and locally. Global update is to update only the pheromone on the rectangular part of the global optimal packing sequence. The rules are as follows:

$$\eta(i) = \min f(A_i)(1 - \mu) \cdot \tau(i) + \mu \cdot \Delta\tau(i) \quad (13)$$

If ψ is the pheromone density left on the rectangular piece ρ . ψ is the Volatilization Coefficient of pheromone, Q is a pheromone intensity constant, area is the sum of the areas to be discharged into the rectangular parts, and area represents the rectangular plate area under the height contour of the layout map corresponding to the global optimal layout sequence. Local update can dynamically change the pheromone concentration left on each rectangular piece, and better provide search information for other ants. The update rule is as follows:

$$\tau(i) = (1 - \psi\rho)_{\text{Area}} \cdot [Q\tau(i) + \psi \cdot \eta(i)]_{\text{Areal}} \quad (14)$$

For a given rectangular piece emission sequence $\{1,2,3,4\}$, the emission processes of BL algorithm and lower step algorithm are shown in Figure 5 respectively.

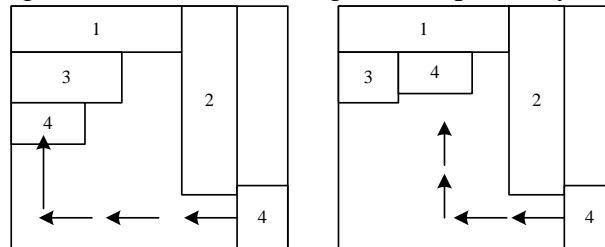


Fig.5 Matrix Discharge Process

It can be seen from the figure that even for the same emission sequence, different emission algorithms get different nesting results. Obviously, the lower step algorithm also meets the ant colony path condition. For the optimal nesting graph shown in the figure, the corresponding sequence obtained by the lower step algorithm is $\{-1,2,3,4, -5, -6, -7,8\}$. In ant colony algorithm, a certain scale of chromosomes constitute the population; First, evaluate the advantages and disadvantages of each chromosome to get its fitness value; Then, the chromosomes are genetically manipulated to produce new chromosomes, and their advantages and disadvantages are evaluated; Finally, the better chromosomes were selected from the old and new generations to form a new population. After several generations of evolution, an optimal chromosome is obtained, which is the optimal or near optimal solution of the problem.

3. Analysis of Experimental Results

In order to verify the practicability of the developed automatic nesting software system for rectangular pieces of panel furniture, a certain example test is carried out, and the actual nesting order of the project company is taken as an example to verify it. This example is a cutting order of a

common floor cabinet. The specification of the plates used is 400*500*160. A total of 500 rectangular plates need to be cut. At the same time, there are texture requirements, that is, rectangular plates cannot be rotated. Three batches are randomly selected as a, B and C for sample layout scheme. The total area of batch a is 41.86m^2 , A total of 100 plates; The total area of batch B is 44.52m^2 , 150 plates in total; The total area of batch C is 48.37m^2 , There are 200 pieces of plates in total, and the part information in each batch is imported into Matlab software. After multiple iterative searches of ant colony, the layout optimization scheme for three batch orders is finally completed, and some batch a layout schemes are selected, as shown in the figure 6:

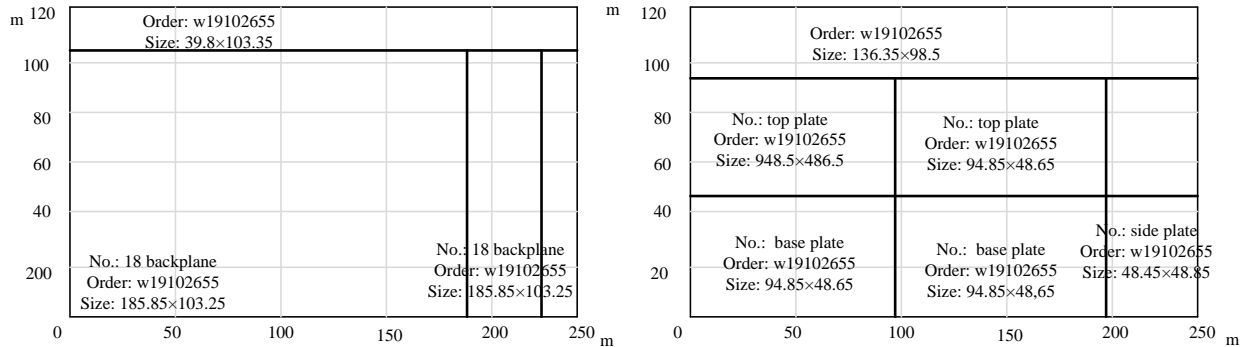


Fig.6 Example of Partial Batch a Layout Scheme

The feasibility and effectiveness of the algorithm are verified by experiments. The selected sheet size is 400*500. In order to make a reasonable evaluation of the algorithm, this paper uses the genetic algorithm and this algorithm on the basis of genetic algorithm to search the lowest horizontal line rotation. Each example is calculated 20 times, and the average value is taken for calculation. The results are as follows:

Table 1 Calculation results of parts layout algorithm

	Genetic ant colony algorithm	Maximum material utilization	Lowest value of material utilization
genetic algorithm	0.8098	0.8477	0.7605
Ant colony	0.9628	0.9846	0.9119

It can be seen from the calculation results that the genetic ant colony algorithm designed in this paper based on the lowest horizontal line rotation search method has a good effect in the rectangular part layout problem. Through the multiple iterative search of ant colony, the raw material utilization rate of the layout scheme finally generated by three batches can be calculated, and compared with the raw material utilization rate of the traditional empirical algorithm, as shown in the table 2 below:

Table 2 Layout Results of Different Examples

Numerical example	Number of plates / piece	the measure of area /m2	Empirical algorithm raw material utilization rate /%	This paper calculates the utilization rate of raw materials /%
1	100	41.86	80.73	90.99
2	150	44.52	81.29	93.69
3	200	48.37	81.45	96.89

When nesting for different examples, the traditional empirical algorithm of clustering and batch sampling of different order data samples and the optimized nesting algorithm in this study are compared. The results show that under the conditions of different order sample sizes, the optimized nesting algorithm in this study significantly improves the raw material utilization rate compared

with the traditional empirical algorithm. Through multiple iterations, the remaining size of the raw material board can be maximized, which improves the utilization rate of raw materials and the economic benefits of the enterprise.

4. Conclusion

Ant colony algorithm is an intelligent algorithm combining distributed computing and positive feedback mechanism, which has strong global search ability. Aiming at maximizing the utilization of raw materials in a batch layout, this study builds a model for the layout problem of customized furniture rectangular parts, combines the structure of ant colony algorithm with the actual information of customized furniture layout problem, designs the pheromone update mechanism and heuristic information definition in ant colony algorithm, and applies it to solve the batch layout problem. The parameter values of ant colony algorithm are determined through many experiments. Finally, an example shows that the algorithm can effectively improve the utilization rate of raw materials in batches, and is effective in solving the layout problem in customized furniture production.

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