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Security Control of Cyber-Physical Systems with Cyber Attacks and Mixed Time-Delays

Xuan Jia[1,a,*]

[1]*College of Electrical and Information Engineering, Lanzhou University of Technology, Lanzhou, 730050, China*
[a]*1015887185@qq.com*
**Corresponding author*

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Abstract: This article is concerned with the security control problem for discrete-time cyber-physical systems with both distributed state delay and time-varying input delay under false data injection attacks. Firstly, by introducing mixed delay and network attack, a polyhedron model is proposed to describe the nonlinear input caused by cyber attacks. Secondly, combined with a piecewise Lyapunov functional and some summation inequalities, sufficient conditions for the exponential stability of the closed-loop system are derived by using linear matrix inequality approach, and the design process of the security controller is given. Finally, an illustrative example is given to demonstrate the effectiveness of the proposed method.

1. Introduction

Cyber-physical systems (CPS) are the integration of physical processes, pervasive computing, efficient communication and network control to realize the effective control of physical processes [1]. In recent years, due to the progress of computers, network communication and related hardware technologies, CPS has developed rapidly and been widely used in the fields of transportation networks, smart grids, medical care and smart home [2]. However, due to the complexity of the system and the openness of communication protocols, CPS is vulnerable to malicious cyber attacks. Where, a false data injection (FDI) attack destroys the authenticity and integrity of information by tampering with the contents of data packets, which can affect the analysis and decision-making of the remote control centre. Therefore, it is a kind of network attack with a high threat to system security, which has attracted the extensive attention of scholars and achieved a series of meaningful research results [3].

Up to now, scholars have mainly studied FDI attacks from three aspects: attack detection, security control and state estimation [4]-[10]. Aiming at the state estimation problem of the system under cyber attacks, Liu et al. studied the state estimation problem of the real-time system under FDI attack by using the least square algorithm [4]. A detection method based on Kalman filter technology to detect the attacked network nodes is adopted in [5] and [6]. The above research realizes the detection of FDI attacks, but with the improvement of CPS complexity, simple attack detection is not enough to meet the needs of system security control. In order to maintain the consistency of network cooperation systems attacked by FDI, Gusrialdi et al. proposed a design method of elastic cooperation control based on the concept of competitive interaction [7]. In addition, in order to satisfy the H-infinity control performance of the system, Wang et al. used the robust model predictive control method to realize the stabilization control for the time-varying system with FDI attacks [8]. Liu et al. studied the H-infinity load frequency control of the power system with FDI attacks [9]. However, most of the above literature only considers the design of control strategy for the systems under FDI attacks but ignores the decline of system performance caused by the inherent inducement factors of the network. For example, in many practical systems, due to network congestion, CPS inevitably has time delay in the process of information interaction, which often leads to the decline or even instability of CPS performance [11][12].

In response to the aforementioned discussion, this paper studies the security control of a class of CPS under cyber attacks and mixed time delays. 1) A piecewise Lyapunov function is used to eliminate the traditional time-delay constraint for FDI attacks input time-delay systems. 2) By introducing a distributed delay term, a multipatch model is proposed to handle the non-linear input caused by FDI attacks. 3) Sufficient conditions for asymptotic stability of CPS with cyber attack, state and input delay are established, and a security controller is designed.

The rest of this article is organized as follows. Section 2 is the problem description and preliminaries. The sufficient conditions for the exponential stability of the closed-loop system are derived, and the safety controller is designed in Section 3. Section 4 provides a numerical example to illustrate the effectiveness of our results. Finally, Section 5 concludes the paper.

Notations: Denotes that P is a real, symmetric, and positive definite matrix. R^n is the n-dimensional Euclidean space. Meanwhile, denoting $\|\bullet\|$ as the Euclidean norm for vectors. $K_{(l)}$ is the *l-th* row of the matrix K. I is the identity matrix with compatible dimension.

2. Problem Formulation

Consider the following discrete-time cyber-physical systems with distributed state delays and fast-varying input delays under cyber attacks.

$$\begin{cases} x(k+1) = Ax(k) + A_d \sum_{i=1}^{+\infty} \mu_i x(k-i) + Bf\left(u(k-\tau_k)\right) \\ x(k) = \phi(k), k \in (-\infty, 0] \end{cases} \tag{1}$$

where $x(k) \in R^n$ represents the system state; $u(k) \in R^m$ represents control input; $\phi(k) \in R^n$ is the initial condition. A, A_d is a known constant matrix with appropriate digits. $\sum_{i=1}^{+\infty} \mu_i x(k-i)$ represents distributed state delay item, τ_k indicates that the fast-varying input delay is satisfied $0 \le \tau_k \le \tau$ (τ is a positive integer). $f\left(u(k-\tau_k)\right)$ represents the nonlinear function caused by FDI attacks.

Assumption 1. For a constant matrix U, FDI attack signals $\tilde{f}\left(u(k-\tau_k)\right)$ and satisfy the following conditions:

$$\left\| f\left(u(k-\tau_k)\right) \right\|_2 \le \left\| Uu(k-\tau_k) \right\|_2$$

where U is a matrix given with appropriate dimensions, representing the upper bound of FDI attacks.

Based on the above analysis, the following state feedback controller is constructed:

$$u(k) = Kx(k), k \ge 0 \tag{2}$$

where K represents the controller gain matrices.

For system (1), we make the following assumptions

Assumption 2. For the given scalars $\mu_i > 0 (i = 1, 2, ...)$, there is the positive scalar $0 \le \lambda \le 1$ such that the following inequality holds

$$\sum_{i=1}^{+\infty} \mu_i \lambda^{-i} < \sum_{j=1}^{+\infty} \sum_{i=1}^{j} \mu_i \lambda^{-i} < \sum_{l=1}^{+\infty} \sum_{j=1}^{l} \sum_{i=1}^{j} \mu_i \lambda^{-i} < +\infty$$

In addition, the delay dependent sector condition is introduced to deal with the actuator end nonlinearity caused by cyber attacks [13,14].

$$f^T\left(u(k-\tau_k)\right) H\left[f\left(u(k-\tau_k)\right) - Kx(k-\tau_k)\right] \le 0 \tag{3}$$

According to condition (3) one has

$$-f^T\left(u(k-\tau_k)\right) H\left[f\left(u(k-\tau_k)\right) - Kx(k-\tau_k)\right] \ge 0 \tag{4}$$

According to condition (2) and (4). Therefore, the system (1) is equivalent to

$$x(k+1) = Ax(k) + BKx(k-\tau_k) + A_d \sum_{i=1}^{+\infty} \mu_i x(k-i) - Bf\left(u(k-\tau_k)\right) \tag{5}$$

Then, we will introduce two lemmas to assist the follow-up work.

Lemma 1 ([15]). Let $v \in R^{\bar{m}}$ be such that $\|v\|_\infty \le 1$, where $\bar{m} = m^{2m-1}$. Let the elements in D_m be labelled as $D_i\left(i \in \left[1, 2^m\right]\right)$, where D_m is a set of $m \times m$ diagonal matrices with diagonal elements being either 1 or 0, and the function f_m be defined as $f_m(0) = 0$ and

$$f_m(i) = \begin{cases} f_m(i-1) + 1, D_i + D_j \ne I_m, \forall j \in [1, i] \\ f_m(j), D_i + D_j = I_m, \exists j \in [1, i] \end{cases}$$

Then, for any $u \in R^m$, there holds $f(u) \in \mathrm{co}\left\{D_i u + \tilde{D}_i^- v : i \in \left[1, 2^m\right]\right\}$, where "co" denotes the convex hull and $\tilde{D}_i^- \in R^{m \times m}$ is defined as $\tilde{D}_i^- = e_{2^{m-1}, f_m} \otimes D_i^-$ with $D_i^- = I - D_i$.

Lemma 2 ([16], [17]). Let $0 < Z \in R^{n \times n}$, $x_i \in R^n$ and the scalars $\mu_i \ge 0 (i, j = 1, 2, \cdots)$ be given. Then, we have

$$\left(\sum_{i=1}^{+\infty} \mu_i x_i\right)^T Z \left(\sum_{i=1}^{+\infty} \mu_i x_i\right) \le \left(\sum_{i=1}^{+\infty} \mu_i \lambda_i^-\right)\left(\sum_{i=1}^{+\infty} \mu_i \lambda_i x_i^T Z x_i\right)$$

$$\left(\sum_{j=1}^{+\infty} \sum_{i=1}^{j} \mu_j x_i\right)^T Z \left(\sum_{j=1}^{+\infty} \sum_{i=1}^{j} \mu_j x_i\right) \le \left(\sum_{j=1}^{+\infty} \sum_{i=1}^{j} \mu_j \lambda_i^-\right)\left(\sum_{j=1}^{+\infty} \sum_{i=1}^{j} \mu_j \lambda_i x_i^T Z x_i\right)$$

Let U, V and W be $\bar{m} \times n$ matrices and denote that

$$\vartheta(k) \triangleq Ux(k) + V\sum_{i=1}^{+\infty} \mu_i x(k-i) + W\sum_{j=1}^{+\infty} \mu_j \sum_{i=k-j}^{k-1} x(i) \tag{6}$$

Assumption 3. The following constraints are assumed:

$$\|v(k)\|_\infty \le 1, k \in [0, +\infty) \tag{7}$$

According to Lemma 1, the nonlinearity $f\left(u(k-\tau_k)\right)$ can be expressed as

$$f\left(u(k-\tau_k)\right) = \sum_{s=1}^{2^m} \varpi_s^k \left[D_s u(k-\tau_k) + D_s^- \vartheta(k)\right] \tag{8}$$

Considering condition (4) and (5), the closed-loop system can be obtained as follows:

$$x(k+1) = \sum_{s=1}^{2^m} \varpi_s^k \left\{\left(A + BD_s^- U\right)x(k) + BKD_s x(k-\tau_k) + \left(A_d + BD_s^- V\right)\sum_{i=1}^{+\infty} \mu_i x(k-i)\right.$$

$$+BD_s^- W\sum_{j=1}^{+\infty}\mu_j\sum_{i=k-j}^{k-1}x(i)\Big\},k\in[0,+\infty) \tag{9}$$

The purpose of this paper is to design the controller (2) to make the closed-loop system (9) exponentially stable.

3. Main Results

In order to facilitate the subsequent proof, the following piecewise augmented Lyapunov functional is proposed.

$$V(k)=V_1(k)+V_2(k),k\in[0,+\infty) \tag{10}$$

where

$$\begin{aligned}V_\alpha(k)=\eta^T(k)P_\alpha\eta(k)+\sum_{i=k-\tau}^{k-1}\lambda_\alpha^{k-i-1}x^T(i)Q_\alpha x(i)\\
+\sum_{j=1}^{+\infty}\mu_j\sum_{i=k-j}^{k-i}\lambda_\alpha^{k-i-1}x^T(i)S_{\alpha1}x(i)\\
+\sum_{l=1}^{+\infty}\mu_l\sum_{j=1}^{l}\sum_{i=k-j}^{k-1}\lambda_\alpha^{k-i-1}x^T(i)S_{\alpha2}x(i)\\
+\tau\sum_{j=-\tau}^{-1}\sum_{i=k+j}^{k-1}\lambda_\alpha^{k-i-1}y^T(i)(R_{\alpha1}+R_{\alpha2})y(i)\\
+\sum_{l=1}^{+\infty}\mu_l\sum_{j=1}^{l}\sum_{i=k-j}^{k-1}\lambda_\alpha^{k-i-1}y^T(i)Z_\alpha y(i),\alpha=1,2\end{aligned}$$

with $P_\alpha>0$, $Q_\alpha>0$, $S_{\alpha1}>0$, $S_{\alpha2}>0$, $R_{\alpha1}>0$, $R_{\alpha2}>0$, $Z_\alpha>0$, $0<\lambda_1\le 1$, $\lambda_2>1$, $y(k)=x(k+1)-x(k)$ and $\eta(k)=\left[x^T(k)\sum_{i=k-\tau}^{k-1}x^T(i)\quad \sum_{j=1}^{+\infty}\mu_j\sum_{i=k-\tau}^{k-1}x^T(i)\right]^T$.

For convenience of subsequent presentation, we define

$$\breve{\tau}_k\triangleq\tau_k+1,\ \hat{\tau}_k\triangleq\tau-\tau_k+1,\ \tilde{\tau}\triangleq\tau+1,$$

$$\varepsilon_0(k)\triangleq\left[\sum_{i=1}^{+\infty}\mu_i x^T(k-i)\sum_{j=1}^{+\infty}\mu_j\sum_{i=k-\tau}^{k-1}x^T(i)y^T(k)\right]^T,$$

$$\varepsilon_1(k)\triangleq\left[x^T(k)x^T(k-1)x^T(k-\tau)1/\breve{\tau}_k\sum_{i=k-\tau_k}^{k}x^T(i)1/\hat{\tau}_k\sum_{i=k-\tau_k}^{k-\tau_k}x^T(i)\varepsilon_0^T(k)\right]^T,$$

$$\varepsilon_2(k)\triangleq\left[x^T(k)x^T(k-\tau_k)x^T(k-\tau)1/\tilde{\tau}\sum_{i=k-\tau}^{k}x^T(i)\varepsilon_0^T(k)u^T(k-\tau_k)\right]^T,$$

$$\varepsilon_3(k)\triangleq\left[x^T(k)x^T(k-\tau)1/\tilde{\tau}\sum_{i=k-\tau}^{k}x^T(i)\varepsilon_0^T(k)\right]^T,$$

$$\Gamma_1\triangleq\begin{bmatrix}I&0&0&0&0&0&0&I\\0&-I&-I&\breve{\tau}_kI&\hat{\tau}_kI&0&0&0\\\kappa I&0&0&0&0&-I&I&0\end{bmatrix},\ \Gamma_2\triangleq\begin{bmatrix}I&0&0&0&0&0&0&0\\-I&-I&0&\breve{\tau}_kI&\hat{\tau}_kI&0&0&0\\0&0&0&0&0&0&I&0\end{bmatrix},\ \Gamma_6\triangleq\begin{bmatrix}I&0&0&0&0&0\\0&-I&\tilde{\tau}I&0&0&0\\0&0&0&0&I&0\end{bmatrix},$$

$$\Gamma_3\triangleq\begin{bmatrix}I&0&0&0&0&0&I&0\\0&0&-I&\tilde{\tau}I&0&0&0&0\\\kappa I&0&0&0&-I&I&0&0\end{bmatrix},\ \Gamma_4\triangleq\begin{bmatrix}I&0&0&0&0&0&0&0\\-I&0&0&\tilde{\tau}I&0&0&0&0\\0&0&0&0&0&I&0&0\end{bmatrix},\ \Gamma_5\triangleq\begin{bmatrix}I&0&0&0&0&I\\0&-I&\tilde{\tau}I&0&0&0\\\kappa I&0&0&-I&I&0\end{bmatrix},$$

$$\Phi_1 \triangleq \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ I & I & 0 & -2I & 0 & 0 & 0 & 0 \\ 0 & I & -I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & I & 0 & -2I & 0 & 0 & 0 \end{bmatrix},\ \Phi_3 \triangleq \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & -I & 0 & 0 & 0 & 0 & 0 \end{bmatrix},\ \Phi_7 \triangleq \begin{bmatrix} \kappa I & 0 & 0 & -I & 0 & 0 \end{bmatrix},$$

$$\Phi_2 \triangleq \begin{bmatrix} \kappa I & 0 & 0 & 0 & 0 & -I & 0 & 0 \end{bmatrix},\ \Phi_5 \triangleq \begin{bmatrix} \kappa I & 0 & 0 & 0 & -I & 0 & 0 & 0 \end{bmatrix},\ \Phi_8 \triangleq \begin{bmatrix} \tau I & -I & 0 & 0 \end{bmatrix},$$

$$\Phi_4 \triangleq \begin{bmatrix} I & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ I & 0 & I & -2I & 0 & 0 & 0 & 0 \end{bmatrix},\ \Phi_6 \triangleq \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 \\ I & I & -2I & 0 & 0 & 0 \end{bmatrix},\ \Phi_9 \triangleq \begin{bmatrix} \kappa I & 0 & 0 & -I \end{bmatrix},$$

$$R_\alpha \triangleq R_{\alpha 1} + R_{\alpha 2},\ \bar{R}_\alpha \triangleq \bar{R}_{\alpha 1} + \bar{R}_{\alpha 2},\ \kappa \triangleq \sum_{i=1}^{+\infty} \mu_i,\ \tilde{\kappa}_\alpha \triangleq \sum_{i=1}^{+\infty} \mu_i \lambda_\alpha^{-i},\ \sigma \triangleq \sum_{i=1}^{+\infty} i\mu_i,\ \tilde{\sigma} \triangleq \sum_{j=1}^{+\infty} \sum_{i=1}^{j} \mu_j \lambda_\alpha^{-i},$$

$$Sym(T) \triangleq T^T + T,\ \varphi_1 \triangleq \frac{\lambda_2^\tau - 1}{\lambda_2 - 1},\ \varphi_4 \triangleq \frac{(1-\lambda_2)\tau - \lambda_2 + \lambda_2^{\tau+1}}{(\lambda_2 - 1)^2},\ \varphi_{2\alpha} \triangleq \sum_{j=1}^{+\infty} \sum_{i=1}^{j} \mu_j \lambda_\alpha^{-i},\ \varphi_{3\alpha} \triangleq \sum_{l=1}^{+\infty} \sum_{j=1}^{l} \sum_{i=1}^{j} \mu_l \lambda_\alpha^{i-1}.$$

Theorem 1. Let the scalars $0 < \lambda_1 \le 1$, $\lambda_2 > 1$, $v > 0$, the integer $k \ge 1$ and the matrix K be given. Assume that there exist matrices $0 < P_\alpha \in R^{3n \times 3n}$, $0 < Q_\alpha \in R^{n \times n}$, $0 < S_{\alpha j} \in R^{n \times n}$, $0 < R_{\alpha j} \in R^{n \times n}$, $0 < Z_\alpha \in R^{n \times n}$, $T_{ij} \in R^{n \times n}\ (\alpha, j = 1,2.i = 1,2)$, $M_1 \in R^{2n \times 2n}$, $M_2 \in R^{n \times n}$, $U \in R^{m \times n}$, $V \in R^{m \times n}$, $W \in R^{m \times n}$ and the diagonal matrices $0 < H \in R^{m \times n}$ such that, for $\tau_k = 0, \tau$, $\forall s \in [1, 2^m]$, $\forall l \in [1, \vec{m}]$, the matrix inequalities

$$\Lambda_1 \triangleq \begin{bmatrix} \breve{R}_1 & M_1 \\ M_1^T & \breve{R}_1 \end{bmatrix} > 0,\ \Lambda_2 \triangleq \begin{bmatrix} R_{21} & M_2 \\ M_2^T & R_{21} \end{bmatrix} > 0 \tag{11}$$

$$\Xi_1(\tau_k, s) \triangleq \Gamma_1^T P_1 \Gamma_1 - \lambda_1 \Gamma_2^T P_1 \Gamma_2 - \lambda_1^\tau \Phi_1^T \Lambda_1 \Phi_1 - \Phi_2^T (Z_1/\tilde{\sigma}_1) \Phi_2 + Sym(T_1 \Sigma_1) + \Psi_1 < 0 \tag{12}$$

$$\Xi_2 \triangleq \Gamma_3^T P_2 \Gamma_3 - \lambda_2 \left(\Gamma_4^T P_2 \Gamma_4 + \Phi_3^T \Lambda_2 \Phi_3 + \Phi_4^T \breve{R}_{22}^\tau \Phi_4 \right) - \Phi_5^T (Z_2/\tilde{\sigma}_2) \Phi_5 + Sym(T_2 \Sigma_2 + T_4 \Sigma_4) + \Psi_2 < 0 \tag{13}$$

$$\Xi_3 \triangleq \Gamma_5^T P_1 \Gamma_5 - \lambda_2 \Gamma_6^T P_2 \Gamma_6 - \lambda_2 \Phi_6^T R_2^\tau \Phi_6 - \Phi_7^T (Z_2/\tilde{\sigma}_2) \Phi_7 + Sym(T_3 \Sigma_3) + \Psi_3 < 0 \tag{14}$$

$$\begin{cases} P_1 \le vP_2, Q_1 \le vQ_2, S_{1j} \le vS_{2j} \\ R_{1j} \le vR_{2j}\ (j = 1,2), Z_1 \le vZ_2 \end{cases} \tag{15}$$

$$\Xi_4(l) \triangleq \begin{bmatrix} 1/v\lambda_2^k & N_{(l)} \\ N_{(l)}^T & diag\{P_1, 0\} + \Psi_4/\lambda_1 \end{bmatrix} \ge 0 \tag{16}$$

where $N_{(l)} = \begin{bmatrix} U_{(l)} & 0 & W_{(l)} & V_{(l)} \end{bmatrix}$, $\Psi_1 = diag\{Q_1 + \kappa S_{11} + \sigma S_{12}, 0, -\lambda_1^\tau Q_1, 0, 0, -S_{11}/\tilde{\kappa}_1, -S_{12}/\tilde{\sigma}_1, \tau^2 R_1 + \sigma Z_1\}$, $\Psi_2 = diag\{Q_2 + \kappa S_{21} + \sigma S_{22}, 0, -\lambda_2^\tau Q_2, 0, -S_{21}/\tilde{\kappa}_2, -S_{22}/\tilde{\sigma}_2, \tau^2 R_2 + \sigma Z_2, 0\}$, $\Psi_3 = diag\{Q_2 + \kappa S_{21} + \sigma S_{22}, -\lambda_2^\tau Q_2, 0, -S_{21}/\tilde{\kappa}_2, -S_{22}/\tilde{\sigma}_2, \tau^2 R_2 + \sigma Z_2\}$, $\Psi_4 = diag\{0, (\lambda_1^\tau/\tau) Q_1, S_{12}/\tilde{\sigma}_1, S_{11}/\tilde{\kappa}_1\} + (2\lambda_1^\tau/\tilde{\tau}) \Phi_8^T R_1 \Phi_8 + (1/\tilde{\sigma}_1) \Phi_9^T Z_1 \Phi_9$, $T_1 = \begin{bmatrix} T_{11}^T & 0 & 0 & 0 & 0 & 0 & 0 & T_{12}^T \end{bmatrix}^T$, $T_3 = \begin{bmatrix} T_{31}^T & 0 & 0 & 0 & 0 & T_{32}^T \end{bmatrix}^T$, $T_2 = \begin{bmatrix} T_{21}^T & 0 & 0 & 0 & 0 & 0 & T_{22}^T & 0 \end{bmatrix}^T$, $T_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & H^T \end{bmatrix}^T$, $\Sigma_1 = \begin{bmatrix} A + BD_s^- U - I & BKD_s & 0 & 0 & 0 & A_d + BD_s^- V & BD_s^- W & -I \end{bmatrix}$, $\Sigma_2 = \begin{bmatrix} A - I & BK & 0 & 0 & A_d & 0 & -I & -B \end{bmatrix}$, $\Sigma_3 = \begin{bmatrix} A - I & 0 & 0 & A_d & 0 & -I \end{bmatrix}$, $\Sigma_4 = \begin{bmatrix} 0 & K & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix}$, $\breve{R}_1 \triangleq diag\{R_1, 3R_1\}$, $\breve{R}_2^\tau \triangleq diag\{R_2, 3\varphi_\tau R_2\}$, $\breve{R}_{22}^\tau \triangleq diag\{R_{22}, 3\varphi_\tau R_{22}\}, (\varphi_\tau \triangleq (\tau+1)/(\tau-1)(\tau > 1), \varphi_1 \triangleq 1)$.

Then, for any initial condition $\phi(k) \in [0, +\infty)$ satisfying $V(0) \le 1$, the closed-loop system (9) is exponentially stable.

Proof. Let $\Delta V_\alpha(k) \triangleq V_\alpha(k+1) - \lambda_\alpha V_\alpha(k)$, by calculations and using Lemma 2, it is obtained

$$\begin{aligned}\Delta V_\alpha(k) \le \eta^T(k+1)P_\alpha\eta(k+1) - \lambda_\alpha\eta^T(k)P_\alpha\eta(k)\\ +x^T(k)(Q_\alpha + \kappa S_{\alpha 1} + \sigma S_{\alpha 2})x(k) + y^T(k)\\ \times(\tau^2 R_\alpha + \sigma Z_\alpha)y(k) - \lambda_\alpha^\tau x^T(k-\tau)Q_\alpha x(k-\tau)\\ -\varsigma_1^T(k)(S_{\alpha 1}/\tilde{\kappa}_\alpha)\varsigma_1(k) - \varsigma_2^T(k)(S_{\alpha 2}/\tilde{\sigma}_\alpha)\varsigma_2(k)\\ -\tau\tilde{\lambda}_\alpha\sum_{i=k-\tau}^{k-1} y^T(i)R_\alpha y(i) - \varsigma_3^T(k)(Z_\alpha/\tilde{\sigma}_\alpha)\varsigma_3(k)\end{aligned} \tag{17}$$

where $\tilde{\lambda}_1 = \lambda_1^\tau$, $\tilde{\lambda}_2 = \lambda_2$, $\varsigma_1(k) = \sum_{i=1}^{+\infty}\mu_i x(k-i)$, $\varsigma_2(k) = \sum_{j=1}^{+\infty}\sum_{i=1}^{j}\mu_j x(k-i)$, $\varsigma_3(k) = \sum_{j=1}^{+\infty}\sum_{i=1}^{j}\mu_j y(k-i)$.

Using the discrete Wirtinger-based inequality [16], [18] and interactive convex combinatorial inequality [18], and $\sum_{i=k-\tau}^{k-1}(*) = \sum_{i=k-\tau_k}^{k-1}(*) + \sum_{i=k-\tau}^{k-\tau_k-1}(*)$, we have

$$\tau\sum_{i=k-\tau}^{k-1} y^T(i)R_1 y(i) \ge \varepsilon_1^T(k)\Phi_1^T\Lambda_1\Phi_1\varepsilon_1(k) \tag{18}$$

Since $\Lambda_1 > 0$, similar to (18), Jensen inequality [16] and cross convex combinatorial inequality [18] can be used to obtain

$$\tau\sum_{i=k-\tau}^{k-1} y^T(i)R_1 y(i) \ge \varepsilon_2^T(k)\Phi_3^T\Lambda_2\Phi_3\varepsilon_2(k) \tag{19}$$

Because $\Lambda_2 > 0$, by directly using the discrete Wirtinger-based inequality [16], which yields

$$\tau\sum_{i=k-\tau}^{k-1} y^T(i)R_{22} y(i) \ge \varepsilon_2^T(k)\Phi_4^T R_{22}^\tau\Phi_4\varepsilon_2(k) \tag{20}$$

$$\tau\sum_{i=k-\tau}^{k-1} y^T(i)R_2 y(i) \ge \varepsilon_3^T(k)\Phi_6^T R_2^\tau\Phi_6\varepsilon_3(k) \tag{21}$$

According to (9), combined with matrix spaces T_i and $\Sigma_i\,(i = 1,2,3)$, it can be obtained

$$2\varepsilon_1^T(k)T_1\Sigma_1\varepsilon_1(k) = 0 \tag{22}$$

$$2\varepsilon_2^T(k)T_2\Sigma_2\varepsilon_2(k) = 0 \tag{23}$$

$$2\varepsilon_3^T(k)T_3\Sigma_3\varepsilon_3(k) = 0 \tag{24}$$

In addition, it can be obtained from the sector condition (3), one has

$$-2\varepsilon_2^T(k)T_4\Sigma_4\varepsilon_2(k) \ge 0 \tag{25}$$

Adding the left-hand side of (21) to $\Delta V_1(k)$, and using (18) and $\sum_{j=1}^{+\infty}\sum_{i=1}^{j}\mu_j y(k-i) = Kx(k) - \sum_{i=1}^{+\infty}\mu_j y(k-i)$, we obtain

$$\Delta V_1(k) \le \sum_{s=1}^{2^m}\varpi_s^k\varepsilon_1^T(k)\Xi_1(\tau_k, s)\varepsilon_1(k) \tag{26}$$

Similar to (26), add the left side of (23) and e (25) to $\Delta V_2(k)$, and comprehensively consider (19) and (20), one has

$$\Delta V_2(k) \le \varepsilon_2^T(k)\Xi_2\varepsilon_2(k) \tag{27}$$

In addition, by adding the left of (24) and applying inequality (21), we can obtain

$$\Delta V_2(k) \le \varepsilon_3^T(k)\Xi_3\varepsilon_3(k) \tag{28}$$

For $\tau_k = 0, \tau$ and $\forall s \in [1, 2^m]$, if matrix inequalities (11)-(14) hold, then (26)-(28) can be obtained

$$V_1(k+1) \le \lambda_1 V_1(k) \tag{29}$$

$$V_2(k+1) \le \lambda_2 V_2(k) \tag{30}$$

Considering (29), (30) and (15), we can obtain

$$V_1(k) \le v V_2(k), k \ge 0 \tag{31}$$

According to conditions (29)-(31), it can be obtained

$$V_1(k) \le \lambda_1^k \left[v V_2(0) \right], k \in [0, +\infty) \tag{32}$$

It is assumed that

$$\sum_{j=1}^{+\infty} \mu_j \sum_{i=k-j}^{k-1} f(i) \ge \sum_{j=1}^{+\infty} \mu_j f(k-j), \ \sum_{l=1}^{+\infty} \mu_l \sum_{j=1}^{l} \sum_{i=k-j}^{k-1} f(i) \ge \sum_{l=1}^{+\infty} \mu_l \sum_{j=1}^{l} f(k-j)$$

hold, where $f(i)$ is a positive real function, then the following inequalities can be obtained by using lemma 1 and Jensen inequality [16]

$$V_1(k) \ge \tilde{\eta}^T(k)\left[diag\{P_1, 0\} + \Psi_4/\lambda_1 \right]\tilde{\eta}(k) \tag{33}$$

where $\tilde{\eta}(k) = \left[\tilde{\eta}^T(k) \quad \sum_{i=1}^{+\infty} \mu_i x^T(k-i) \right]^T$.

Using Schur lemma to (17), we can obtain

$$diag\{P_1, 0\} + \Psi_4/\lambda_1 \ge v\lambda_2^k N_{(l)}^T N_{(l)}, l \in [1, \vec{m}] \tag{34}$$

Considering (32)-(34), we can obtain

$$\left|\vartheta_l(k)\right|^2 = \tilde{\eta}^T(k) N_{(l)}^T N_{(l)} \tilde{\eta}(k) \le \left(1/\left(v\lambda_2^k\right)\right) V_1(k) \le V_2(0), l \in [1, \vec{m}], k \in [0, +\infty) \tag{35}$$

For any initial condition $\phi(k)$ is satisfied $V_1(0) \le V_2(0) \le 1$, it is not difficult to obtain from equation (35) and condition (7) holds. In addition, can be obtained from equation (32), and the closed-loop system (9) is exponentially stable. This completes the proof.

Theorem2. Let the scalars $0 < \lambda_1 \le 1$, $\lambda_2 > 1$, $v > 0$, $\delta_i \ne 0 (i = 1,2,3)$ the integer $k \ge 1$ be given. Assume that there exist matrices $0 < \bar{P}_\alpha \in R^{3n \times 3n}$, $0 < \bar{Q}_\alpha \in R^{n \times n}$, $0 < \bar{S}_{\alpha j} \in R^{n \times n}$, $0 < \bar{R}_{\alpha j} \in R^{n \times n}$, $0 < \bar{Z}_\alpha \in R^{n \times n} (\alpha, j = 1,2.i = 1,2)$, $\bar{M}_1 \in R^{2n \times 2n}$, $\bar{M}_2 \in R^{n \times n}$, $X \in R^{n \times n}$, $Y \in R^{n \times n}$, $\bar{U} \in R^{m \times n}$, $\bar{V} \in R^{m \times n}$, $\bar{W} \in R^{m \times n}$,and the diagonal matrix $0 < \bar{H} \in R^{m \times n}$ such that, for $\tau_k = 0, \tau$, $\forall s \in [1, 2^m]$, $\forall l \in [1, \vec{m}]$, the matrix inequalities

$$\bar{\Lambda}_1 \triangleq \begin{bmatrix} \hat{R}_1 & \bar{M}_1 \\ \bar{M}_1^T & \hat{R}_1 \end{bmatrix} > 0, \ \Lambda_2 \triangleq \begin{bmatrix} \bar{R}_{21} & \bar{M}_2 \\ \bar{M}_2^T & \bar{R}_{21} \end{bmatrix} > 0 \tag{36}$$

$$\Xi_1(\tau_k, s) \triangleq \Gamma_1^T \bar{P}_1 \Gamma_1 - \lambda_1 \Gamma_2^T \bar{P}_1 \Gamma_2 - \lambda_1^\tau \Phi_1^T \bar{\Lambda}_1 \Phi_1 - \Phi_2^T \left(\bar{Z}_1/\tilde{\sigma}_1 \right) \Phi_2 + Sym\left(\bar{T}_1 \Sigma_1 \right) + \bar{\Psi}_1 < 0 \tag{37}$$

$$\bar{\Xi}_2 \triangleq \Gamma_3^T \bar{P}_2 \Gamma_3 - \lambda_2 \left(\Gamma_4^T \bar{P}_2 \Gamma_4 + \Phi_3^T \bar{\Lambda}_2 \Phi_3 + \Phi_4^T \hat{R}_{22}^\tau \Phi_4 \right) - \Phi_5^T \left(\bar{Z}_2/\tilde{\sigma}_2 \right) \Phi_5 + Sym\left(\bar{T}_2 \Sigma_2 + \bar{T}_4 \Sigma_4 \right) + \bar{\Psi}_2 < 0 \tag{38}$$

$$\bar{\Xi}_3 \triangleq \Gamma_5^T \bar{P}_1 \Gamma_5 - \lambda_2 \Gamma_6^T \bar{P}_2 \Gamma_6 - \lambda_2 \Phi_6^T \hat{R}_2^\tau \Phi_6 - \Phi_7^T \left(\bar{Z}_2/\tilde{\sigma}_2 \right) \Phi_7 + Sym\left(\bar{T}_3 \Sigma_3 \right) + \bar{\Psi}_3 < 0 \tag{39}$$

$$\begin{cases} \bar{P}_1 \le v\bar{P}_2, \bar{Q}_1 \le v\bar{Q}_2, \bar{S}_{1j} \le v\bar{S}_{2j} \\ \bar{R}_{1j} \le v\bar{R}_{2j}\left(j=1,2\right), \bar{Z}_1 \le v\bar{Z}_2 \end{cases} \tag{40}$$

$$\Xi_4(l) \triangleq \begin{bmatrix} 1/v\lambda_2^k & \bar{N}_{(l)} \\ \bar{N}_{(l)}^T & diag\{\bar{P}_1, 0\} + \Psi_4/\lambda_1 \end{bmatrix} \ge 0 \tag{41}$$

where $\bar{N}_{(l)} = \left[\bar{U}_{(l)} \quad 0 \quad \bar{W}_{(l)} \quad \bar{V}_{(l)}\right]$, $\Psi_1 = diag\{\bar{Q}_1 + \kappa\bar{S}_{11} + \sigma\bar{S}_{12}, 0, -\lambda_1^\tau\bar{Q}_1, 0, 0, -\bar{S}_{11}/\tilde{\kappa}_1, -\bar{S}_{12}/\tilde{\sigma}_1, \tau^2\bar{R}_1 + \sigma\bar{Z}_1\}$, $\Psi_2 = diag\{\bar{Q}_2 + \kappa\bar{S}_{21} + \sigma\bar{S}_{22}, 0, -\lambda_2^\tau\bar{Q}_2, 0, -\bar{S}_{21}/\tilde{\kappa}_2, -\bar{S}_{22}/\tilde{\sigma}_2, \tau^2\bar{R}_2 + \sigma\bar{Z}_2, 0\}$, $\Psi_3 = diag\{\bar{Q}_2 + \kappa\bar{S}_{21} + \sigma\bar{S}_{22}, -\lambda_2^\tau\bar{Q}_2, 0, -\bar{S}_{21}/\tilde{\kappa}_2, -\bar{S}_{22}/\tilde{\sigma}_2, \tau^2\bar{R}_2 + \sigma\bar{Z}_2\}$, $\Psi_4 = diag\{0, (\lambda_1^\tau/\tau)\bar{Q}_1, \bar{S}_{12}/\tilde{\sigma}_1, \bar{S}_{11}/\tilde{\kappa}_1\} + (2\lambda_1^\tau/\tilde{\tau})\Phi_8^T\bar{R}_1\Phi_8 + (1/\tilde{\sigma}_1)\Phi_9^T\bar{Z}_1\Phi_9$, $\bar{T}_1 = [I \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \delta_1 I]^T$, $\bar{T}_3 = [I \quad 0 \quad 0 \quad 0 \quad 0 \quad \delta_3 I]^T$, $\bar{T}_2 = [I \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \delta_2 I \quad 0]^T$, $\bar{T}_4 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad I]^T$, $\bar{\Sigma}_1 = \left[(A-I)X^T + BD_s^-\bar{U} \quad BD_s Y \quad 0 \quad 0 \quad 0 \quad A_d X^T + BD_s^-\bar{V} \quad BD_s^-\bar{W} \quad -X^T\right]$, $\bar{\Sigma}_2 = \left[(A-I)X^T \quad BY \quad 0 \quad 0 \quad A_d X^T \quad 0 \quad -X^T \quad -B\bar{H}\right]$, $\bar{\Sigma}_3 = \left[(A-I)X^T \quad 0 \quad 0 \quad A_d X^T \quad 0 \quad -X^T\right]$, $\bar{\Sigma}_4 = \left[0 \quad Y \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -\bar{H}\right]$, $\hat{R}_1 \triangleq diag\{\bar{R}_1, 3\bar{R}_1\}$, $\hat{R}_2^\tau \triangleq diag\{\bar{R}_2, 3\varphi_\tau\bar{R}_2\}$, $\hat{R}_{22}^\tau \triangleq diag\{\bar{R}_{22}, 3\varphi_\tau\bar{R}_{22}\}, (\varphi_\tau \triangleq (\tau+1)/(\tau-1)(\tau>1), \varphi_1 \triangleq 1)$.

Then, for any initial condition $\phi(k) \in [0, +\infty)$ satisfying $V(0) \le 1$, and (2) the gain of the controller is $K = YX^{-1}$, the closed-loop system (9) is exponentially stable.

Proof. Assuming that (36)-(38) are satisfied, the matrix X is nonsingular. Define the following equation

$$\begin{cases} P_\alpha \triangleq \breve{X}^{-1}\bar{P}_\alpha\breve{X}^{-T}\left(\breve{X} \triangleq diag\{X, X, X\}\right) \\ Q_\alpha \triangleq X^{-1}\bar{Q}_\alpha X^{-T}, S_{\alpha j} \triangleq X^{-1}\bar{S}_{\alpha j}X^{-T} \\ R_{\alpha j} \triangleq X^{-1}\bar{R}_{\alpha j}X^{-T}, Z_\alpha \triangleq X^{-1}\bar{Z}_\alpha X^{-T} \\ T_{i1} \triangleq X^{-1}, T_{i2} \triangleq \delta_i X^{-1}, \alpha, j = 1,2, i = 1,2,3 \\ M_1 \triangleq \hat{X}^{-1}\bar{M}_1\hat{X}^{-T}\left(\hat{X} \triangleq diag\{X, X\}\right) \\ M_2 \triangleq X^{-1}\bar{M}_2 X^{-T}, H \triangleq \bar{H}^T, K \triangleq YX^{-T} \\ U \triangleq \bar{U}X^{-T}, V \triangleq \bar{V}X^{-T}, W \triangleq \bar{W}X^{-T} \end{cases} \tag{42}$$

Contract transformation is performed on conditions (36)-(41) and (42), and (11)-(16) can be obtained respectively. This completes the proof.

4. Numerical Simulation (Heading 4)

In this section, a numerical example is used to illustrate the effectiveness of the proposed approach. Consider the following two-dimensional CPS:

$$A = \begin{bmatrix} 1.10 & 0.15 \\ 0.03 & 0.80 \end{bmatrix},\ A_d = \begin{bmatrix} 1 & -0.1 \\ 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0.1 \\ 0.1 & 1 \end{bmatrix},\ \mu^i = 2^{-i},\ \bar{u}_1 = 15,\ 0 \le \tau_k = 2 + (-1)^k \le 3,$$

For this example, the parameters $\delta_1 = \delta_2 = \delta_3 = 4$, $\lambda_1 = 1$, $\lambda_2 = 1.24$, $v = 0.97$ and FDI attack signals $f(u(k), k) = cos(2(k-\tau_k))$ are selected according to Theorem 2, and the controller gain can be obtained by solving (36)-(41) under given initial conditions

$$K = \begin{bmatrix} 2.4855 & 0.2495 \\ -0.3337 & -0.0439 \end{bmatrix}$$

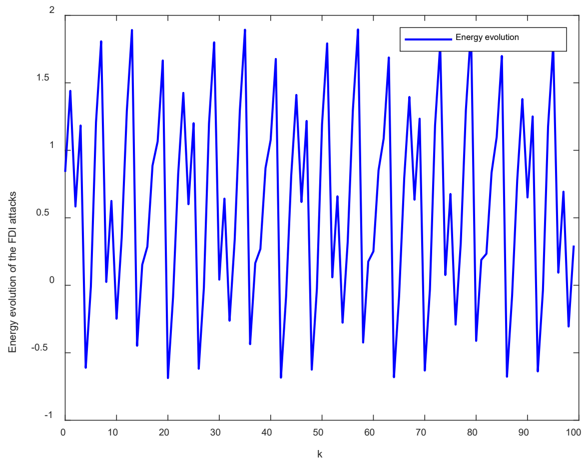

Figure 1: Energy evolutions of the FDI attacks.

The FDI attacks' energy evolution trajectory is shown in Figure 1. Figure 2 shows the state trajectory the closed-loop system. It is assumed that the initial condition of the system is $x_0 = [2 \quad -2]$. As can be seen from Figure 2, under the influence of FDI attacks and mixed delays, the state of the closed-loop system still tends to equilibrium although it fluctuates to a certain extent. Therefore, the control method proposed in this article can make the system state converge to the equilibrium state and has certain control performance.

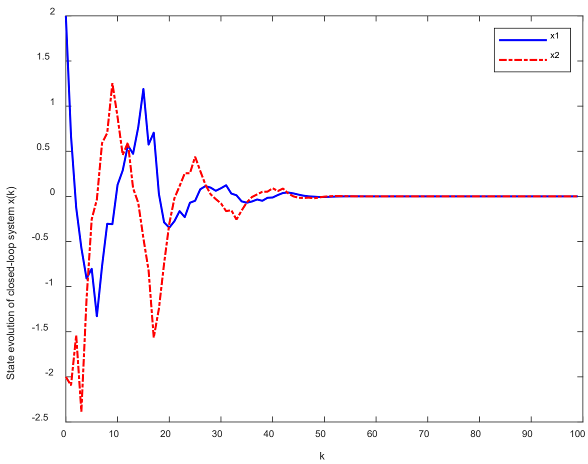

Figure 2: State evolution of the closed-loop system.

5. Conclusions

In this article, the security control problem of discrete CPS with distributed state delay and rapidly varying input delay under network attack is studied. Using a polyhedron model, piecewise Lyapunov functional and some summation inequalities, sufficient conditions for asymptotic stability of linear matrix inequalities are established, and the safety controller is designed. At the same time, it is also our future work to consider the security control of systems with distributed input delays and complex cyber attacks.

Acknowledgements

If any, should be placed before the references section without numbering.

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