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GM (1,1) and Quantitative Trading Decision Model

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Abstract: Depending on the data of recent years, it is understood that Bitcoin and gold are finan-cial products worth looking forward to receiving. How to make the right choice between Bitcoin, gold, and continue to hold cash at the right time node at the right time and with the right funds to achieve the greatest economic benefits is the most important research object for the establishment of the mathematical model. For Model I, We establish a price prediction model based on GM (1,1). Firstly, we conduct a quasi-exponential test on the prices of gold and bitcoin. After the test, we estab-lish a GM (1,1) grey differential model and combine it with Metabolism GM (1,1) to predict the future trading prices of gold and bitcoin by using MATLAB. Then according to the actual transaction price, the residuals are calculated to quantify the financial risk by Ljung-Box Q test and LM statistical test. Finally, it is found that the prediction results of gold and bitcoin are ' no sequence correlation ' and there are GARCH errors. For Model II, We establish a strategy making model based on genetic algorithm. We first consider using Trend tracking strategy to make decisions. Since the trend tracking strategy is difficult to determine the trend, only using the past results to make decisions, we consider the combination of the previous prediction results, using genetic algorithm to op-timize the trend tracking strategy. Finally, it is concluded that from September 11,2016 to September 10,2021, the value of up to $ 75,000 cans be generated on the basis of $ 1000, and with further iteration no long-er change, the optimal value is achieved. In addition, We provide evidence from Model I and Model II to prove that we have given the highest economic value. For Model I, we add some disturbances to the actual price of gold and bitcoin and then make multiple predictions and error analysis, and the ac-curacy of the final prediction is not greatly affected. For Model II, we add some disturb-ances to the original optimal decision scheme to calculate the investment value, and find that the investment value of the disturbed scheme is lower than that before the disturbance. Eventually, We conducted a sensitivity analysis of Model I and Model II and found that the number of gold transactions decreased significantly as transaction costs increased, but bitcoin transactions were not significantly affected.

1. Introduction

1.1. Problem Background

"The success of an investment is based on the knowledge and experience you already have!" — Roy Newberg This famous saying reveals the nature of investing. How to buy suitable financial prod-ucts with the right funds at the right time is critical to a profitable investment. Depending on the data of recent years, it is understood that Bitcoin and gold are financial products worth looking forward to receiving. How to make the right choice between Bitcoin, gold, and continue to hold cash at the right time node at the right time and with the right funds to achieve the greatest economic benefits is the most important research object for the estab-lishment of the mathematical model.

1.2. Restatement of the Problem

Taking into account the background of the problem and the constraints given in the ti-tle, we need to address the following issues:

Problem 1: Build a mathematical model that uses only the price of the day's previous days, and then use the mathematical model to determine whether the day's transaction should buy, hold or sell the assets in its portfolio.

Problem 2: Build a mathematical model that can give the best trading strategy for the day based on the price data as of the current day. Beginning at $1,000, between September 11, 2016 and September 10, 2021, you will invest in Bitcoin, gold, and the U.S. dollar using the established model. Built on the mathematical model established, calculate what the ini-tial $1,000 investment value was as of October 9, 2021. In addition to this, it is demonstrat-ed that the model provides the best strategy, and the sensitivity of the strategy to transaction costs is determined, explaining how transaction costs affect the strategy and results.

2. Assumptions and Justifications

To simplify the problem, we make the following basic assumptions, each of which is properly justified.

Assumption 1: All transactions on gold and bitcoin can be completed on the same day, and the transaction costs on that day are calculated at the same price only.

Assumption 2: Transaction costs arising from all transactions are borne only by the traders.

Assumption 3: Gold and bitcoin reserves enough to have no impact on trading decisions.

Assumption 4: Cash holdings do not add value, that is, the interest rate on cash is 0.

3. Notations

The key mathematical notations used in this paper are listed in Table 1.

Table 1: Notations Description

Sign	Implication
$x^{(0)}$	Output of raw data
$x^{(1)}$	One-time cumulative yield
$z^{(1)}$	Adjacent to the generated sequence
-a	development coefficient
b	grey action quantity
σ	class ratio
ρ	evenness
ε_r	residual error

4. Question Analysis and Model Preparation

4.1. Question Analysis

Combined with the gold and bitcoin trading process in actual financial activities, we divide the problem analysis required by the investment process into how to accurately predict the price change trend of gold and bitcoin, and plan the daily gold and bitcoin transaction amount in a certain period of time based on the prediction results to maximize the sum of cash, gold and bitcoin in this time period, and to get as close as possible to the global maximum profit by constantly looking for local maximum profit.

4.2. Time series chart of Gold's USD(PM) and Bitcoin Value

We first draw the time series of gold and bitcoin, which are shown in Figure 1 and Figure 2.

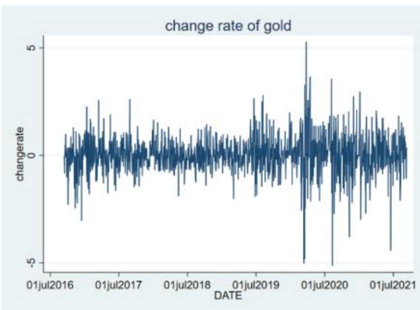
Figure 1: Time series of Bitcoin price

Figure 2: Time series of Gold price

It can be seen that gold and bitcoin showed an upward trend during the five years. In contrast, gold prices rose relatively flat, reaching a maximum of more than 2100 USD (PM) around August 2000 and began to fall steadily. The price of bitcoin remained stable until 2020, but it showed an explosive growth after November 2020. The value of a single bitcoin once exceeded USD 60,000, and then began to fall rapidly and rise again after falling to USD 30,000 in 2021.

4.3. Time series chart of Change rate

Then, we draw the time series chart of Change rate of gold and bitcoin, which are illustrated in Figure 3 and Figure 4 and Table 2



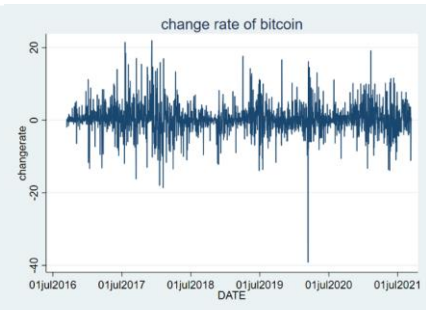


Figure 3: Change rate of Bitcoin

Figure 4: Change rate of Gold

Table 2: Descriptive statistics for gold and bitcoin

Variable	Obs	Mean	Std. Dev.	Min	Max
Change rate of Gold	1,264	0.028	0.866	5.128	5.267
Change rate of Bitcoin	1,825	0.323	4.148	39.140	21.867

In the analysis of the change rate of daily gold and the closing price of bitcoin, it can be seen that the change amplitude and frequency of bitcoin are also much higher than those of gold. Next, we further test whether the yield series have unit roots, and use Stata to do the ADF test. The results are as follows in Table and Table 4.

Table 3: Check if Gold closing price rate r is unit root

Dickey-Fuller test for unit root   Number of obs = 1263

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-34.808	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

Table 4: Check if Bitcoin closing price rate r is unit root

Dickey-Fuller test for unit root   Number of obs = 1824

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-44.222	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

The above table shows that gold and bitcoin p-value = 0. The original hypothesis is rejected at 99 % confidence level, so the sequence is stable.

5. Model I: Price Forecasting Model Based on Gray Forecast Model

In the actual trading process of Financial Products, traders cannot accurately obtain the price changes of the traded Financial Products before deciding the trading strategy of the day, and traders can only obtain the historical data before the opening day. How to provide reference and as accurate as possible for the price of the actual transaction process according to historical data plays a very important role in the formulation of the maximum profit trading strategy.

5.1. Introduction of GM (1,1) Model

Before the investment activities of Financial Products. How to predict the price of Financial Products has always been a hot research field for domestic and foreign economic and financial researchers and industry investors. Taking stock prices as an example, researchers in past dynasties have developed many prediction methods, such as Autoregressive Integrated Moving Average (ARIMA) and Generalized Au-to regressive Conditional Heteroskedasticity Model (GARCH). The premise of these methods to predict the time series is that the time series satisfies the linear hypothesis. However, the actual financial activities will be interfered by many external factors that are difficult to quantify, such as natural disasters and national policies, which makes it difficult for the financial time series to meet the linear and stationary assumptions. Thus, the prediction effect of the financial time series is not ideal. Machine learning and deep learning methods suitable for extracting nonlinear characteristics gradually occupy the mainstream of stock price prediction.

Gray Forecast Model is a prediction method to establish mathematical model and make prediction through a small amount of incomplete information. Common GM (1,1) model and GM (1, N) model. Gray Forecast Model does not need to predict the sample has a strong regularity, for the financial time series which contains uncertainty factors of the sample short-term prediction has a good prediction effect. At the same time, due to the fast update frequency of Financial Products, it is necessary to predict many times in the decision-making process. The Gray Forecast Model has a small amount of calculation and does not consume too much time. To a certain extent, it reduces the possibility that the late decision-making process is too slow and lags behind the market change due to the high frequency change of Financial Products.

5.2. Establishment of GM (1,1) Model

(1) creating process

GM (1,1) model prediction principle is the use of a certain range of fluctuations in the gray amount of data sequence generated by the accumulation of data sequence is basically linear distribution. The first ' 1 ' means that the equation is first-order, and the second ' 1 ' means that there is only one variable. In this paper, when predicting the closing prices of gold and bitcoin, since the only factor involved is the transaction price, the specific prediction process of GM (1,1) model is shown in the following figure 5:

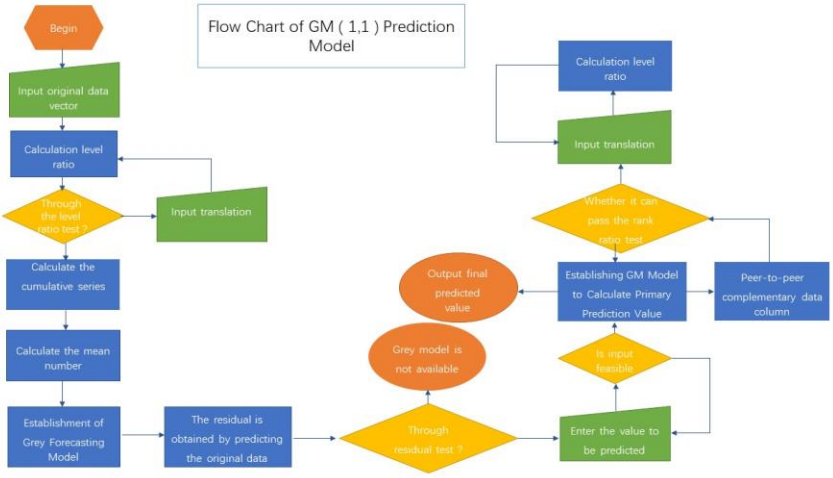


Figure 5: Flow Chart of GM (1,1) Prediction Model

We assume the transaction price is x , definition

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots x^{(0)}(n))$$

Is the original array given by the topic. Here we use 30 days of initial data for each prediction. quasi n=30; The above (0) means the original data.

Data column given to a topic by processing data $x^{(0)}$ Carry out a cumulative new generation list

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n))$$

In the array,

$$x^{(1)}(m) = \sum_{i=1}^{m} x^{(0)}(i), m = 1, 2, \cdots, n$$

.

$$z^{(1)} = (z^{(1)}(2), z^{(1)}(3), \cdots, z^{(1)}(n)),$$

In this column

$z^{(1)}(m) = \delta x^{(1)}(m) + (1-\delta) x^{(1)}(m-1), m = 2, 3, \cdots n$, In this article $\delta = 0.5$

We call equations $x^0(k)+az^{(1)}(k)=b$ for $GM(1,1)$ The basic form of the model. In this equation b It represents the amount of grey action. $-a$ represents the coefficient of development.

introducing appropriate matrix

$$\nu=(a,b)^T, Y=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, \quad B=\begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}$$

Therefore, $GM(1,1)$ model $x^0(k)+az^{(1)}k=b$ can be written as

$Y=B\nu$

We can use the least square method to obtain a,b the estimate is

$$\hat{\nu}=\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix}=(B^TB)^{-1}B^TY$$

In addition. If $x^{(0)}(m)$ when $m=2,3,\cdots n$ as a continuous variable t ,now $x^{(1)}$ as t 's function,denoted by $\hat{x}^{(1)}(t)$,to $x^{(0)}(k)$ corresponding to derivative $\frac{d\hat{x}^{(1)}(t)}{dt}$, $z^{(1)}(k)$ corresponding to $x^{(1)}(t)$,Then, the relative grey equation can be established $GM(1,1)$'s white differential equation: $\frac{d\hat{x}^{(1)}(t)}{dt}+\hat{x}^{(1)}(t)=b$, we call $GM(1,1)$'s whitening equation.

(2) model solving

Test of quasi-exponential law Before formally using the model prediction results, we need to test the data quasi exponential law. Quasi - exponential Law of Data is the Theoretical Basis of Grey System Modeling.

If ∀k, interval length $\sigma(k)\epsilon[a,b]$,Simultaneous interval length b-a<0.5, Accumulate the sequences after r times has quasi exponential law.

For $GM(1,1)$ Model, we just need to judge the sequence after one accumulation $x^{(1)}$ is there a quasi-exponential law.

Define a cumulative sequence $x^{(1)}$'s stepwise ratio

$$\sigma(k)=\frac{x^{(r)}(k)}{x^{(r)}(k-1)}=\frac{x^{(0)}(k)}{x^{(1)}(k-1)}+1, k=2,3,\cdots,n$$

Define the original sequence $x^{(0)}$'s smooth ration

$$\rho(k)=\frac{x^{(0)}(k)}{x^{(1)}(k-1)}$$

With the increase of k, finally, $\rho(k)$ will gradually approach 0, so to make $x^{(1)}$ with quasi - exponential law.

Quasi

Make $z^{(1)}$ as $x^{(1)}$'s adjacent mean generates an array. In other words $\forall k$,length of interval b-a<0.5,just guarantee $\rho(k)\epsilon(0,0.5)$ is OK,now sequence $x^{(1)}$'s grade ratio $\sigma(k)\epsilon(1,1.5)$.

In the actual modeling, we need to calculate $\rho(k)\epsilon(0,0.5)$'s proportion. In this article, if the whole $x^{(0)}(k)$'s smooth ration less than 0.5 's proportion exceeds 60%,when removal $\rho(2)$ and $\rho(3)$'s smooth ration less than 0.5removal 90%,it is considered that the data meet the requirements of the quasi-exponential law.

After testing, gold and bitcoin any continuous 30 days of trading prices meet the quasi-exponential law.

Prediction Model Training Here, we test the model with 30 groups of data per ounce of gold from September 12 to October 21, 2016 and 30 groups of data per ounce of bitcoin from September 11 to October 10, 2016. Each time, we observe the prediction results of the model for the next three days and observe the fitness of the model for data.

Here we show the prediction results of the next ten days in Figure 6 and Figure 7.

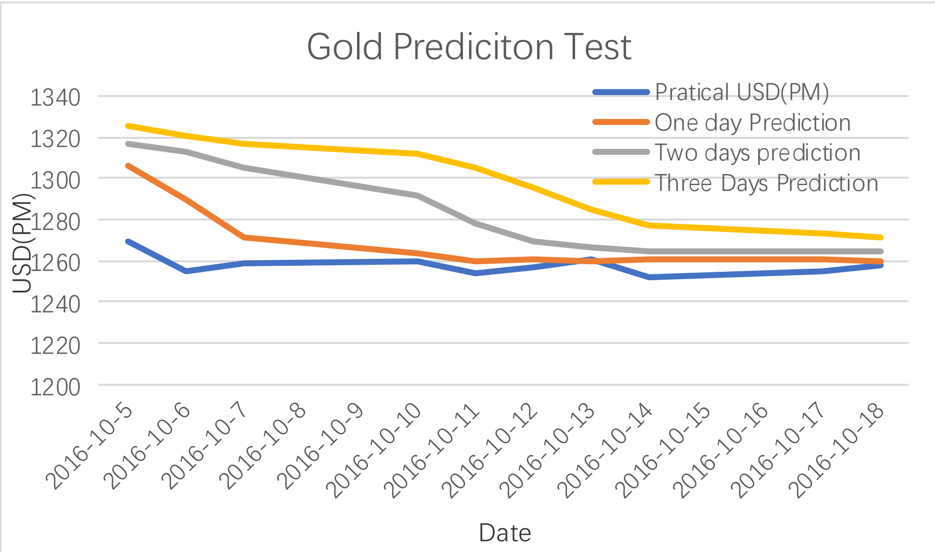


Figure 6: Gold prediction results of the next ten days

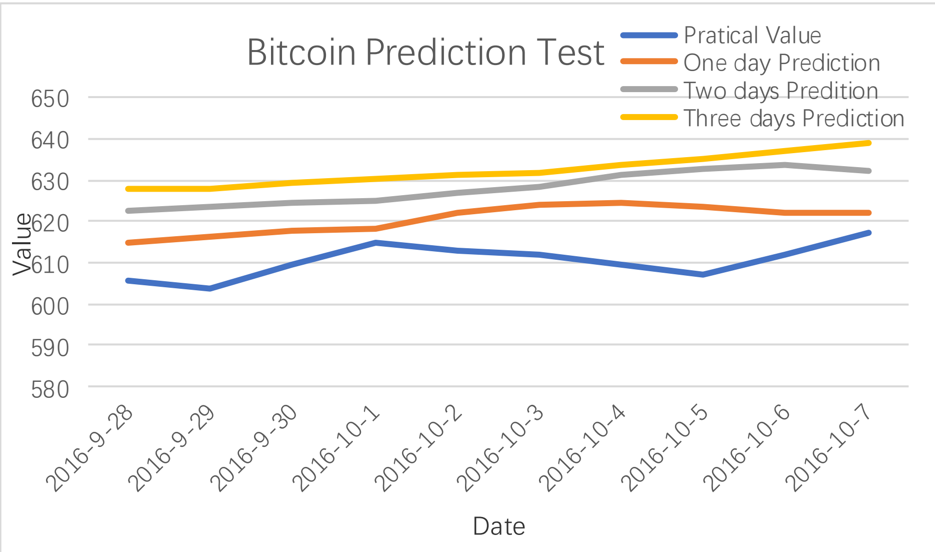


Figure 7: Bitcoin prediction results of the next ten days

It can be seen that the prediction effect of gold is better than that of bitcoin, and their errors are acceptable.

Solution of prediction results If you take the initial value $\hat{x}^{(1)}(t)|_{t=1}=x^{(0)}(1)$,the corresponding solution can be obtained as

$$\hat{x}^{(1)}(t)=\left[x^{(0)}(1)-\frac{b}{a}\right]e^{-a(t-1)}+\frac{b}{a}$$

Then it can be known that $GM(1,1)$ model $x^{0}(k)+az^{(1)}(k)=b$'s solution is

$$\hat{x}^{(1)}(m+1)=\left[x^{(0)}(1)-\frac{b}{a}\right]e^{-am}+\frac{b}{a},m=1,2,\cdots,l-1$$

From the above mathematical formulas $x^{(0)}$'s value of simulation is:

$$\hat{x}^{(0)}(m+1)=\hat{x}^{(1)}(m+1)-\hat{x}^{(1)}(m)=(1-e^{a})\left[x^{(0)}(1)-\frac{b}{a}\right]e^{-am},m=1,2,\cdots.l-1$$

If the original data need to be predicted, as long as the above formula is taken $m\geqslant l$.

In this paper, each time gold and bitcoin are backward to predict the closing price of three days, so l 's value is 4.

(3) model improvement

With Grey Forecast Model's development, without conventional GM (1,1) model, also derived some other's GM (1,1) model. This article also uses New Information GM (1,1) and Metabolism GM (1,1) model to predict the closing price of future gold and bitcoin. closing price forecast.

New Information GM(1,1)is in GM(1,1)model After each prediction of the price of one day, use the price of the new forecast as raw data for the next forecast $x^{(0)}$'s the last one,making $x^{(0)}$ More new prediction information ; Metabolism GM (1,1) really New Information GM (1,1) on the basis of removing $x^{(0)}(1)$,the number of data in the original data column of each prediction is ensured to be constant.

Because of three GM (1,1) for different raw data prediction accuracy is different, to ensure the highest accuracy of prediction model. Each time $x^{(0)}$'s last three groups are used as the test group to calculate the sum of error squares and SSE of each model about the test group. Each time, the minimum group of SSEs is taken as the GM (1,1) model used for the day prediction.

5.3. Model calibration

Use $GM(1,1)$ when predicting the data, the following two test methods can be used to test the prediction results.:

error check: the definition of residual is as follows:

absolute residual: $\varepsilon(k)=x^{(0)}(k)-\hat{x}^{(0)}(k),k=2,3,\cdots,n$

relative residuals: $\varepsilon_r(k)=\dfrac{|x^{(0)}(k)-\hat{x}^{(0)}(k)|}{\hat{x}^{(0)}(k)}\times 100\%,\ k=2,3,\cdots,n$

Here, we take the residual error of the price prediction one day after the closing price of gold as an example to illustrate the analysis process of the residual error.

First, we draw the distribution histogram of the residual sequence in Figure 8,

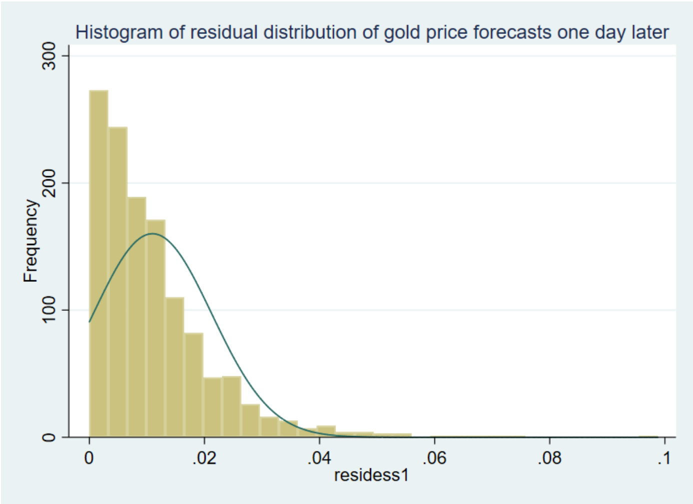


Figure 8: Gold distribution histogram of the residual sequence

Then, we use the Ljung-Box Q test to test the validity of the prediction model, and the test results are shown in the Table 5:

Table 5: Results of Ljung-Box Q test

```
Portmanteau test for white noise
---------------------------------------
 Portmanteau (Q) statistic =   952.0217
 Prob > chi2(12)           =     0.0000
```

According to the results of observation, it is not difficult to find that the significant level of the test value with lag of 12 items is p-value = 0 less than 5 %, and we can safely reject the null hypothesis of "no serial autocorrelation".

We also used LM statistics $\varepsilon_r(k)$ to test whether there is GARCH error to evaluate the accuracy of the prediction results. Since gold is traded only on weekdays, we construct the following regression equation

$$\varepsilon_r^2(k)=a_0+a_1\varepsilon_{r-1}^2(k)+a_2\varepsilon_{r-2}^2(k)+a_3\varepsilon_{r-3}^2(k)+a_4\varepsilon_{r-4}^2(k)+a_5\varepsilon_{r-5}^2(k)+u_t$$

Null hypothesis $H_0: a_1=a_2=a_3=a_4=a_5=0$,significance level $\alpha=0.05$
If the original assumption is true, then we do not think there is GARCH error, otherwise there is.

Table 6: Results of GARCH regression

Source	SS	df	MS			
				Number of obs	=	1,250
				F(5, 1244)	=	152.64
Model	.000134619	5	.000026924	Prob > F	=	0.0000
Residual	.000219431	1,244	1.7639e-07	R-squared	=	0.3802
				Adj R-squared	=	0.3777
Total	.00035405	1,249	2.8347e-07	Root MSE	=	.00042

Results in Table 6 from Stata regression we can know that $\{\varepsilon_r^2(k)\}$ The F-statistic of the original assumption that all coefficients of the lag value are equal to 0 is equal to 152.64, When the molecular degree of freedom is 5 and the denominator degree of freedom is 1244, p-value is 0.000. In addition, LM statistics considering regression are obtained in Stata:

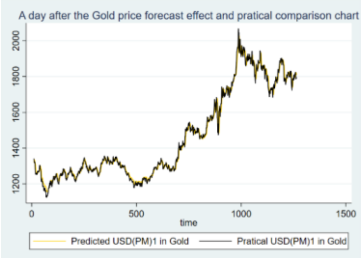
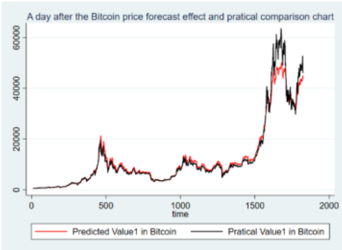
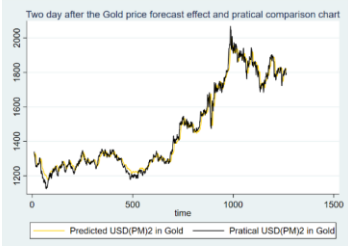
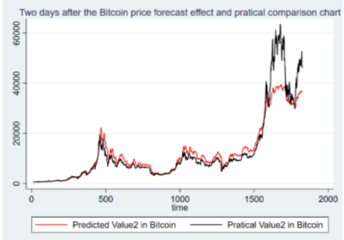
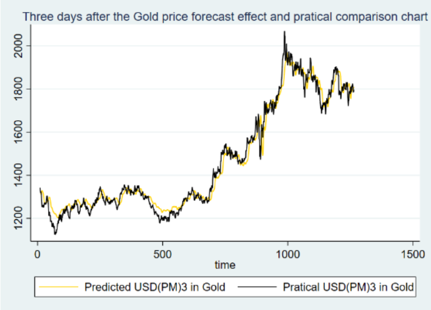
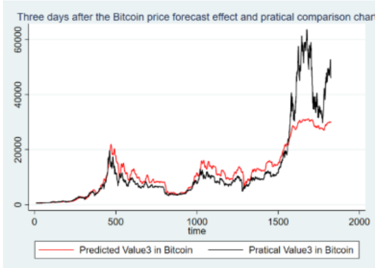
LM=475.28372   p-value=1.72e-100

Because p-value is far less than 0.05, we reject the original assumption that there is GARCH error in the price of gold one day after the closing price.

5.4. Results of Model I

According to Model I, the results of one day, two days and three days forecast for gold and bitcoin are summarized in Table 7.

Table 7: Results of Model I

	Gold’s USD(PM) Forecast	Bitcoin Value Forecast
One day after		
Two days after		
Three days after		

6. Model II: Improved Trend Tracking Strategy Based on Genetic Algorithm

The most important transaction strategy is to determine when to buy and when to sell. In order to solve this problem, many trading strategies are proposed, such as futures index, mean recovery strategy, trend tracking strategy, among which trend tracking strategy is the most common. How to judge whether there has been a rising or falling trend is the key to the effectiveness of the trend tracking strategy. [1]

6.1. Trend Tracking Strategy Based on Genetic Algorithm

Trend tracking strategy is based on the price trends over the past period of time to determine whether to buy or sell. However, in the implementation process of trend tracking strategy, it is often difficult to grasp the trend, that is, the date of tracking and the transaction ratio are difficult to determine. At the same time, the traditional trend tracking strategy only relies on the past data, and judges the future trend only through the past rules, without predictability. In the previous paper, we have predicted the future gold price and Bitcoin trend through the grey model. Here, we can combine it with the trend strategy to make up for its unforeseen defects to some extent. At the same time, we use genetic algorithm to assist in exploring the optimal parameters of trend tracking strategy. As a method to solve complex combinatorial optimization problems, genetic algorithm is combined with trend tracking strategy, so that the trend tracking strategy can be continuously optimized according to the actual market situation, and it is also convenient to further determine the number of tracking days and the specific value of transaction ratio.

6.2. Establishment of Trend Tracking Strategy Based on Genetic Algorithm

(1) Creating Process

Genetic algorithm essentially adopts the idea of evolution. In the process of dealing with problems, it often adopts the process of discretization. Suppose the number of evolutions is $t = 1 \cdots T$;Suppose the length is n chromosomes are represented as symbol strings $X = x_1 x_2 \cdots x_n$,In this: $x_i\,(i = 1, 2, \cdots, n)$ symbolizing a genetic gene. All possible values of genetic genes are called alleles; All alleles form the basic space of the solution:

$$A = x_1 \times x_2 \times \cdots \times x_n = \prod{}^{n}{}_{i=1} x_i$$

In this model, we input the gold purchase trend tracking days, ratio, gold sales trend tracking days, ratio, bitcoin purchase trend tracking days, ratio, bitcoin sales trend tracking days, ratio as a set of overall data. Pop ,Substitute in all subsequent genetic algorithm calculations. In genetic algorithm. Be in a certain position t groups of evolutionary times written as $A(t)$;and environmental records corresponding to the number of evolutions $B(t)$,and it is assumed that the environment corresponding to the number of evolutions is $B(t)$ each other is independent;group adaptability to environment is $C(t)$;natural selection and genetic mechanism d_t is new solution set generated under action is $A(t+1)$;

$$A(t+1) = d_t(A(t), C(t))$$

The fittest survived, the weak eliminated. Environment plays a very important role in evolution. So, we should put groups in a dynamic environment for further research. The above model only considers that the population ' s adaptability to the environment corresponding to the current number of evolutions is unreasonable, so we introduce a new variable $E_B(t) = \langle C(1), C(2), \cdots, C(t-1) \rangle$

This reflects the historical information of population and environment dynamic adaptation before the evolution times. Therefore, the above model is changed into

$$A(t+1) = d_t(A(t), C(t), E_B(t))$$

In this formula, $E_B(t+1)$'s Generation reflects the processing methods of natural selection and genetic mechanism in the natural evolution of populations. It is worth noting that the genetic mechanism contains many different mechanisms, in this model, we use two special genetic mechanisms of hybridization and mutation. Hybridization, that is, different data may be exchanged, so that the whole data group can be combined with each other for combinatorial optimization. Mutation, that is, the data may be mutated from one number to another. Mutation ensures the richness of the data, so that the genetic algorithm will not fall into local optimum. It can introduce new variables so that the genetic algorithm is not limited by the initial input data.

$$E_B(t+1) = d_t(E_B(t), C(t))$$

The generation of new solution set can be understood as a random process. For finite solution set space, $X = [x_1, x_2 \cdots, x_i, \cdots x_n]$, $n = |X|$,when $A(t) = x_j (j = 1, 2, \cdots, n)$,natural selection and genetic mechanism d_t is generating new solution sets under action $A(t+1) = \{x_q | q = 1, 2, \cdots, n\}$,probability is

$$P(t) = \left\{ p_{i,q}(t) | \sum_{q=1}^{n} p_{i,q}(t) = 1 \right\}$$

The above model can be rewritten to

$$P(t+1) = dt(A(t), C(t), E_B(t))$$

Population $A(t)$ for environment $B(t)$ is adaptive measures are generally represented by real numbers greater than zero, expressed as a

$$\mu_B(A(t)) = \mu_{B,t}(A(t), B(t))$$

Quorum: $\mu_{B,t}$ indicates that t Population of evolutionary times $A(t)$ for environment $B(t)$ level of adaptation。 Here, we take the previous data Pop as input, and bring the trend tracking strategy to calculate the final return. Here, we simulate the data provided between 2016 and 2021 according to the results of each group of data, and observe the changes in earnings as a measure of the suitability of the group of data. It is worth noting that the trend tracking strategy we use here considers the previous prediction results, and the prediction results are also part of the trend tracking. Once the group and environment have changed, the ability of the group to adapt to the environment will also change accordingly, and the adaptability measurement function will also change accordingly.

$$\mu_{B,t+1} = d_t(\mu_{B,t}, A(t), C(t))$$

In t the adaptability of evolutionary population to environment $C(t)$ can be expressed as:

$$C(t) = \mu_{B,t}(A(t))$$

With the continuous iteration of the data, the final population and fitness will gradually converge, that is, the optimal strategy for trend tracking we need.

(2) Model Solving

In the process of solving, we determined a total of seven groups of data, including initial population size, problem solution interval, binary encoding length, iteration number, hybridization rate, selection rate and mutation rate. The initial population is randomly generated, and the range is within the problem solution interval. Figure 9 shows the process of Genetic Algorithm.

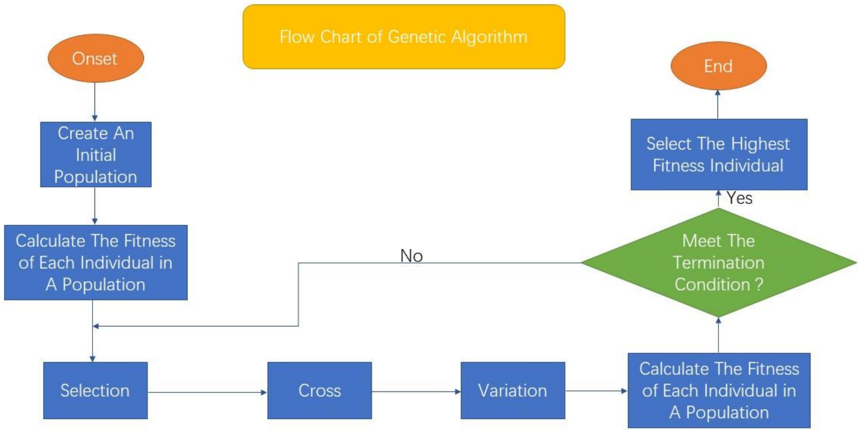


Figure 9: Flow Chart of Genetic Algorithm

Firstly, the initial population is substituted into the fitness calculation to obtain the fitness generated by each group of data.

Secondly, the population selection and elimination, which we use the roulette selection method.

That is, the fitness value of a part x_i is expressed as $f(x_i)$,The probability of this part being selected is $p(x_i)$,The cumulative probability is $q(x_i)$,The corresponding calculation formula is as follows ::

$$p(x_i) = \frac{f(x_i)}{\sum_{j=1}^{N} f_j}$$

$$q(x_i) = \sum_{j=1}^{i} p(x_j)$$

(1) Calculate the probability of each individual being selected $p(x_i)$

(2) Calculate the cumulative probability of each part $q(x_i)$

(3) Generate an array randomly m, the value range of elements in the array is between 0 and 1, and it is sorted from small to large. If cumulative probability $q(x_i)$ greater than elements in an array m(i),individual x_i is selected,If less than m(i),compare the next individual x_{i+1} until select an individual. Repeated N times。

After that, the results are binary coded, and then simulated mutation and crossover are carried out by generating random numbers, so that the data group generates new offspring.

Finally, the data is decoded to get a new initial population. Repeat the process until it reaches the iteration limit.

(3) Model Result

The Fitness of each iteration and Results of Model II are shown in Figure 10 and Table 8 respectively.

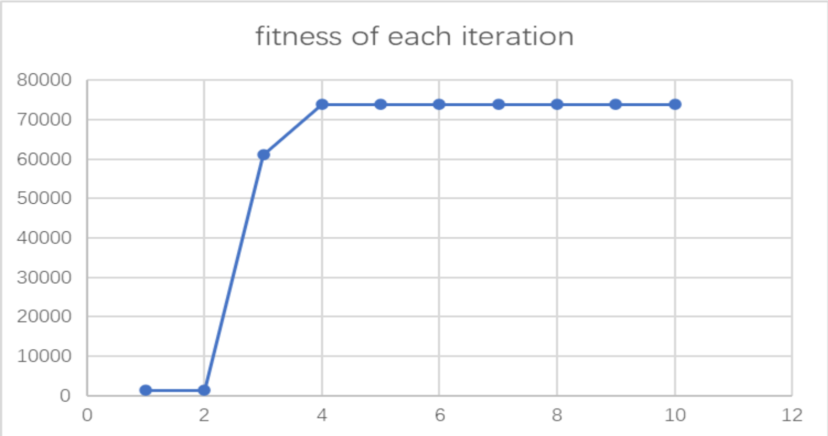


Figure 10: Fitness of each iteration

Table 8: Results of Model II

Gold purchase days	6
Gold purchase ratio	1.14
Gold sold days	7
Gold sold ratio	0.63
Bitcoin purchase days	1
Bitcoin purchase ratio	1.15
Bitcoin sold days	3
Bitcoin sales ratio	0.73

7. Sensitivity Analysis of the Model

Considering that the transaction costs of the market are not necessarily fixed in real life, we consider changing the transaction costs of gold and bitcoin, and conduct a sensitivity analysis of transaction costs, and obtain the following results shown in Figure 11 and Figure 12 respectively.

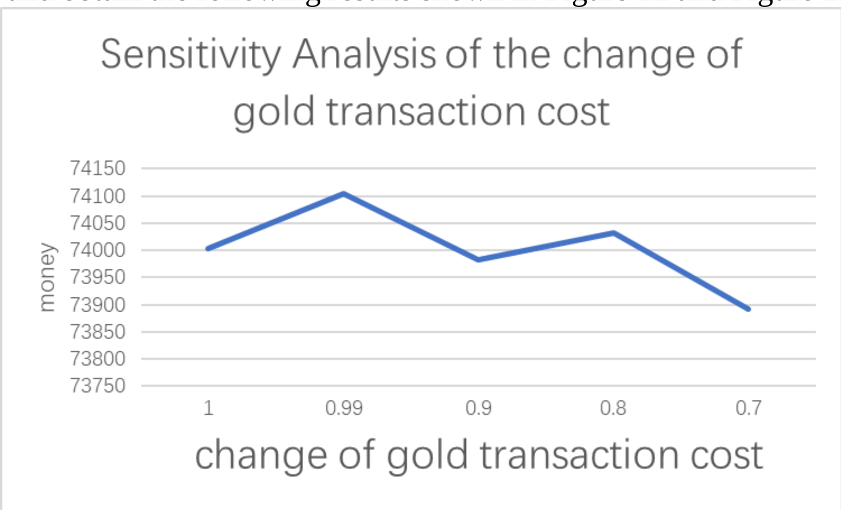


Figure 11: Sensitivity Analysis of the change of gold transaction cost

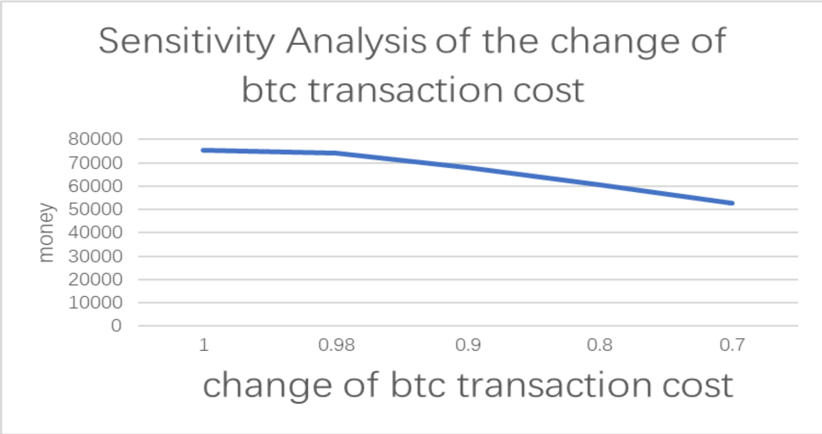


Figure 12: Sensitivity Analysis of the change of btc transaction cost

We can find that changing the transaction cost of gold price has no obvious impact on the final return, while changing the transaction cost of bitcoin can significantly change the final return. It is found that due to the small growth rate and slow growth rate of gold, with the increase of transaction cost, traders will tend to stop gold trading, resulting in no matter how the transaction cost of gold changes, the final results of the income gap is not big; for Bitcoin, its growth rate and growth rate are far higher than those of gold. Unless extreme situations are formed (such as when the transaction cost of Bitcoin is 100 %), traders will eventually choose to buy Bitcoin, resulting in a significant impact of transaction cost on the final result.

8. Model Evaluation and Further Discussion

8.1. Model I Evaluation

(1) Strengths

Samples do not need to be distributed with strong regularity: Compared with traditional time series, GM (1,1) model does not require the sample data to meet the linear hypothesis and stationary hypothesis, and GM (1,1) model has a good prediction effect for the data types such as Financial Products with large fluctuation range, many influencing factors and difficult to quantify.

Check before and after the forecast repeatedly to reduce financial risks as much as possible: Before using the forecast, we did a lot of analysis on the price and change rate of gold and bitcoin to ensure that the trading price of these two Financial Products passed the use conditions of GM (1,1) model. After the forecast, we conducted ljung-box Q test and Q test on residuals to quantify Financial risks as much as possible

Three GM (1,1) models are compared and selected to improve the accuracy of prediction: Different from the traditional grey prediction model, we also used Metabolism GM (1,1) as the New Information GM (1,1) in the prediction. By calculating the sum of error squares of the three models to get the minimum value each time, the prediction model adopted for the day was determined, and the prediction accuracy was further improved by optimizing the prediction.

(2) Weakness

The number of predictions is too small: since GM (1,1) is generally applicable to the analysis of data with a small number of samples, in order to ensure that the accuracy of prediction will not decrease too much, the number of days for backward prediction is very limited at each time. As a result, only the local optimal strategy can be selected in the later planning, but the global optimal strategy cannot be obtained

The data were analyzed and tested before the prediction, which did not conform to the

reality: Before predicting the price of gold and bitcoin, this paper analyzed the data provided by the title in advance. Since traders generally cannot obtain the price of Financial Products to be traded in advance in actual Financial activities, the analysis and test of gold and bitcoin prices before prediction did not conform to the actual situation.

(3) Further Discussion

In this paper, only one prediction model is used to predict the price fluctuations of gold and bitcoin, and it is difficult to avoid financial risks in some financial emergencies. In actual financial activities, some financial time series prediction models based on machine learning algorithms can be added to the original basis, such as RNN neural network and LSTM neural network, so as to avoid financial risks by comparing various prediction models.

In this paper, only financial risks are described, and no financial risks are added into the later planning model as the basis for decision making. Multiple regression analysis can be used to establish the functional relationship between investment return, price change rate and risk coefficient, providing more reference for decision-making.

8.2. Model II Evaluation

(1) Strengths

Avoid falling into local optimum: Genetic algorithm is adopted to make trend tracking strategy get rid of the large amount of calculation and falling into local optimum that the traditional optimization algorithm may produce.

Comprehensive Prediction model: Grey prediction model is added to the traditional trend tracking strategy, making trend tracking strategy not limited to the summary of the past trend, which is more scientific.

(2) Weaknesses

Difficult to obtain the global optimal solution: only single trend tracking strategy is considered in the strategy selection, and the optimal achieved is only the optimal result under the trend tracking strategy, which cannot be guaranteed to be the optimal solution in the complete sense.

Large amount of calculation: Due to the need to bring the data into the simulated transaction, the data scale is large, the computer calculation takes a long time, and even there is a certain risk of collapse.

Genetic algorithm has randomness: Genetic algorithm has a certain randomness, and the results of each operation are different from each other to a certain extent, which can not be completely accurate to the theoretical limit.

(3) Further Discussion

This paper only considers the theoretical conditions combined with historical data to obtain the best model method, often a large amount of cash investment. In actual financial activities, a certain proportion of cash is often needed to avoid risks, so a certain proportion of cash surplus can be set in the model to enable traders to have a certain ability to resist risks.

In addition, this paper only considers the price fluctuations of gold and Bitcoin. In real life, the value of cash also fluctuates with market changes. Historical data of cash value in the past can be found and a prediction and planning model of three Financial Products can be established.

9. Conclusion

9.1. Result of Problem 1

On the premise that traders trade in accordance with the trading strategy we gave them every day, we find that the revenue eventually converges to about $75,000 and there is no obvious fluctuation, which can be considered as the optimal result.

9.2. Result of Problem 2

Since the problem-solving process of this paper mainly starts from prediction and planning, we also prove that our model provides the best strategy respectively from these two models.

Prediction Model

We artificially added some perturbations to the gold and bitcoin prices given in the question to see if the accuracy of the prediction could be guaranteed when the data fluctuated.

The Table 9 and Table 10 is obtained by analyzing the residuals of the gold and Bitcoin predictions after three days.

Table 9: Residuals of the gold predictions after three days.

Gold	1(No disturbance)	2	3	4	5	6	7	8	9	10
average of residual	0.161	0.257	0.244	0.230	0.181	0.261	0.198	0.189	0.231	0.172
variance of residual	0.115	0.154	0.157	0.174	0.214	0.251	0.294	0.335	0.342	0.377

Table 10: Residuals of the Bitcoin predictions after three days.

Bitcoin	1(No disturbance)	2	3	4	5	6	7	8	9	10
average of residual	0.020	0.025	0.034	0.034	0.041	0.051	0.052	0.056	0.060	0.065
variance of residual	0.018	0.021	0.021	0.021	0.025	0.030	0.030	0.031	0.034	0.036

As can be seen, even if artificial perturbations are added to the raw data, the effect of prediction is not greatly affected. Therefore, it can be considered that prediction model decisions provide accurate prediction results.

Trend-following model

To prove the optimality of the model, we provide the highest investment value that can be generated under the current forecast. We can consider adding some perturbations to the decision scheme and show that the result of the perturbation is no longer the highest investment value. The results of multiple tests are shown in Table 11:

Table 11: Results of multiple tests

	1(No disturbance)	2	3	4	5	6	7	8	9	10
Returns	74857	74379	74038	73915	73549	72563	72423	71568	70878	70461

It can be seen that the investment value of gold and bitcoin trading schemes after adding volatility is not as good as that before adding volatility. It can be considered that the trend-tracking model adopted by us provides the scheme with the highest economic value creation.

9.3. Result of Problem 3

We conducted sensitivity analysis on the transaction costs of gold and Bitcoin. According to Chapter 7 of this paper, it can be seen that when the transaction costs of gold increase, the transaction times of gold decrease significantly, but when the transaction costs of Bitcoin increase, the transaction times do not decrease significantly.

See Chapter 7 above for specific analysis results.

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