

Research on Shape Adjustment of “FAST” Active Reflector

Jifeng Zhao^{1,*}, Tian Liang², Yan Gao³

¹*Department of Environmental and Safety Engineering, Qingdao University of Science & Technology, Qingdao 266042, China*

²*Department of Chemical Engineering Institute, Qingdao University of Science & Technology, Qingdao 266042, China*

³*Department of Chemistry and Molecular Engineering, Qingdao University of Science & Technology, Qingdao 266042, China*

**Corresponding author: 1463410660@qq.com*

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Abstract: In this paper, we investigate the shape adjustment problem of “FAST” active reflectors. Specifically, taking the reference sphere center C as the coordinate origin, establishing a space rectangular coordinate system, establishing a single-objective optimization model, and using the A* algorithm to solve it through Matlab. Secondly, through the coordinate rotation transformation, the rotated working paraboloid is obtained, and a plane is determined by using the intersection of the projection parabola and the projection of the reference spherical surface and the direction of the plane normal vector, and by traversing all nodes, finding out that each node is below this plane. The position coordinates of the nodes are obtained, and the number of the main cable node and the expansion and contraction amount of each actuator are obtained. Finally, the paraboloid and the feed cabin are projected onto the yCz plane for analysis. According to the critical conditions of electromagnetic wave reflection, two critical reflection parabolas are determined, and the number of nodes that satisfy this slope range is searched through Matlab to obtain the receiving ratio of the feed cabin.

1. Research Background

FAST consists of an active reflector, a signal receiving system (feed cabin) and related control, measurement and support systems. It is an adjustable spherical surface composed of main components such as actuator and supporting structure. The main cable net is composed of flexible main cables according to the geodesic triangular grid, which is used to support the reflective panel (including the back frame structure). Each main cable node is connected to a pull-down cable, and the lower end of the pull-down cable is connected to the actuator fixed on the surface to realize the shape control of the main cable network. There is a certain gap between the reflective panels, which can ensure that the reflective panels will not be deformed by being squeezed or pulled during displacement.

The active reflector can be divided into two states: the reference state and the working state. In

the reference state, the reflecting surface is a spherical surface with a radius of about 300 meters and a diameter of 500 meters (reference spherical surface). In the working state, the shape of the reflecting surface is adjusted to an approximate rotating paraboloid (working paraboloid) with a diameter of 300 meters. The effective area of the feed cabin to receive signals is a central disk with a diameter of 1 meter. When FAST observes the celestial target S in a certain direction, the center of the receiving plane of the feed cabin is moved to the intersection P of the straight line SC and the focal plane, and the partial reflective panel on the reference spherical surface is adjusted to form the line SC as the symmetry axis and the focal point of P is an approximate rotating paraboloid, so that the parallel electromagnetic wave reflections from the target celestial body are concentrated into the effective area of the feed cabin.

Adjusting the reflective surface to a working paraboloid is the key to the active reflective surface technology. The length of the drop cable is fixed. The actuator is installed radially along the reference spherical surface, its bottom end is fixed on the ground, and the top end can expand and contract radially along the reference spherical surface to complete the adjustment of the pull-down cable, thereby adjusting the position of the reflective panel, and finally forming a working paraboloid.

In this paper, an ideal paraboloid is determined under the adjustment constraints of the reflective panel, and then the reflective surface is adjusted to a working paraboloid by adjusting the radial expansion and contraction of the actuator, so that the paraboloid is as close to the ideal paraboloid as possible, and the best receiving effect of celestial electromagnetic wave reflected by the reflector can be obtained.

2. Single-Objective Optimization Model

2.1. Solving the paraboloid equation of revolution

Taking the center C of the reference sphere as the coordinate origin, a space rectangular coordinate system is established, and the paraboloid of revolution is projected onto the plane to obtain a parabola^[1], as shown in Fig.1.

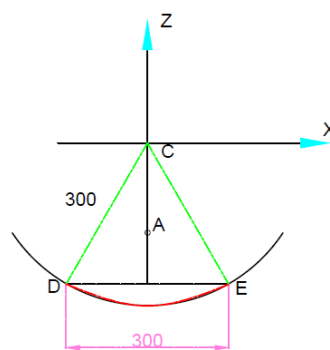


Figure 1 Projection of a paraboloid of revolution

In order to more clearly show the relationship between the ideal paraboloid and the static spherical surface, Fig. 1 is enlarged to obtain as shown in Fig. 2.

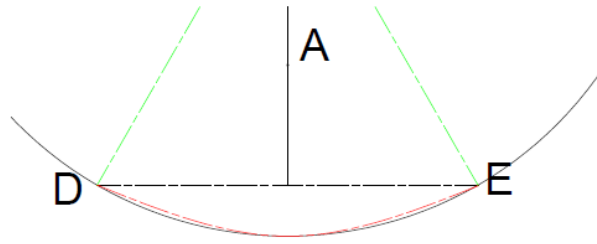


Figure 2 Partial enlarged view of the paraboloid of revolution

The focus A is on a sphere with C as the center and a radius of 300, and CDE is an equilateral triangle with side length 300. According to the geometric relationship, the coordinates of the focus point are (0, -139.8), the coordinates of a point D on the parabola can be determined according to the pythagorean theorem, and the coordinates of point D are (-150, -150√3).

Let the parabola equation be $x^2 = 2pz + c$, the value of p is 279.6 determined by the focal length, bring the coordinates of point D into the parabola equation, and solve $c=12587140.2$, the parabola equation is:

$$z_0 = \frac{x^2}{559.2} + 300.03 \quad (1)$$

Rotating the projection parabola around the axis z to obtain the required paraboloid of revolution. The equation of paraboloid of revolution is:

$$z_0 = \frac{x^2+y^2}{559.2} + 300.03 \quad (2)$$

The image of the rotating paraboloid is shown in Fig. 3.

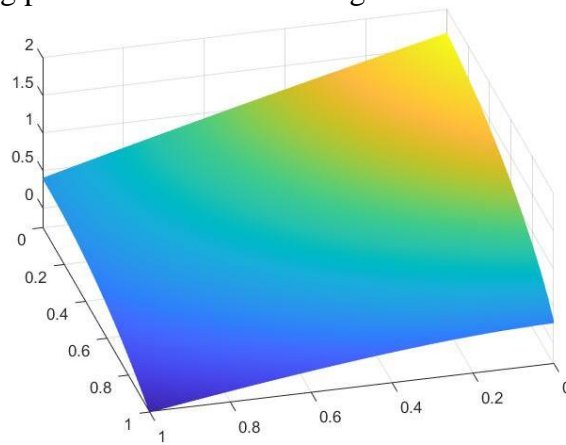


Figure 3 Rotating Paraboloid Image

2.2 Establishment and solution of single-objective optimization model

The radial expansion and contraction range of the top of the actuator is -0.6 to +0.6 meters, that is:

$$-0.6 \leq \Delta d \leq 0.6 \quad (3)$$

where Δd is the radial expansion and contraction of the actuator tip, $\Delta d = \sqrt{(z - z_0)^2 + (x - x_0)^2}$

The shape of an ideal paraboloid should satisfy the equation of an elliptical paraboloid, which

can be expressed as:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 300.3 = z^2 \quad (4)$$

The shape of the ideal paraboloid is as close as possible to the paraboloid of revolution, which can be transformed into the smallest square of the difference between the ideal paraboloid and the paraboloid of revolution at any point, which can be expressed as:

$$\min[z(x, y) - z_0(x, y)]^2 \quad (5)$$

where $z(x, y)$ is the desired ideal paraboloid, and $z_0(x, y)$ is the rotating paraboloid that the ideal paraboloid should approach.

The shape of the ideal paraboloid should satisfy the parameters in the equation of the elliptic paraboloid as the decision variable, and the minimum square of the difference between the ideal paraboloid and the rotating paraboloid is the objective function, and the single-objective optimization model is established. The model is:

$$\min[z(x, y) - z_0(x, y)]^2 \quad (6)$$

$$\text{s. t. } \begin{cases} -0.6 \leq \Delta d \leq 0.6 \\ \frac{x^2}{a^2} + \frac{y^2}{b^2} - 300.3 = z^2 \end{cases} \quad (7)$$

The value of $[z(x, y) - z_0(x, y)]^2$ can only be calculated by giving a certain point at a certain position. The value of the optimal solution a and b obtained at this time can only make the ideal paraboloid close to the rotating paraboloid at this point, but at other positions in the space, finding the obtained ideal paraboloid may have a large gap with the rotating paraboloid image, that is, the obtained a and b are a local optimal solution. In order to find the global optimal solution, this paper uses the A^* algorithm to solve the optimization model [5].

Finally, the ideal paraboloid equation obtained by the model solution is:

$$z = \frac{x^2}{94037} + \frac{y^2}{0.0093} - 300.3 \quad (8)$$

The analysis results obtained are shown in Fig. 4.

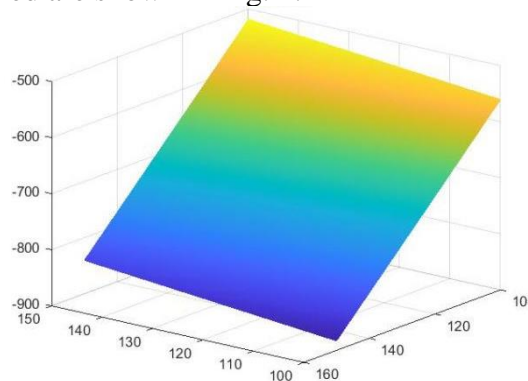


Figure 4 Obtained ideal paraboloid results

Residual analysis is performed on the results of the model solution. For the different values of a and b, the residual results are shown in table 1.

The residual calculation formula is $\hat{\varepsilon} = z - \hat{z}$, it can be seen from table 1 that the given a and b values are 2.39, and the residual is small. Through the residual analysis and the drawn ideal paraboloid image and the unadjusted constrained rotating paraboloid image are synthesized by comparison, it can be shown that the ideal paraboloid obtained is better.

Table 1 Changes in the mean value of residuals caused by different groups a & b

serial number	(a, b)	residual mean
1	(0.00326, 10.39)	2.39
2	(0.0187, 4.33)	20.59
3	(0.945, 5.37×10^{-6})	188
4	(0.00223, 2.20)	55
5	(0.00253, 3.15)	24.8

The center of the position of the feed cabin can only move on a spherical surface that is concentric with the reference spherical surface, so the line segment CF and CF' are the radius of the focal plane, and the radius of the focal plane is set as r , then CFF' is an isosceles triangle with waist length r and vertex angle θ , as shown in Fig. 5.

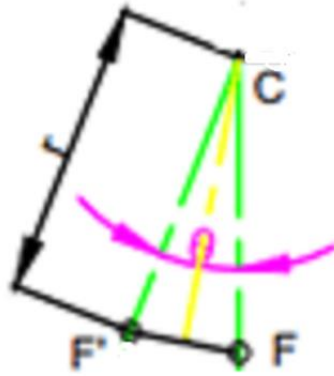


Figure 5 Partial enlarged view after parabolic rotation

Point F is the focal point before the parabola does not rotate, the coordinate of point F is $(0, -139.8)$, r is the radius of the focal plane, which is 139.8. θ is the rotation angle of the parabola, and the size of the rotation angle is 9.5224° . Using the geometric relationship, the F coordinate can be obtained as $(-r \sin \theta, -r \cos \theta)$, which is $(-23.13, -137.87)$.

The parabola focus and the parabola vertex are on the same line, so the angle between the line connecting the parabola focus and the parabola vertex and the z -axis is the angle between CF and the z -axis, which can be expressed as: $r = (90^\circ - \theta) = 80.04776^\circ$.

The eccentricity of the parabola $e=1$, bring $g = -23.13, h = -137.87, \gamma = 80.04776^\circ, p = 139.8$ into the unified equation of the plane conic curve, and determine the equation of the parabola at this time as:

$$\left| (x+23.13)\cos 80.04776^\circ + (z+137.87)\sin 80.04776^\circ + 139.8 \right| - \sqrt{(x+23.13)^2 + (z+137.87)^2} = 0 \quad (9)$$

The paraboloid of revolution is formed by rotating the projection parabola around its symmetry axis, then the equation of the trajectory circle of the point rotating around the straight line is:

$$\begin{cases} x^2 + y^2 + z^2 = x_1^2 + z_1^2 \\ wx + ky + tz = wx_1 + tz_1 \end{cases} \quad (10)$$

M point is a moving point on the straight line, then the total space formed by the whole rotating circle is set as Π , and Π can be expressed as:

$$\begin{cases} x^2 + y^2 + z^2 = x_1^2 + z_1^2 \\ wx + ky + tz = wx_1 + tz_1 \\ x_1 = g(i) \\ y_1 = 0 \\ z_1 = h(i) \end{cases} \quad (11)$$

The equation satisfied by the paraboloid of revolution can be obtained by taking the relevant parameters into the simplification, and the coordinates of the vertex can be obtained by using the paraboloid equation of revolution as (-49.3406,-36.9048,-293.9109).

According to the information^[2], after a point in space rotates $\left(\frac{\pi}{2} - \beta\right)$ counterclockwise around the y-axis, and then rotates α clockwise around the z-axis, the relationship between the new coordinates (x, y, z) and the old coordinates (x', y', z') is:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \cos \alpha & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (12)$$

By performing the above coordinate transformation on the obtained ideal paraboloid, the working paraboloid equation can be obtained as:

$$z = 3.82x + 2.6y - 3642 \times \sqrt{(-0.002x^2 + 0.0055xy + 0.0021x - 0.004y^2 + 0.0016y + 1.123) + 3564.6} \quad (13)$$

Let the vector (m, n, p) be the direction vector of the straight line l_{cs} , then from the geometric relationship, the coordinate of any point on the straight line l_{cs} is $(s \cos \beta \cos \alpha, s \cos \beta \sin \alpha, s \sin \beta)$, where s is the distance from the coordinate origin C to the point, so the direction vector of the straight line is $(s \cos \beta \cos \alpha, s \cos \beta \sin \alpha, s \sin \beta)$. By the properties of the paraboloid, it can be seen that the direction vector of the straight line l_{cs} is the normal vector of the desired plane, and the coordinates of the intersection of the projected parabola and the projection of the reference sphere are (x_0, y_0) . Therefore, the equation of the desired plane is $m(x - x_0) + n(y - y_0) + mpz = 0$. By bringing in the relevant parameters, the final plane equation is :

$$42.67x + 32.93y + 254.28z + 6403 = 0 \quad (14)$$

Traverse all nodes on the search sphere, bring the abscissa and ordinate of the node into the equation of the plane, determine whether the node is under the plane, search for all the nodes under the plane, determine its number by the coordinates, and then find its Actuator retraction amount.

3. Acceptance Ratio of Feed Cabin

3.1. Determination of critical reflection parabola

According to the knowledge of electromagnetic wave reflection, two critical reflection parabolas can be determined, that is, when the y value is the same, the slope of the point on the working parabola should be between the slopes of the corresponding points of the two critical reflection parabolas^[3]. The focus of the two critical reflection parabolas can be determined by the electromagnetic wave reflection relationship and the definition of the parabola, and then the equations of the two critical reflection parabolas can be determined, as shown in Fig. 6.

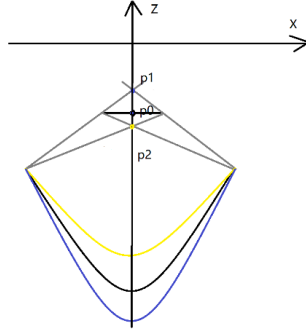


Figure 6 Schematic diagram of critical reflection parabola confirmation

where p_0 is the position of the feed cabin, p_1 is the focus of the lower critical parabola, and p_2 is the focus of the upper critical parabola.

The upper and lower critical reflection parabola equations are obtained as:

$$\begin{aligned} z_{11} &= \frac{y^2}{280} + 254950 \\ z_{12} &= \frac{y^2}{279.9} + 25512 \end{aligned} \quad (15)$$

3.2. Solving the coordinates of the “inflection point” on the projected parabola

The projection parabola is a parabola determined by a number of nodes. On the right half of the projection parabola ($y > 0$), the adjacent nodes projected on the parabola are connected by a straight line. At this time, the slope of the straight line represents the space of the mirror corresponding to the node. In the left half branch of the projection parabola ($y < 0$), taking the projection of the midline of the mirror where the node is located through the node on this plane in space, and find the slope of this projection to describe the inclination of the mirror corresponding to the node in space^[4].

3.3. Search all nodes to find nodes that satisfy the slope constraints

Taking any node on the projection paraboloid, set the coordinate as (y_0, z_0) , finding the slope k of the straight line describing the inclination of the mirror in space corresponding to the node, and the slopes k_{11} and k_{12} at the ordinate y_0 , if $k_{11} < k < k_{12}$, record the node. If the node does not satisfy $k_{11} < k < k_{12}$, another node is randomly selected, and the process is repeated until all nodes are traversed. Calculating the ratio of all nodes that meet the conditions to the total number of nodes, that is, to find the receiving ratio.

The model is solved by Matlab, and the final result of the solution is that the receiving ratio of the reflection surface within the 300-meter aperture is 49.3%. That is to say, the ratio of the reflected signal received in the effective area of the feed cabin to that of the reflection surface within the 300-meter aperture is 49.3%, and the receiving ratio of the reference sphere is 17.3%, that is, the reflected signal received in the effective area of the feed cabin is the same as that of the 300-meter-diameter reflected signal. The ratio of the reflected signal of the reflecting surface inside the aperture is 17%.

4. Conclusion

In this paper, the difference of the three-dimensional surface function values is used as the objective function to obtain the equation of the ideal paraboloid which is in line with the reality. In addition, the equation of the ideal paraboloid after the azimuth of the star has been changed is successfully obtained by using the method of space coordinate transformation, which greatly reduces the amount of model calculation. At the same time, the working paraboloid equation that best approximates the ideal paraboloid is successfully obtained by using the idea of A* algorithm. Finally, the vertex coordinates of the reflection panel are the coordinates of the main cable node, that is, a reflection panel can correspond to a main cable point, so that the signal ratio is accurately quantified and a more reasonable result is obtained.

References

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