

A Study on the Allocation Strategy of Hydroelectric Power in Five Continents under Linear Planning

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Abstract: The balance of hydroelectric power generation has a large impact on the realization of economic, social and other benefits maximization. In this paper, we analyzed the demand of five continents, the supply of water from river dams, and combined with the maximization of benefits to make a reasonable planning of water resources. We performed a sensitivity analysis, considering the effect of the parameters in the model on the satisfaction, and wrote summary articles for managers, explaining the results and giving our recommendations.

1. Introduction

1.1 Problem Background

With the continuous development of science and technology, people's requirements for economy are also increasing, among which the hydropower problem is the top priority. However, in recent years, with the change of climate, there are more and more dry weather, which leads to the situation of "short supply of water resources" in many areas. In this case, the government has also established a reservoir to solve the problem through water diversion. Therefore, the rational allocation of water resources and maintaining the balance of hydropower have become the most important issues to be considered at present.

1.2 Restatement of the problem

This is a problem of hydropower balance. We need to reasonably allocate the water regulation according to the water storage and power generation capacity of Colorado River, Glen Canyon Dam and Hoover Dam, connecting with the water and power demand of agriculture and industry on five continents, so as to achieve the goal of maximizing benefits.

For the first question of question 1, we need to develop a model and calculate it under normal conditions (with fixed water supply and demand conditions). That is: when the water level of the lake Meade is M and the water level of Powell lake is P , how much water should allocate from these two lakes to meet the demand. For the second question of question 1, we should use the newly developed model under dynamic conditions, with the limitation that the demand is fixed and no

additional water is provided. But the water consumption is constantly changing. We need to calculate how long it will take to meet the demand under this condition. For the third question of question1, we need to calculate: with the time going, according to the model, how much additional water must be supplied to ensure that the fixed demand is met.

For problem 2, we need to establish an optimization algorithm to solve the allocation problem of water resources according to the model. We need to consider the agricultural and industrial uses after allocating the water of the five continents, comparing their respective economic and social benefits, so as to calculate the weight and achieve the goal of allocating water resources under the maximization of benefits.

For problem 3, we need to consider how to minimize the loss caused by insufficient water resources, so as to allocate water resources reasonably.

The problem 4 is the specific application of the model. The first question of problem 4 is to evaluate the sensitivity and accuracy of the model over time. It mainly assesses the balance of hydropower generation when the population, agriculture and industry in the areas affected by the two dams grow or shrink. The second question of problem 4 is how to adjust the supply and keep the balance assuming that the proportion of renewable energy technology increases and reduces the burden of dam water supply. The third question of problem 4 is that we need to adjust the demand according to the model to meet the balance after the implementation of more water and power saving measures.

1.3 Literature Review

According to the literature, the current methods of water resources optimization mainly include grey system evaluation [1]and linear programming model. It can be seen from the literature that the structure of each scheme of the grey system evaluation model can not be changed in operation. At the same time, with the time going, the accuracy of the functional relationship between elements will also shift. Therefore, this paper compares the model with the long-term results to select a more appropriate optimization model.

2. Terminology and Definitions as shown in Table 1

Table 1: Notations used in this paper

Symbol	Description	Symbol	Description
a	Five continents total daily water consumption	h_1	Lake Powell has falling water level
b	The amount of water the Glen Canyon Dam uses hourly to generate electricity	h_2	The falling water level of Lake Mead
c	Glen Canyon Dam installed capacity	x_3	The amount of water the Glen Canyon Dam supplies to the Hoover Dam
d	The amount of water the Hoover Dam uses hourly to generate electricity	x_1	The amount of water the Glen Canyon Dam provides to the five states
e	Installed capacity of the Hoover Dam	y_1	The amount of water that the Hoover Dam provides to the five states
f	Five continents total daily electricity consumption	x_2	The Glen Canyon Dam supplies electricity to the five states
g	The Glen Canyon Dam drops the one meter of water lost	y_2	Electricity supplied by the Hoover Dam to the five states
l	The Hoover Dam drops the one meter of water lost	h_{01}	Lake Powell specifies the water level
P	The current water level of Lake Powell	h_{02}	Lake Mead specifies the water level
M	Meade Lake water level		

3. Problem Analysis

In the first part, we use the objective programming model to allocate water, and obtain the optimal solution through the objective function of the degree of hydropower resources. Then on this basis, Monte Carlo is used to generate a random matrix to calculate the relationship between daily water demand and days, so as to obtain the days required to meet the maximum demand. Finally, by debugging the number of days in the model, we can get the additional water supply to meet the fixed demand.

For the second part, the analytic hierarchy process is used to maximize the benefits as the goal, and the weight of water distribution is calculated according to the collected GDP value of each state and the data of industry, agriculture, housing and power production. Then the ideal point method is used to solve the multi-objective programming and find the standard to solve the competitive interest.

For the third part, the method of multi-objective linear programming is adopted. Comprehensively consider the emphasis of water and electricity, and adjust the model coefficient to meet the requirements.

For the fourth part, the optimal solution can be obtained by adjusting the objective programming function.

Finally, we wrote a summary to the water infrastructure managers in the southwest of the United States, giving and explaining our suggestions.

4. Model Assumptions

- 1) It is assumed that in the process of water transmission and electricity transmission, the loss of hydropower is 0.
- 2) It is assumed that the speed of the pump is constant.
- 3) It is assumed that daily precipitation has no effect on daily water demand.
- 4) Ignore the transmission time of hydropower from the two dams to the five states.

5. Models and Solutions

5.1 For Part I

We use the goal programming model to allocate water^[2]. Define an objective function of the degree of hydropower resources obtained by five continents. The greater the degree, the higher the satisfaction of the five states with the allocated hydropower resources, and the linear programming model is obtained as follows:

Objective function:

$$y = \frac{(x_1+y_1)}{a} + \frac{(\frac{x_2}{b} * c + \frac{y_2}{d} * e)}{f} \quad (1)$$

Constraint condition:

$$h_1 \leq P - h_{01} \quad (2)$$

$$h_2 \leq M - h_{02} + x_3/S \quad (3)$$

$$-x_1 - x_2 \leq -a \quad (4)$$

$$-\frac{x_2}{b} * c - \frac{y_2}{d} * e \leq -f \quad (5)$$

$$-h_1g + x_1 + x_2 + x_3 = 0 \quad (6)$$

$$-h_2l + y_1 + y_2 - x_3 = 0 \quad (7)$$

The maximum value in the figure is 58324. It is assumed that a pump with a pumping rate of 80 cubic meters/s is used to pump water to meet the maximum demand. Because there is an overlap in time when pumping water and delivering water to the five states, the time for delivering water to the five states is ignored. Finally, it is calculated that the time required to meet the maximum demand is about 32 days.

The conclusion is as follows: It takes about 32 days to meet the maximum water demand in one day because of the use of one pump. In other words, managers need to deliver water to the five states one month in advance and only meet the demand for one day, so the number of pumps needs to be increased to 32. In this way, managers can deliver water to five continents one month in advance to meet the demand for one month, and if they want to shorten the time, they only need to increase the number of pumps appropriately.

Adjusting the number of days in the model in Question 1 shows that the current water level of the two lakes can only meet the needs of six days in five states. Therefore, to meet the demand for one month, it needs to provide four times more water and electricity, that is, 231,600 million gallons of water and 5,028 million kilowatt-hours of electricity. The final conversion of electricity to water: At least 556,551.2 million gallons of water will be required.

5.2 For Part II

Based on the above single objective programming model, We find the optimal solution to meet the water and electricity demand of all continents, Because every continent is in agriculture, The economic benefits of using water for industry, housing and power generation are different. At the same time, according to the certain tendency in the process of transferring water from Powell Lake and Meade Lake, according to the GDP situation of each continent, the general water use channels such as agriculture, industry and residence in each continent, and the economic benefits generated by power generation, the weight of water use and electricity consumption of each continent and industry is determined by analytic hierarchy process.

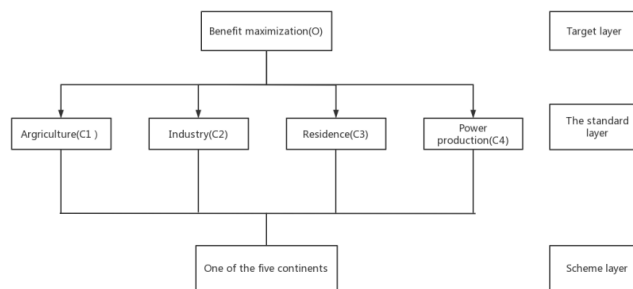


Figure 1 Hierarchical diagram of water distribution planning decision-making

First of all, it is necessary to clarify the decision-making objective (O), After taking the maximization of benefits as our goal layer, We can get four criterion layers, There are four influencing factors in the criterion layer: agricultural water consumption (C1), industrial water consumption (C2), residential water consumption (C3) and electric power water consumption (C4). In the scheme layer, we can take a state as an example to study, and constantly adjust the influence

degree of the criterion layer on the target layer, so as to get the water distribution data of different states and even five continents[3]. As shown in Figure 1.

At the same time, according to the US nominal GDP data provided by the US Bureau of Economic Analysis, we have also compiled the ranking and specific values of the US nominal GDP in 2020, and the table is as follows Table 2.

Table 2 The 2020 US Substate nominal GDP(Only five continents are studied here)

State area	Nominal GDP (US \$100 million)	Rank
AZ	30918.72	1
CA	3900.99	16
WY	3724.61	19
NM	1003.10	38
CO	362.42	50

We wanted to calculate the distribution of water resources between the two dams to the five continents, and before using the hierarchical analysis method, to simplify the model and clarify the thinking, we made the following assumptions:

① Assuming that all water resources are allocated to five continents and fully utilized, the sovereignty of surplus water distribution is not considered;

② assumes that GDP reflects most of the proportion of benefits, and the water for agriculture and residential water and electricity generation are approximately positive, and the following weights only consider the distribution of total water volume and do not subdivide the specific uses.

We construct:

Object function:

$$y = \frac{(x_1 + y_1)}{a} + \frac{\left(\frac{x_2}{b} * c + \frac{y_2}{d} * e\right)}{f} \quad (8)$$

$$Y = P_1(x_1 + y_1) + P_2(x_2 + y_2) \quad (9)$$

Constraint condition:

$$x_1 \leq P - 01 \quad (10)$$

$$x_2 \leq M - 02 + x_3/S \quad (11)$$

$$-x_1 - x_2 \leq -a \quad (12)$$

$$-\frac{x_2}{b} * c - \frac{y_2}{d} * e \leq -f \quad (13)$$

$$-g + x_1 + x_2 + x_3 = 0 \quad (14)$$

$$-l + y_1 + y_2 - x_3 = 0 \quad (15)$$

5.3 For Part III

The third question is based on how to use the model solution if there is not enough water to distribute, and the overall power generation efficiency and water use efficiency of the five continents need to be considered here. If the economic benefits of water and power generation are higher than those of agricultural housing and industry, the water for power generation should be satisfied as far as possible, and the domestic agricultural water and industrial water should be supplemented with as little as possible.

Our constraints are as follows:

Object function:

$$y = \frac{(x_1 + y_1)}{a} + \frac{\left(\frac{x_2}{b} * c + \frac{y_2}{d} * e\right)}{f} \quad (16)$$

$$Q = f - 0.34x_2 - 0.47y_2 \quad (17)$$

Constraint condition:

$$h_1 \leq P - h_{01} \quad (18)$$

$$h_2 \leq M - h_{02} + x_3/S \quad (19)$$

$$-x_1 - x_2 \leq -a \quad (20)$$

$$-\frac{x_2}{b} * c - \frac{y_2}{d} * e \leq -f \quad (21)$$

$$-h_1g + x_1 + x_2 + x_3 = 0 \quad (22)$$

$$-h_2l + y_1 + y_2 - x_3 = 0 \quad (23)$$

The objective functions y and Q get good solutions here and use fuzzy mathematical method.

A fuzzy number M is called the L R type fuzzy number is simply called the L R number if $\mu_M(x) = L(x, a)$, , $x \leq m, \alpha > 0$, $R(x - m, \beta)$ $x \geq m, \beta > 0$ where L, R is a reference function. Since their different status is called L left branch R right branch m is called main value a, b is called left and right exhibition shape, respectively. The fuzzy number brief notation of L R type is $M = (m, a, b)$ LR[5].

5.4 For Part IV

With the industry and agriculture, population, and the development of science and technology. Facing the changing demand of water and electricity, we need to adjust the target planning function to obtain the corresponding solution. Since the previous analyses allocated water resources based on conventional conditions, we developed rational distribution strategies based on the changing relationship of monthly precipitation dynamics across continents.

The reason for this conclusion is that when the precipitation is basically rich, the previous month's distribution generally can still meet the water demand of the month. We can draw from the table that the allocation strategy will be redistributed every three or four months from January to August, and the model will be rerun more frequently after September to the following January, and

once every two months or so. In the end, we concluded that the operation of Lake Powell and Lake Mead throughout the year.

6. Conclusions---Article for Drought and Thirst magazine

With certain water levels in both Lake Powell and Lake Mead, we can reasonably allocate water resources through the satisfaction of water and electricity on all continents. At the current height of the two reservoirs, when Glen Canyon Dam is reduced by 2.9800m, Hoover Dam decreases by 5.2000m, and Glen Canyon Dam supplies 4.8863×10^4 million gallons to Hoover Dam, Glen Canyon Dam needs only 2.3928×10^3 million degrees of electricity to five continents, and Hoover Dam provides 57,900 million gallons of water and 7.8845×10^4 million degrees to meet the prescribed daily demand.

Moreover, in stable weather, if we use only one pump, it will take about 32 days to meet the maximum water demand of a day. That said, managers need to deliver water to the five states a month in advance to meet a day's demand. As expected, at least the number of pumps to 32, so managers can deliver water to five continents one month in advance to meet a month's demand. And if you want to continue to shorten the time, you just need to increase the number of pumps appropriately.

Based on state monthly precipitation, evaporation, and topography, we conclude that the allocation strategy is redistributed every three or four months from January to August, and that the model reruns more frequently after September to the following January, and runs every two months or so.

As the climate changes abnormally, the Colorado River water decreases dramatically. And because the Colorado River originates in Colorado and passes through seven U. S. states, irrigating coastal crops and supplying hydroelectric power plants, raising about 36 million people and various endangered wildlife, all continents need to properly allocate water from Lake Powell and Lake Mead. Policy makers may allocate water without considering the Colorado River flow to Mexico.

7. Sensitivity Analysis of Languages

In the above model, we assume that the current water level, daily water demand, and daily electricity demand are fixed, but in practice, these parameters are not unchanged, so some defects in our results are inevitable.

Therefore, we conducted a sensitivity analysis of the linear planning model[6], considered the impact of changes in these parameters on satisfaction, and drew the relationship between these parameters and satisfaction with MATLAB software. Due to the positive impact of changes in satisfaction: reduced water level, time and additional water volume provided. Therefore, only these parameters need to be analyzed here.

It can be seen from the figure that as the current water level decreases, the satisfaction of daily demand increases is almost straight line, and when the current water level is unchanged, we get a more exact solution: $h_1 = 2.9761$, $h_2 = 5.1959$, $y = 1.1001$, And thus we obtain the exact solution of the remaining unknown number, which is not calculated here.

When we use Matlab to randomly generate a set of values of a_i, b_i , we can find the corresponding ideal solution. In this way the model can be applied not only in the conventional case but also for dynamic water resources conditions.

8. Strengths and Weaknesses

Strengths: Under the condition of the assumption, the linear planning model provides managers with more accurate hydropower distribution planning. And managers can modify the model parameters according to the actual situation to deal with the changes in daily and monthly demand.

Weaknesses: The weight value obtained by using the hierarchical analysis method is more random, and different people may make different comparison analysis of the same problem matrices, resulting in weight bias.

9. Future Work

After the analysis and solution of the above model, we want to get a more accurate strategy for managers to use, and reduce the time of manager analysis strategy, therefore, in the future, we hope to further optimize the model and data collection and processing more accurate, consider more influencing factors for managers to provide a strategy for each year, which greatly improve the efficiency.

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