

Hydropower balance under resource allocation

Chengzhi Zhao

College of Mathematics and Physics, Wenzhou University, Wenzhou, Zhejiang 325035, China

Keywords: multi-objective programming model of water resources allocation, resource benefit, sensitivity analysis, comprehensive evaluation

Abstract: With the ecosystem being gradually changed, the Colorado river is facing the trials of water shortage and disconnection, which not only has a serious influence on the plants and animals around the area, but also altered the production and life of people impressively. Therefore, it is urgent to redefine the water allocation plan. With some data on the current situation of the basin, a multi-objective planning model for water allocation was developed to achieve a rational allocation of water resources, aiming at the demand of each industry in each surrounding states. In this model, multidimensional knowledge capabilities are involved, integrating other disciplines such as academics, physics, and economics, and how frequently the model is rerun by formulating a rule to make the model results more realistic, we also test our parameters and results through robustness analysis.

1. Introduction

Since the 1990s, hydropower has been paid more and more attention by various countries. People build dams in superior geographical locations such as steep mountains and fast rivers as the basic equipment of hydropower. Because the construction of dams can bring practical economic problems, not all rivers can generate electricity by building dams. Therefore, it is necessary to make good use of the only existing hydraulic equipment to achieve the balance of hydropower generation. With the change of climate, the amount of water from rivers and lakes is changing. In most areas, this change is unfavorable [1]. The reduction of water directly leads to the shortage of water demand everywhere, and indirectly leads to the reduction of electricity generated by hydropower, resulting in the shortage of electricity demand everywhere. When the water volume of the river is lower than a certain water level, it is impossible to generate electricity by hydropower.

An agreement has been reached on the allocation of water resources of the Colorado river hundreds of years ago. Since some areas do not use all the water resources allocated, even if the water resources are decreasing, the contradictions in various areas have not intensified. However, with the continuous change of water resources and environment, if no change measures are made and the original distribution mode is maintained, the water volume will not be able to meet the living needs and power needs of people in all regions [2]. Natural resources officials in Arizona (AZ), California (CA), Wyoming (WY), New Mexico (nm) and Colorado (CO) are also negotiating how to manage the water and power production of Glen Canyon and Hoover Dam. Therefore, it is necessary to formulate a scientific water resources allocation plan to deal with the change of water resources.

This theme requires us to meet the water and electricity needs of the western United States for

five weeks over a period of time by establishing a water distribution plan. Our work mainly includes the following contents: By assuming that the flow rate of water is fixed, water resources give priority to meeting the basic water and electricity needs of residents, simplify the hydropower process, and replace the electric energy generated by hydropower with gravitational potential energy [3]. By increasing the weight, calculating the weight between different water in each state, determining the standard of competitive interests, and using the model to solve the competitive interests of general use and water resource availability of hydropower. If there is not enough water to meet the needs of water and electricity, reduce the allocation of water resources in each week in equal proportion. At the same time, the way of transferring water from the east to the west is adopted to meet the water demand of various states to a certain extent. Using the model obtained before, explore the problems in different situations, and implement water-saving and power-saving measures to better meet the water shortage of various states.

2. Multi objective optimization model of water resources allocation

For the water use planning of lake mead and lake powell, it is necessary to meet the needs of five regions for water and power resources at the same time. Similarly, in the process of pumping, it is necessary to consider that lake mead and lake powell need to be within the specified height. For such a system with complex constraints, the method of linear programming is applied to solve it. The relationship between the pumping capacity of the two lakes can be found by the method of linear programming. It is the most appropriate to solve the pumping capacity of the two lakes by satisfaction.

The water level of lake mead is M and the water level of lake powell is P . through data investigation, it is found that the daily water resource needs of residents in Arizona (AZ), California (CA), Wyoming (WY), New Mexico (nm) and Colorado (CO) are a_1, a_2, a_3, a_4 and a_5 respectively, and the daily power resources of residents are b_1, b_2, b_3, b_4 and b_5 the daily water demand of agriculture is c_1, c_2, c_3, c_4 and c_5 respectively, and that of industry is d_1, d_2, d_3, d_4 and d_5 respectively:

$$a = \sum_{i=1}^5 a_i \quad (1)$$

$$b = \sum_{j=1}^5 b_j \quad (2)$$

2.1. Linear programming model

We can see that the residential water used in the five continents is $m_1 + n_1$. Since the distribution of water resources must meet the residential water use of the five continents, so:

$$\sum_{i=1}^5 a_i \leq m_1 + n_1 \quad (3)$$

Given that the water level of lake mead is M and the water level of lake powell is P , the water volume of lake mead and lake powell can be obtained.

For the water volume of lake powell, since no other lake flows to lake powell, the total water volume of lake powell is the total water volume of its own lake:

$$W_b = \frac{1}{3}(A_1 + \sqrt{A_1 A_2} + A_2)P \quad (4)$$

For the water volume of lake mead, since the water of lake powell will flow to lake mead, the total water volume of lake mead is the sum of the total water volume of its own lake and the total water volume of lake powell flowing to lake mead:

$$W_m = \frac{1}{3}(A_3 + \sqrt{A_3 A_4} + A_4)M \quad (5)$$

According to the law of conservation of mass, the total water volume of lake powell and lake mead is equal to the total water resource allocation of the two lakes.

$$W_b = \sum_{i=1}^6 m_i \quad (6)$$

$$W_m = \sum_{i=1}^5 n_i \quad (7)$$

According to Newton's theorem, the electric energy generated by hydropower is equal to the gravitational potential energy of water.

$$W_{electric} = W_{electric1} + W_{electric2} = m_4 \rho_{water} (h_1 + P)g + n_4 \rho_{water} (h_2 + M)g \quad (8)$$

Safe water level range of lake powell $[u_1, u_2]$, safe water level range of lake mead $[u_3, u_4]$. Regard $a\%$ as the "importance" of an industry to a state

$$\min: z = \left(\sum_{i=1}^4 m_i + \sum_{j=1}^4 n_i - \sum_{i=1}^5 a_i - \sum_{i=1}^5 b_i - \sum_{i=1}^5 c_i - \sum_{i=1}^5 d_i \right)^2 \quad (9)$$

$$\max: y = \sum_{j=1}^5 \left[(b_{j1} - b_j) \frac{b_j}{a_j + b_j + c_j + d_j} + (c_{j1} - c_j) \frac{c_j}{a_j + b_j + c_j + d_j} + (d_{j1} - d_j) \frac{d_j}{a_j + b_j + c_j + d_j} \right] \quad (10)$$

In order to maximize the benefits of demand and maximize the state's sense of satisfaction, we should first reasonably meet all aspects of demand, and then maximize the sense of satisfaction through the objective function.

If $z=0$ it is solved in the model, it shows that pumping water from the lake can meet the specified demand. If $z>0$ it is, it shows that pumping water from the lake can not meet the specified demand. When pumping water can meet the specified demand, it can further reasonably allocate water resources by maximizing the sense of satisfaction

2.2. Water supply cycle

If additional water resources are not provided, for each state, under the condition of precipitation, the residents of each state will reduce their dependence on the water resources of the lake. Under the condition of losing precipitation to obtain water resources, the residents of each state will increase their dependence on the water resources of the lake, which is due to $m_{true} = m_{rain} + m_{laker}$. When m_{rain} disappear the water resources demand missing from precipitation needs to be supplemented by m_{laker} . Therefore, the water resources required by the five states are Arizona (AZ), California (CA), Wyoming (WY), New Mexico (nm) and Colorado (CO). The daily precipitation is d_1, d_2, d_3, d_4 and d_5 respectively, so the actual water resource requirements of these states are $a_1 + d_1, a_2 + d_2, a_3 + d_3, a_4 + d_4$ and $a_5 + d_5$ respectively. Since the impact of precipitation on various industries in the state is different, the impact of precipitation on various industries in the state can be determined through data survey. Agriculture in each state is obviously the industry most affected by precipitation, while industrial and residential water are relatively less affected [4]. Therefore, these missing precipitation will eventually affect agriculture, industry and residential water, by weighting the agricultural coverage area and population of each state, we can get the impact of precipitation on various industries of each state. Here, take (state) as an example to reflect the impact of precipitation on general use, and then supplement the general use demand on this basis. UI is recorded as the increased residential water consumption of each state due to lack of precipitation. Since the impact of precipitation on each industry in each state is different, the impact

of precipitation on each industry in each state can be determined through data survey. Agriculture in each state is obviously the industry most affected by precipitation, while industrial and residential water are less affected. Therefore, these missing precipitation will eventually affect agriculture, industry and residential water, by weighting the agricultural coverage and population of each state, the impact of precipitation on various industries of each state can be obtained, which can be expressed as:

$$\min: z = \left(\sum_{i=1}^4 m_i + \sum_{j=1}^4 n_j - \sum_{i=1}^5 a_i - \sum_{i=1}^5 b_i - \sum_{i=1}^5 c_i - \sum_{i=1}^5 d_i \right)^2 \quad (11)$$

$$\max: y = \sum_{j=1}^5 \left[(b_{j1} - 1.44 \times b_j) \frac{1.44 \times b_j}{a_j + b_j + c_j + d_j} + (c_{j1} - 1.723 \times c_j) \frac{1.723 \times c_j}{a_j + b_j + c_j + d_j} + (d_{j1} - 1.137 \times d_j) \frac{1.137 \times d_j}{a_j + b_j + c_j + d_j} \right] \quad (12)$$

At this time, the demand of agriculture, industry and residents in all States has increased. The influence degree of the three is [0.723,0.137,0.140] calculated by MATLAB, so as to obtain the water demand of agriculture, industry and residents in new states. Through the solution of the above model, the water resource allocation of lake mead and lake powell to various industries in various states can be calculated. The drainage rate of lake mead and lake powell can be obtained through data investigation. The actual time can be obtained by dividing the water resource demand of each state by the drainage rate of the two lakes.

2.3. Competitive benefits of water resources

Based on the above model, to solve the competitive interests of water resources availability for general (agricultural, industrial and residential) use and power production, first consider the living needs of each state and must be met first. In order to balance the domestic water needs of each state, first allocate the necessary water resources for residents to each state according to the population, and then allocate other parts. However, according to the geographical location, Wyoming (WY), New Mexico (nm) and Colorado (CO) are within the scope of lake powell, while Arizona (AZ) and California (CA) are within the scope of lake mead [5]. Due to the topographic factors of the two places, considering the direct transfer of the water of lake powell to Arizona (AZ) California (CA) and Wyoming (WY), New Mexico (nm) and Colorado (CO), which directly transfer the water of lake mead, are uneconomical. Therefore, priority is given to the water supply of lake powell to Wyoming (WY), New Mexico (nm) and Colorado (CO), and the water supply of lake mead to Arizona (AZ) California (CA). The water resource efficiency of each industry is represented by the ratio of the water resources used by each industry to the economic benefits generated. The water resource efficiency of each industry is represented by the ratio of water resources used by each industry to the economic benefits generated by the state, the water resource efficiency of the state is represented by the ratio of water resources used by the state to the economic benefits generated by the state, and the industrial and agricultural benefits of the state are represented by the ratio of industrial water resources to agricultural water resources. The results in shown in Fig.1 and Fig.2.

$$a\% = \frac{\text{Water used in agriculture in Wyoming}}{\text{Economic Benefit of Agriculture in Wyoming}} \quad (13)$$

$$b\% = \frac{\text{Water used by states in the United States}}{\text{Economic benefits generated by U.S. states}} \quad (14)$$

$$\varepsilon = \frac{\text{Economic benefit of water per unit of water in the industry}}{\text{Economic benefit of industrial agriculture and electricity production per unit of water resources}} \quad (15)$$

State	agriculture	industry	Electricity production
Arizona	8.15	0.22	1.01
California	9.01	0.11	1.35
Wyoming	7.50	0.35	2.21
New Mexico	8.61	1.42	3.57
Colorado	6.64	0.94	1.75

The lower the value, the higher the benefit

Figure 1 Economic benefit chart

State	b
Arizona	30.21%
California	9.51%
Wyoming	89.05%
New Mexico	72.38%
Colorado	29.92%

The lower the value, the higher the benefit

Figure 2 Percentage chart of economic benefit chart

Taking ε as the standard for the state to allocate water resources in this industry. The water allocated by Wyoming (WY), New Mexico (nm) and Colorado (CO) is recorded as r_1 , r_2 and r_3 . Taking Wyoming as an example, the water resources allocated by Wyoming are:

$$r_1 = m_2 \times b\% \quad (16)$$

Using MATLAB to maximize the economic benefits and realize the distribution, the premise is to meet the water and electricity needs of residents, and compare the benefits of agriculture, industry and power production, so as to maximize the sum of the economies of the five states. Then subdivided into various industries:

$$\min: z = \left(\sum_{i=1}^4 m_i + \sum_{j=1}^4 n_j - \sum_{i=1}^5 a_i - \sum_{i=1}^5 b_i - \sum_{i=1}^5 c_i - \sum_{i=1}^5 d_i \right)^2 \quad (17)$$

$$\max: y = \sum_{j=1}^5 \left[\frac{b'_j}{a_j:Electricityproduction \%} + \frac{c'_j}{a_j:agriculture \%} + \frac{d'_j}{a_j:industry \%} \right] \quad (18)$$

In order to realize the maximum competitive benefits, the model here transforms the maximization of the original demand satisfaction into the maximization of benefits. The standard to solve the competitive benefits is that which industry can create the maximum value with the least water, first allocate water to it, and then distribute it according to the different value creating abilities of each state.

2.4. Solutions to water shortage

According to the above model, over time, some factors will reduce the water resources available to states. For example, global warming will not only exacerbate glaciers and snowmelt on the top of mountains, but also change some natural phenomena: snowfall may decrease and rainfall may also decrease with global warming, which leads to the continuous reduction of water sources, as

vegetation photosynthesis accelerates, people's demand for water will increase, which will lead to insufficient water to meet all water and power needs. We also calculated and analyzed according to our model and got the following conclusions and suggestions:

When not meeting all water and power needs, according to some data and regulations of Arizona (AZ), California (CA), Wyoming (WY), New Mexico (nm) and Colorado (CO), combined with some actual conditions, in the process of pumping water from the two lakes, the domestic water of urban and rural residents in the United States should be met first, Secondly, we should also take into account the needs of agriculture, industry, ecological environment, water use and shipping. When developing and utilizing water resources in arid and semi-arid areas, full consideration should be given to the water needs of the ecological environment. At the same time, we should also consider the need for electricity, save water and electricity, enhance people's awareness of water conservation, and combine water and electricity for resource recycling. If conditions permit, we can vigorously promote conventional water-saving irrigation technologies such as low-pressure water transmission pipeline and canal lining. The development of water-saving irrigation should be combined with the development of efficient agriculture, so as to mobilize the enthusiasm of farmers to raise funds to build water-saving irrigation projects. To solve the problem of water shortage, we must solve the problem of consent and management of water resources [6]. We must have authoritative institutions to manage water according to law, so that they can organize and coordinate, mobilize the forces of all aspects, dispatch, optimize the allocation and efficient utilization of limited water resources. We should change the situation of water resources division management as soon as possible. The final allocation of water resources should be achieved by balancing the maximization of economic benefits and the level of development among states.

	AZ	CA	WY	NM	CO
02-02	0.0	0.98	0.0	0.0	0.0
02-03	0.05	1.83	0.04	0.0	0.01
02-04	0.68	9.57	7.61	0.12	1.7
02-05	2.92	11.74	11.63	4.24	5.57
02-06	3.08	11.74	16.91	4.61	6.55
02-07	3.28	11.78	20.74	6.48	7.69
02-08	3.32	11.83	22.30	7.47	8.25
02-09	3.51	11.95	23.96	7.99	8.76
02-10	3.75	12.37	25.33	9.00	9.5
02-11	4.05	12.98	26.65	9.43	10.04

Figure 3 Rainfall

3. Sensitivity Analysis

When the water demand of a state changes, whether the established model can be adjusted to meet the water distribution after the change. Therefore, the relationship between the water supply of the dam and the water demand of each state can be approximately estimated through the first-order difference as follows:

$$\frac{\partial \eta}{\partial r} \approx \frac{\eta(r+\Delta r) - \eta(r)}{\Delta r} \quad (19)$$

$$\frac{\partial \eta}{\partial m} \approx \frac{\eta(r+\Delta m) - \eta(m)}{\Delta m} \quad (20)$$

Therefore, the calculation results are shown in the Fig.4.

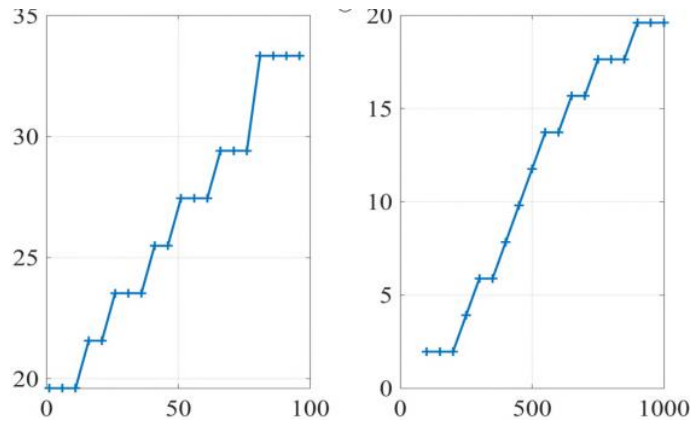


Figure 4 Sensitivity analysis

The data show that when the water demand of a state changes suddenly, the water resource allocation model can well sense the corresponding changes, and adjust the water resource allocation to meet the water demand. This shows that the model has a good sensitivity to changes, can make appropriate adjustments in time, can be better applied to complex and changeable reality, and also proves the rationality of the model.

When the demand for water in a state increases sharply, or when some natural disasters increase the water consumption in an all-round way, whether the model can continue to be used needs to be considered. Therefore, the fitting degree between the two final distributions of the model is calculated by formula:

$$RMSE_{ij} = \sqrt{\frac{1}{N} \sum_{(x,y) \in \Omega} [\phi_i(x,y) - \phi_j(x,y)]^2} \quad (21)$$

Therefore, the calculation results are shown in the Fig.5. The research shows that when the water demand of each state changes, the model can still provide an appropriate amount of water for each state to meet the demand without great deviation. It reflects that the model has strong ability to resist risks and does not need to re-establish a new model because of some subtle changes. The trend of the model obtained through the sensitivity test is also consistent with the actual situation, which proves the rationality and robustness of the water resources allocation model.

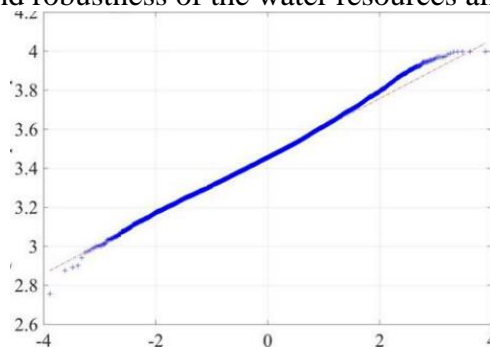


Figure 5 Robustness analysis

4. Conclusion

We have established a multi-objective programming model for water resources allocation and provided a set of relatively reasonable water supply system to solve the water resources allocation under different conditions. By controlling the water resources allocated to agriculture, industry, housing and power production, we can maximize the satisfaction of demand and economic benefits

respectively. Build indicators to determine when you need to re run the model. For Mexico's rights, when the water resources are sufficient and the five States meet all aspects of water use, the remaining water resources flow to Mexico and the Gulf of California. When the water resources are insufficient, give priority to the interests of the five states and then Mexico. Although our model has good economic benefits, it is different from the actual situation in that it can not well safeguard the interests of small states.

References

- [1] Bradley, M.W., *Guidelines for preparation of State water-use estimates for 2015: U.S. Geological Survey Open-File Report.*
- [2] California Department of water resources, *drought information news archive: California department of water resources, 2017.*
- [3] Dieter, C.A., Linsey, K.S., Caldwell, R.R., Harris, M.A., Ivahnenko, T.I., Lovelace, J.K., Maupin, M.A., and Barber, N. L., *Estimated use of water in the United States county-level data for 2015: U.S. Geological Survey data release.*
- [4] Khokha, S. *As Their Wells Run Dry, California residents blame thirsty farms: national public radio, 2014.*
- [5] U.S. Department of Commerce, U.S. Census Bureau, *population change and estimated components of population change, 2015.*
- [6] Stephens, Matt, *California: The no-good, very bad year now, pray for rain: Los Angeles Times, 2015.*