

Optimized Design of Braking System in FSAE Racers

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Abstract: On the basis of meeting the rules of the competition, in order to improve the braking performance of the FSAE racing car, we have given an optimization scheme for designing the braking system of the car. We first use MATLAB to solve the design parameters of the braking system and select the dimensions with reference to relevant standards. Then three-dimensional modeling of key components is carried out through CATIA.

1. Introduction

The formula car race of Chinese college students is called FSAE, a car design and manufacturing competition participated by groups of automotive engineering majors or students specializing in automobile-related majors of institutions of higher learning. Each team participating in the competition shall design and manufacture a small single-seat racing car with excellent performance in all aspects and meeting the competition rules and manufacturing standards within one year^[1]. As an important part of the chassis of the racing car, the braking system undertakes the responsibility of ensuring the safety of the driver and directly affects the maneuverability of the racing car. Therefore, the design of the braking system for the racing car is particularly important. In this article, we will design a braking system that meets race standards and has the best braking performance and lightweight design.

2. Calculation of Design Parameters of Braking System

As an important part of directly ensuring the driver's safety, the braking system of college equation racing generally includes the following parts: brake pedal, balance lever, brake master cylinder, brake pipeline and brake.

2.1. Preliminary Determination of Vehicle Parameters

According to the design experience of racing teams at home and abroad, the vehicle parameters can be preliminarily set, as shown in the table below:

Table 1: Vehicle Parameters.

Parameter	Symbol	Numerical value
curb weight	m_0	260KG(UVW),328KG(LVW)
Gravity	G	2548N(UVW),3214N(LVW)
wheel space	L	1660mm
high center of mass	h_g	330mm
Distance from centroid to rear axle	b	747mm
Distance from centroid to front axle	a	913mm
rear axle load	W_r	55%
front axle load	W_f	45%
lever ratio	i	5

Table 2: Racing Tire Parameters.

Parameter	Symbol	Numerical value
tire size	GB	180/50 R13
rim flange	R_n	8mm
tire width	R_m	223mm
tire contact width	R_{jk}	185mm
Tire	R_w	533mm
tire bore	R_b	244mm
rolling radius	R_e	210mm
tire girth	C	1626mm

2.2. Analysis of Axle Load Distribution of Racing Car

In the process of analyzing the braking performance of the racing car, we use *Matlab* software to visualize the parameters and analyze the relationship between them, such as ground reaction force and adhesion coefficient, I -curve and β -line. The theoretical values of the system parameters are obtained through the above analysis.

The force analysis of the car braking is shown in Figure 1. In the analysis process, we ignore the air resistance rolling resistance couple distance, the rolling resistance couple distance and the inertial force couple distance of the rotating mass, which have little influence on the braking of the racing car.

The torque balance equations of the front and rear wheel grounding points are respectively:

$$\begin{cases} F_{z1}L = Gb + m_0 \frac{du}{dt} h_g \\ F_{z2}L = Ga - m_0 \frac{du}{dt} h_g \end{cases} \quad (1)$$

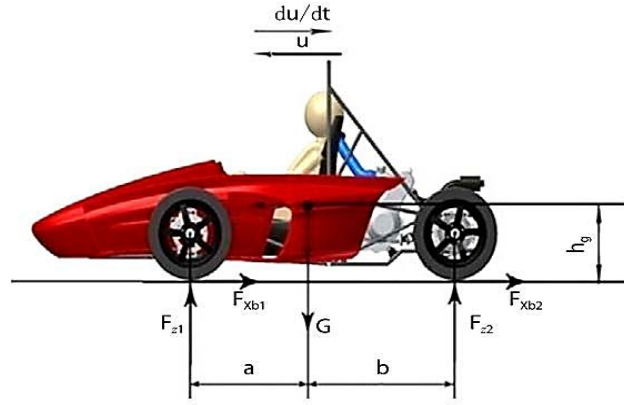


Figure 1: The Force Analysis Diagram of The Racing Car.

Of which: du/dt : Car deceleration (m/s^2);

F_{xb1} : Front wheel ground braking force(N);

F_{xb2} : Rear wheel ground braking force(N);

G : Car and driver total gravity(N);

F_{z1} : Normal reaction force of the front axle(N);

F_{z2} : Normal reaction force of the ground to the rear axle wheel(N).

By solving the system of equation (1):

$$\begin{cases} F_{z1} = \frac{G}{L} (b + zh_g) \\ F_{z2} = \frac{G}{L} (a - zh_g) \end{cases} \quad (2)$$

Of which z : braking efficiency factor, $zg = \frac{du}{dt}$;

If the car is braked on different roads and the front and rear wheels are not locked, the normal reaction force of the horizontal ground to the wheels is:

$$\begin{cases} F_{z1} = \frac{G}{L} (b + \varphi h_g) \\ F_{z2} = \frac{G}{L} (a - \varphi h_g) \end{cases} \quad (3)$$

According to the relationship between the normal reaction forces F_{z1} and F_{z2} of the front and rear wheels on the level surface and the braking forces $F_{\mu1}$ and $F_{\mu2}$ of the brakes, equations (4) can be obtained:

$$\begin{cases} F_{\mu1} + F_{\mu2} = \varphi G \\ F_{\mu1} = \varphi F_{z1} \\ F_{\mu2} = \varphi F_{z2} \end{cases} \quad (4)$$

Combined equations (3) and equations (4), we use *Matlab* to draw the relationship between the normal reaction force between the front and rear wheels of the car and the ground contact as follows:

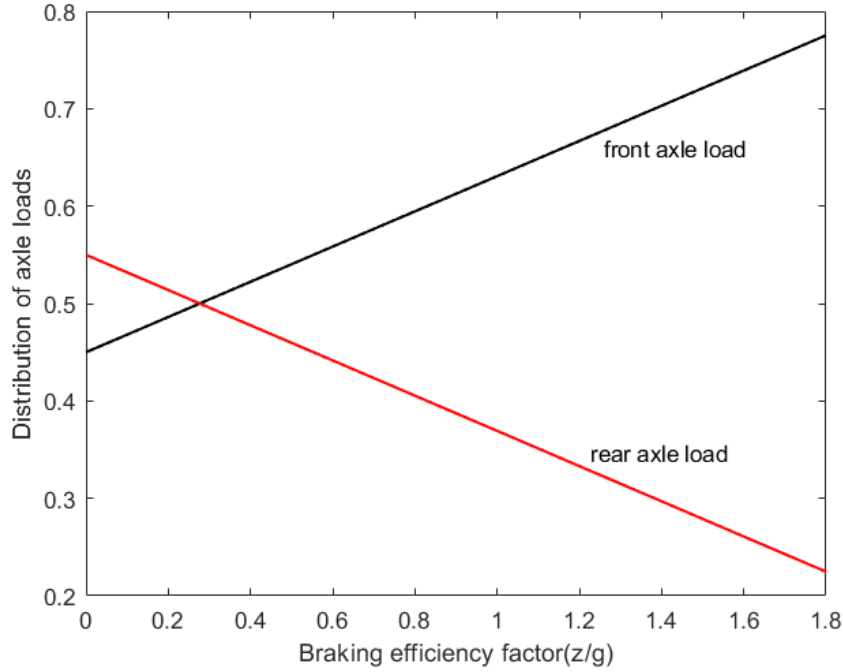


Figure 2: Load Variation Diagram of Front And Rear Axles.

It can be seen from the above figure that the front and rear axle loads change with the braking deceleration during braking. As the braking strength of the car increases, the inertia of the car itself leads to an increase in the load on the front axle and a decrease in the load on the rear axle. This shows that when the braking deceleration of the car is too large, it may lead to excessive load on the front axle, resulting in dangerous situations such as disintegration or out of control of direction. Therefore, in order to avoid such dangerous situations, it is particularly important to set a critical deceleration for the car.

2.3. Distribution of Brake Force

Substitute equation (3) into equation (4) and eliminate φ to get:

$$F_{\mu 2} = 0.5 \left[\frac{G}{h_g} \sqrt{b^2 + 4h_g L \frac{F_{\mu 1}}{G}} - \left(\frac{Gb}{h_g} + 2F_{\mu 1} \right) \right] \quad (5)$$

The ideal braking force distribution curve of front and rear brakes can be drawn from equation (5).

According to the analysis of figure 3, in most cases, the braking force of the rear wheel brake required for car braking is less than that of the front wheel, and the difference between them gradually increases with the increase of the adhesion coefficient.

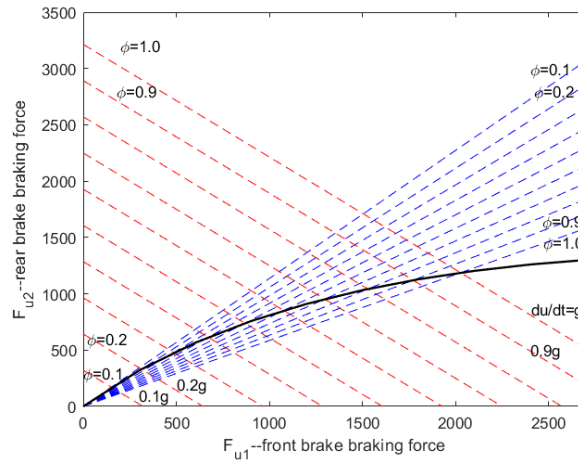


Figure 3: Ideal Braking Force Distribution Diagram of Front and Rear Brakes.

2.4. Determination of Braking Force Distribution Ratio and Synchronous Adhesion Coefficient of Braking System

For many Chinese formula students, the ratio of the braking force of the front and rear brakes is a fixed value. It is usually expressed by the ratio of the braking force of the front wheel brake to the total braking force of the racing car, that is, the braking force distribution coefficient of the brake, and is expressed by the symbol β , namely:

$$\beta = \frac{F_{\mu 1}}{F_{\mu}} \quad (6)$$

And because:

$$\begin{cases} \frac{F_{\mu 1}}{F_{\mu 2}} = \frac{\beta}{1-\beta} \\ \tan \theta = \frac{1-\beta}{\beta} \end{cases} \quad (7)$$

According to equation (5), equation (7) and use *Matlab* to draw the intersection of β -line and I -curve, as shown in the following figure:

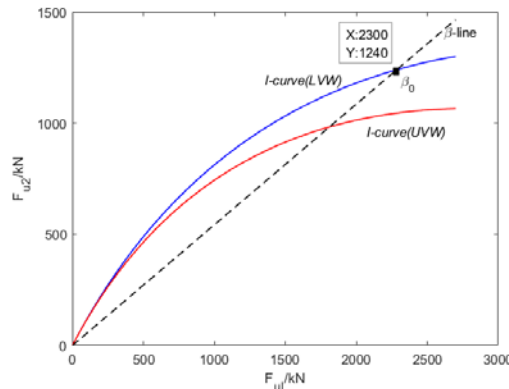


Figure 4: Relationship Between β -line And I -curve.

It can be seen from the above figure that the β -line and the I -curve at full load intersect at a point β_0 , and the coordinates are (2300,1240). For the adhesion coefficient corresponding to the intersection of the β -line and the I -curve, we usually call it the synchronous adhesion coefficient φ_0 , and the corresponding braking acceleration and deceleration is called the critical deceleration a_b [2][3].

And because:

$$\varphi_0 = \frac{L\beta - b}{h_g} \quad (8)$$

$$\varphi_0 \approx 1.105$$

According to the above calculation results, the theoretical critical deceleration a_b of the racing car under this model is:

$$a_b = zg = \varphi_0 g \quad (9)$$

$$a_b \approx 10.83m/s^2$$

From equation (3) and equation (4):

$$\frac{\beta}{1-\beta} = \frac{b + \varphi_0 h_g}{a - \varphi_0 h_g} \quad (10)$$

After finishing, the braking force distribution ratio β is:

$$\beta = 0.65$$

The parameter values obtained by summarizing the above calculation results are as follows:

Table 3: Design Parameters of Braking System.

Parameter	Symbol	Numerical Value
synchronous adhesion coefficient	φ_0	1.105
braking force distribution ratio	β	0.65
lever ratio	i	5
theoretical critical deceleration	a_b	10.83m/s ²

2.5. Force Analysis of Complete Vehicle

Figure 1 is the stress analysis diagram of the car during braking. Through the above calculation, we have calculated the design parameters of the braking system. By substituting the parameters into

equation (3), the maximum normal reaction force of the ground facing the tire when the car reaches the maximum deceleration can be calculated as:

$$\begin{cases} F_{z1} = \frac{3214}{1660} \times (747 + 1.105 \times 300) = 2088.1N \\ F_{z2} = \frac{3214}{1660} \times (913 - 1.105 \times 300) = 1125.9N \end{cases}$$

When braking a racing car, if the front and rear wheels lock up at the same time, the required braking torque of the front and rear axles is ^[4]:

$$\begin{cases} M_{\mu1} = \frac{G}{L} (a + \varphi_0 h_g) \Psi R_e \varepsilon \\ M_{\mu2} = \frac{G}{L} (b - \varphi_0 h_g) \Psi R_e \varepsilon \end{cases} \quad (11)$$

Of which: $M_{\mu1}$ is Braking torque of front wheel; $M_{\mu2}$ is Braking torque of rear wheel; φ_0 is synchronous adhesion coefficient; R_e is rolling radius; Ψ is Ground adhesion coefficient, The value here is 0.7; ε is correction factor, The value here is 1.

$$\begin{cases} M_{\mu1} = 306.96N \cdot m \\ M_{\mu2} = 165.49N \cdot m \end{cases}$$

It can be calculated from equation (2), equation (4), and the above data.

$$\begin{cases} F_{\mu1} = \varphi_0 F_{z1} = 1.105 \times 2088.1 = 2307.4N \\ F_{\mu2} = \varphi_0 F_{z2} = 1.105 \times 1125.9 = 1243.7N \end{cases}$$

When the racing car brakes, to satisfy the rule that the four wheels are locked at the same time, that is, to satisfy the ground-braking force F_{xb} equal to the brake braking force F_{μ} , the following formula can be used to calculate.

$$\begin{cases} F_{xb1} = F_{\mu1} = \beta Gz \\ F_{xb2} = F_{\mu2} = (1 - \beta) Gz \end{cases} \quad (12)$$

Substitute into $\beta = 0.65$, $z = 1.105$ to solve:

$$\begin{cases} F_{xb1} = 2308.7N \\ F_{xb2} = 1243.1N \end{cases}$$

3. Design of Key Components of Braking System

3.1. Determination of Brake Wheel Cylinder Diameter

According to the literature, when the condition of simultaneous locking of four wheels is satisfied, the clamping force F_n of the brake caliper has the following relationship with the ground braking force:

$$F_{xb} = 2\mu F_n \quad (13)$$

Among them, μ is the friction coefficient between the brake pad and the brake disc ^[5], and the empirical value is 0.45 here.

At the same time, without considering the influence of brake fluid, assuming the same force on the left and right wheels, the relationship between the diameter d of the brake caliper wheel cylinder and the hydraulic pressure P in the brake caliper is as follows:

$$F_n = 2 \times \left(\frac{\pi}{4} \times d^2 \right) \times P \quad (14)$$

Among them, P is the pressure value of a single pipeline, and the empirical value here is 3Mpa .

Combining equations (12), (13) and (14), the theoretical minimum caliper wheel cylinder diameters d_1 and d_2 that satisfy the maximum braking force of the front and rear wheels can be solved.

$$\begin{cases} d_1 = 23.33\text{mm} \\ d_2 = 17.12\text{mm} \end{cases}$$

Finally, according to the industry standard, we selected the diameter of the wheel cylinder piston as 31.75mm.

Because the relationship between the working volume of the brake caliper and its piston diameter is as follows ^[6]:

$$V_w = \frac{\pi}{4} \times \sum_1^n d_w^2 \times \delta \quad (15)$$

Among them, V_w is working volume of the brake caliper; d_w is Piston diameter; n is the number of pistons; δ is the wheel cylinder stroke of a wheel cylinder when braking. Since the gap between the friction pads on both sides and the brake disc is removed by 1mm according to the experience value of predecessors, the value of δ is 0.5mm. At this time, the car is free when the pedal is pressed. Travel is minimal and braking efficiency is highest. At the same time, m is the total number of brake calipers, and the value of m is 4.

According to the above conclusions and combined with the selection of 2 pistons for each brake caliper, one on each side, the total working volume of the brake wheel cylinder V_m can be calculated:

$$V_m = \frac{\pi}{4} \times (31.75^2 \times 2 \times 0.5) \times 4 = 3166.9mm^3$$

3.2. Determination of Brake Master Cylinder Diameter

The relationship between the volume of brake master cylinder required by the brake and the volume of brake wheel cylinder is as follows:

$$V_{zg} = V_m + V' \quad (16)$$

Among them, V_m is the total volume of all-wheel cylinders; V' is the volume change of the brake hose [7]. According to the law that the consumption of brake fluid in the pipeline is relatively small, according to the experience value of predecessors and existing regulations, the safety factor is selected as 1.1. Then the working volume V of the racing brake master cylinder can be expressed as:

$$V = 1.1V_m \quad (17)$$

And because there is a relationship between the size of the front and rear brake master cylinder piston diameter D and the piston stroke S as follows:

$$V = \sum_1^k \left(\frac{\pi}{4} \times D^2 \times S \right) \quad (18)$$

Among them, k is the number of brake master cylinders, where k is 2.

In general, the relationship between the piston diameter D of the master cylinder and the piston stroke is as follows:

$$S = (0.8 \sim 1.2) \times D \quad (19)$$

Here we choose a coefficient of 1.0.

Combined with formula (16), formula (17), formula (18) and formula (19), the diameter of the master cylinder piston required for front and rear wheel braking is

$$D = 13.04mm$$

According to the industry standard, we select the piston diameter of brake master cylinder as 15.96mm.

3.3. Determination of Pedal Assembly Parameters

3.3.1. Calculation of Pedal Effort

Since the brake pedal is affected by the two master cylinders, the expression for the brake pedal effort is:

$$F_p = 2 \times \frac{\pi}{4} \times \frac{D^2 P_1}{i_p i_s \eta_p} \quad (20)$$

Among them, D is diameter of master cylinder piston; P_1 is the oil pressure of the brake line, the value here is $6Mpa$; i_p is the pedal ratio, that is, lever ratio $i_p = i$; i_s is the booster ratio. Since the car has no booster, the i_s value is 1; η_p is the mechanical efficiency of the brake pedal and master cylinder, the value is 0.95. From this, we can solve the required maximum brake pedal effort.

$$F_p = 505.2N$$

According to the requirements of the competition, under the load of $700N$ and above, the brake pedal can still work normally to be considered qualified. Therefore, in general, in order to meet the requirements of the event and the safety of the driver, it is more reasonable for the maximum brake pedal force in the design process to be less than $700N$. In summary, the design meets the requirements of the event and use.

3.3.2. Calculation of Working Stroke of Brake Pedal

Calculation of working stroke of brake pedal:

$$X_p = i_p \times (S_m + X_{\delta 1} + X_{\delta 2}) \quad (21)$$

Among them, X_p is pedal stroke (mm); i_p is pedal ratio, the value is 5; S_m is Working stroke of brake master cylinder (mm), the value is 16; $X_{\delta 1}$ is the main cylinder push rod and piston clearance (mm), because the vertical master cylinder is selected, and the master cylinder push rod and the piston are processed into one body, so the value is 0; $X_{\delta 2}$ empty stroke of main cylinder (mm), the value is 1.

Substitute the parameters into the above formula

$$X_p = 85mm$$

According to the requirements of the competition, the turning angle of the brake pedal should not exceed 30° . Therefore, when the pedal turning angle reaches 30° , the working stroke of the selected pedal component should be greater than 85mm. Only in this way can the performance of the master cylinder be maximized. Therefore the required pedal arm length L_t is:

$$L_t = \frac{X_p}{2 \sin \theta_1} \quad (22)$$

Among them, the value of θ_1 is 15° , and it is calculated by substituting it into the above formula.

$$L_t \approx 165mm$$

This length can also be understood as the junction of the brake master cylinder and the pedal arm,

that is, when the position of the balance rod is at or above this position, the full performance of the brake master cylinder can be exerted, so we initially set the position of the balance rod. The vertical distance from the brake base plate is 170mm. So based on experience, we initially selected the brake pedal arm length L_t as 240mm.

3.3.3. Calculation of Included Angle between Master Cylinder and Base Plate

The following figure shows the force analysis diagram of the brake pedal and the master cylinder:

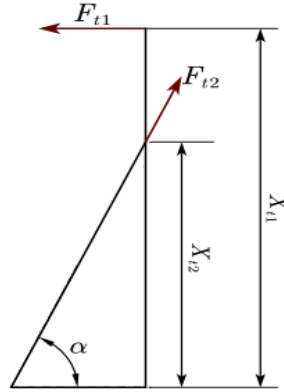


Figure 5: Mechanical Force Diagram of Braking System.

When the mechanical part of the braking system maintains balance, according to the relevant knowledge of theoretical mechanics:

$$i = \frac{F_{t2}}{F_{t1}} = \frac{X_{t1}}{X_{t2} \cos \alpha_1} \quad (23)$$

Among them, i is lever ratio, the value is 5; X_{t1} is the distance from the rotating shaft of the brake pedal to the position where the force is applied to the driver's foot; X_{t2} is the vertical distance from the rotating shaft of the brake pedal to the axis of the stabilizer bar; α_1 is the angle between the axis of the main cylinder block and the plane of the bottom plate; F_{t1} is the force exerted on the driver's feet; F_{t2} is the reaction force of main cylinder to pedal.

Set the value of X_{t1} according to the knowledge of ergonomics, referring to the “GB10000-1988 Human dimensions of Chinese adult”, we choose the value of X_{t1} to be 233mm; using the braking force and the maximum brake pedal force calculated above, the value of F_{t2} can be calculated; at the same time, the maximum force F_{t2} that the driver's feet can generally provide is about 600N; in addition, limited by the size of the brake pedal, and affected by the L_t value, the value of X_{t2} is 170mm.

Through the above analysis, we can get

$$\alpha_1 = \cos^{-1} \left(\frac{X_{t1}}{i \times X_{t2}} \right) = 74.8^\circ$$

4. Finite Element Optimization Analysis of Braking System

4.1. Pedal Stress Analysis and Optimization

According to the requirements of the competition organizing committee, the brake pedal must be able to withstand a force of 2000N and keep working normally. In response to this requirement, we use 7075 aluminum as the raw material to design and process the brake pedal. Meanwhile, in order to reduce the overall mass of the brake pedal as much as possible, we adopt the principle of lightweight design. quality of the pedals. For this reason, we use ANSYS to carry out static structural analysis and carry out multiple optimizations designs according to the stress and strain diagrams of the pedals, and obtain the final design plan and give the stress diagrams and strain diagrams as follows:

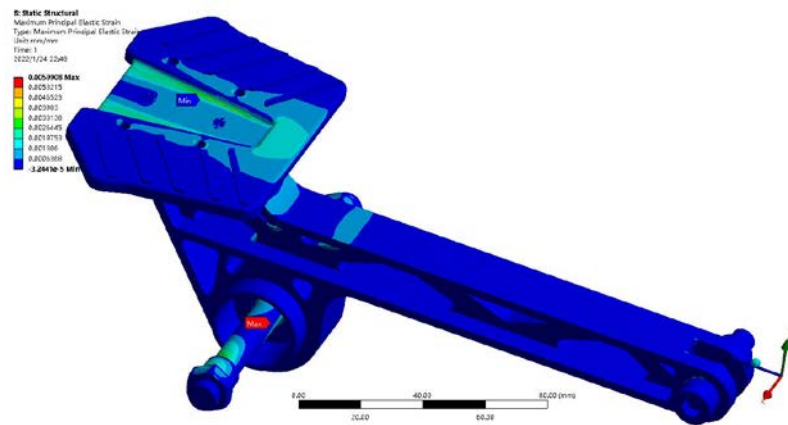


Figure 6: Static Analysis - Pedal Stress Diagram.

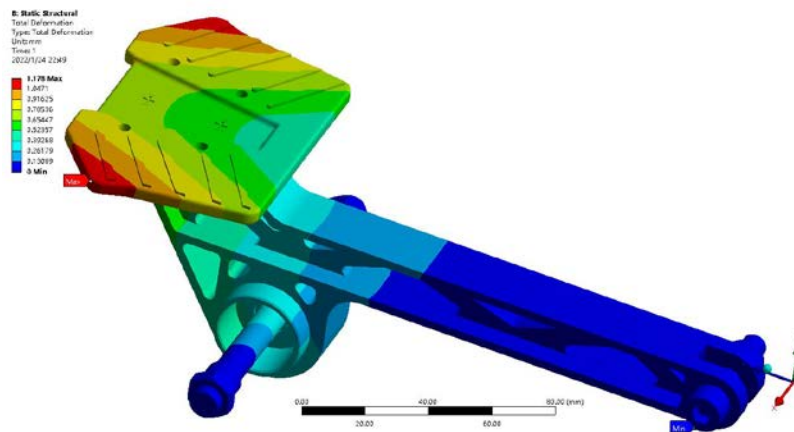


Figure 7: Static Analysis - Pedal Strain Diagram.

By analyzing Figure 7, we can know that when the pedal made of 7075 aluminum is subjected to a load of 2000N for 5 seconds, the maximum deformation is 0.117mm, which meets the international requirements. By consulting the data, we can see that the yield strength limit of 7075 aluminum is 505Mpa . According to experience, we take the safety factor as 1.8, and the allowable stress $[\delta_s]$ is:

$$[\delta_s] = \frac{\delta_s}{n_1} = \frac{505}{1.8} = 280.56Mpa$$

We can know from Figure 6 that the maximum stress on the pedal is about 264.45 Mpa , which is less than the allowable stress of the material and meets the safety requirements.

4.2 Arrangement of Pedal Assembly

Through the above analysis and calculation, we finally give the design scheme of the pedal assembly. The car adopts a vertical structure with the lever ratio of 5:1. The left side is the brake pedal assembly and the right side is the accelerator pedal assembly.

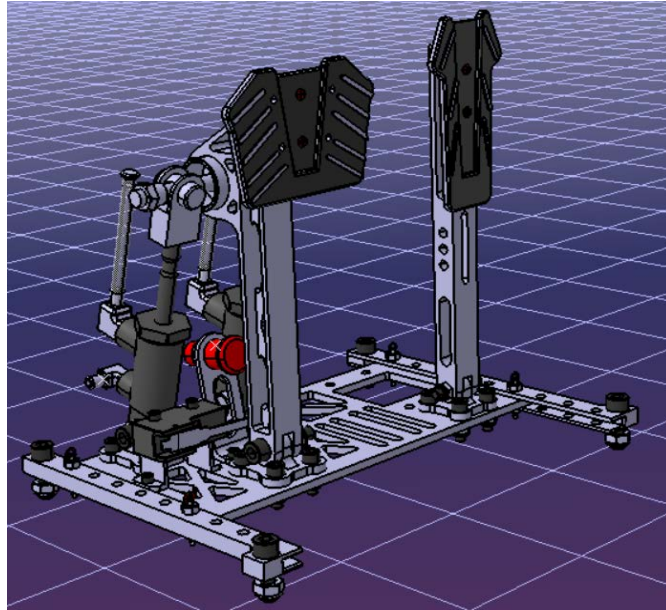


Figure 8: Layout of Pedal Assembly.

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